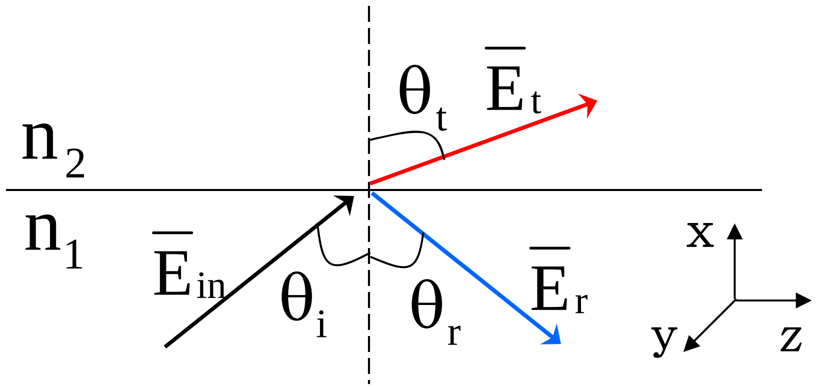
Lect. 6: Oblique Incidence on Dielectric Interface

(Assume E_{in} has only y-component: Perpendicular Polarization)



$$\overline{E}_{in} = \overline{y}E_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{E}_r = \overline{y}E_r e^{jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_t = \overline{y}E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$k_x = n_1 k_0 \cos\theta_i,\ k_z = n_1 k_0 \sin\theta_i$$

$$k_{rx} = n_1 k_0 \cos\theta_r, k_{rz} = n_1 k_0 \sin\theta_r$$

$$k_{tx} = n_2 k_0 \cos\theta_t, k_{tz} = n_2 k_0 \sin\theta_t$$

Unknowns?

$$\theta_r, \theta_t, E_r, E_t$$

How to solve for Unknows?

 W.-Y. Choi

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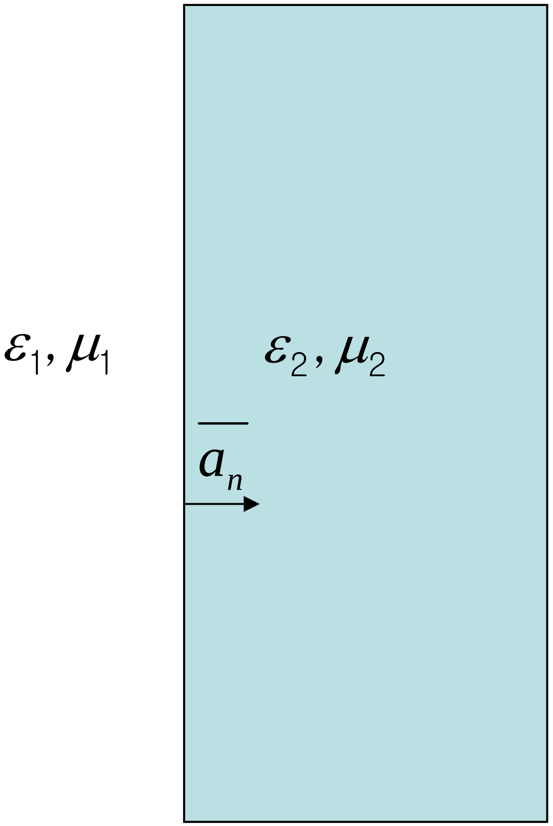
Boundary Conditions: Constraints on E,H fields at a boundary.
Each Maxwell's Eq. provides one constraint on E or H.



$$\nabla \bullet \overline{D} = \rho,$$

$$D_{2,n} - D_{1,n} = \rho_s \Leftrightarrow \boxed{\varepsilon_2 E_{2,n} - \varepsilon_1 E_{1,n} = \rho_s}$$

$$\nabla \times \overline{E} = -\frac{d\overline{B}}{dt},$$

$$\boxed{E_{2,t} - E_{1,t} = 0}$$

$$\nabla \bullet \overline{B} = 0,$$

$$B_{2,n} - B_{1,n} = 0 \Leftrightarrow \boxed{\mu_2 H_{2,n} - \mu_1 H_{1,n} = 0}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{d\overline{D}}{dt},$$

$$\boxed{\overline{a}_n \times (\overline{H}_2 - \overline{H}_1) = \overline{J}_s} \Leftrightarrow H_{2,t} - H_{1,t} = J_s$$

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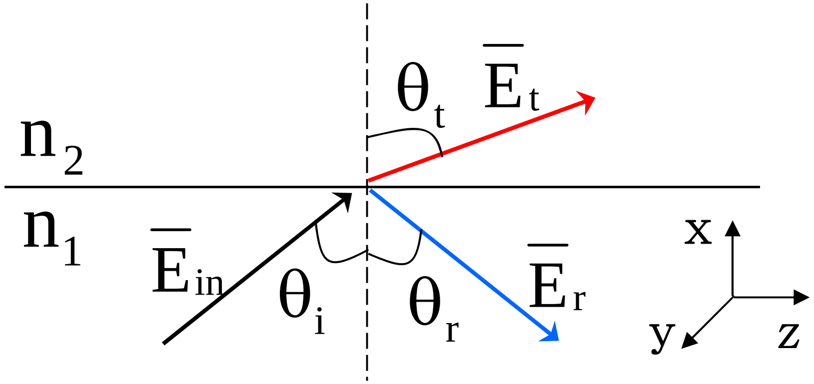

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$$\overline{E}_{t} = \overline{y} E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$k_z = n_1 k_0 \sin\theta_i$$

$$k_{rz} = n_1 k_0 \sin\theta_r$$

$$k_{tz} = n_2 k_0 \sin\theta_t$$

Applying BC on E at $x = 0$,

$$E_i e^{-jk_z \cdot z} + E_r e^{-jk_{rz} \cdot z} = E_t e^{-jk_{tz} \cdot z}$$

=> $k_z = k_{rz} = k_{tz}$ and $E_i + E_r = E_t$

$\therefore\ n_1 \sin\theta_i = n_1 \sin\theta_r = n_2 \sin\theta_t$ ➔ Continuity of k_{tan}

=> $\theta_i = \theta_r$ and $n_1 \sin\theta_i = n_2 \sin\theta_t$ (Snell's Law)

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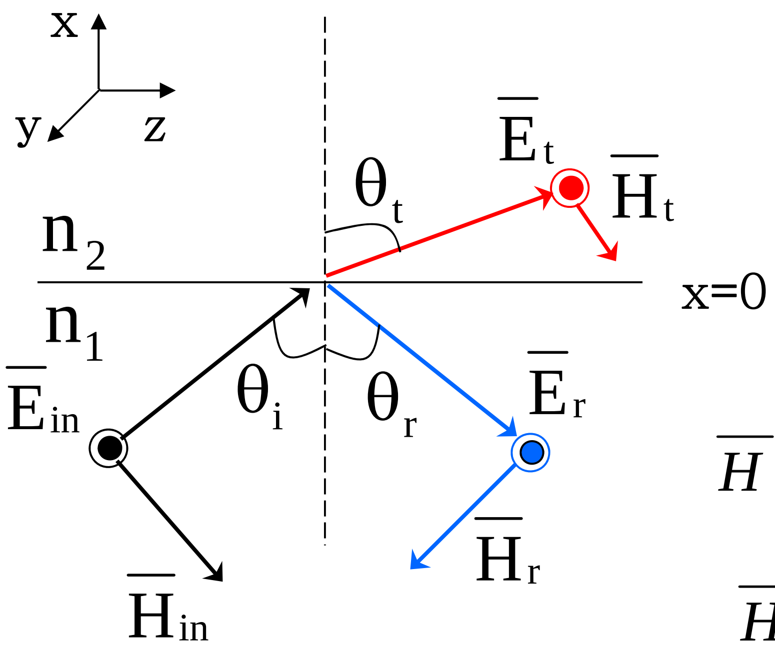

$$\overline{E}_{in} = \overline{y}E_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{E}_{r} = \overline{y}E_r e^{jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_{t} = \overline{y}E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$\overline{H}_{in} = \left(-\overline{x}\sin\theta_i + \overline{z}\cos\theta_i\right) E_i / \eta_1 \, e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{H}_{r} = \left(-\overline{x}\sin\theta_i - \overline{z}\cos\theta_i\right) E_r / \eta_1 \, e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{H}_{t} = \left(-\overline{x}\sin\theta_t + \overline{z}\cos\theta_t\right) E_t / \eta_2 \, e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

Applying BC on H at x=0

$$\left(\cos\theta_i\right) E_i / \eta_1 + \left(-\cos\theta_i\right) E_r / \eta_1 = \left(\cos\theta_t\right) E_t / \eta_2$$

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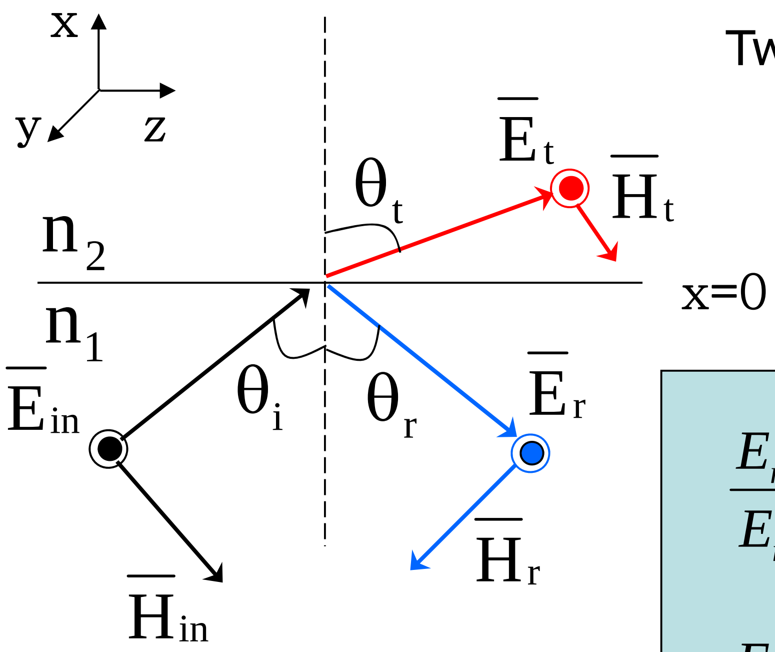


Two simultaneous equations:

$$E_i + E_r = E_t$$

$$(\cos\theta_i)\frac{E_i}{\eta_1} + (-\cos\theta_i)\frac{E_r}{\eta_1} = (\cos\theta_t)\frac{E_t}{\eta_2}$$

$$\frac{E_r}{E_i} = \Gamma_\perp = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$\frac{E_t}{E_i} = \tau_\perp = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}} \qquad \left(n = \frac{n_2}{n_1}\right)$$

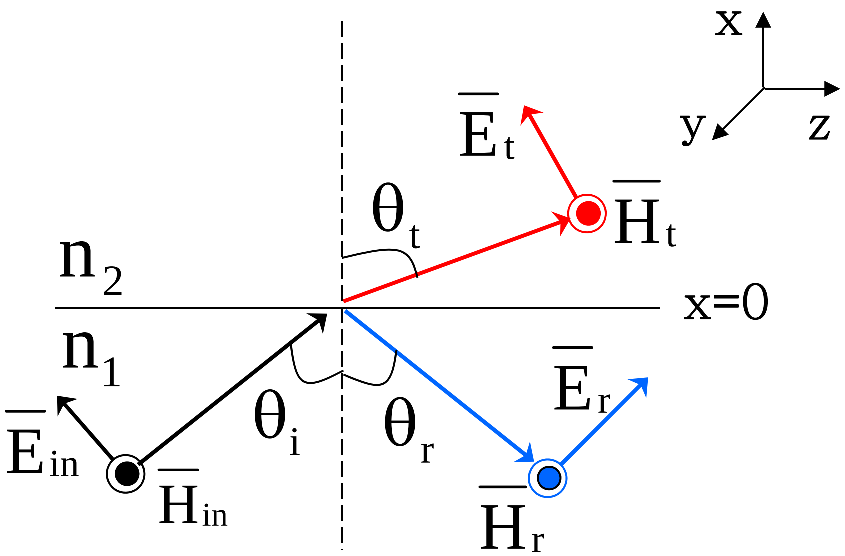
$\theta_r, \theta_t, E_r, E_t$ are all solved!

Lect. 6: Oblique Incidence on Dielectric Interface

Homework:

Consider EM waves incident on the dielectric interface at x=0 shown below.

When incident, reflected, transmitted H-fields are (parallel polarization):



$$\overline{H}_i = \overline{y} H_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{H}_r = \overline{y} H_r e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{H}_t = \overline{y} H_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

where $k_z = n_1 k_0 \sin\theta_i$

$k_{rz} = n_1 k_0 \sin\theta_r$

$k_{tz} = n_2 k_0 \sin\theta_t$

determine two independent equations involving H_i, H_r, H_t.

(See 8-10.3 or 2016-2 전자기2 Lect. 13)