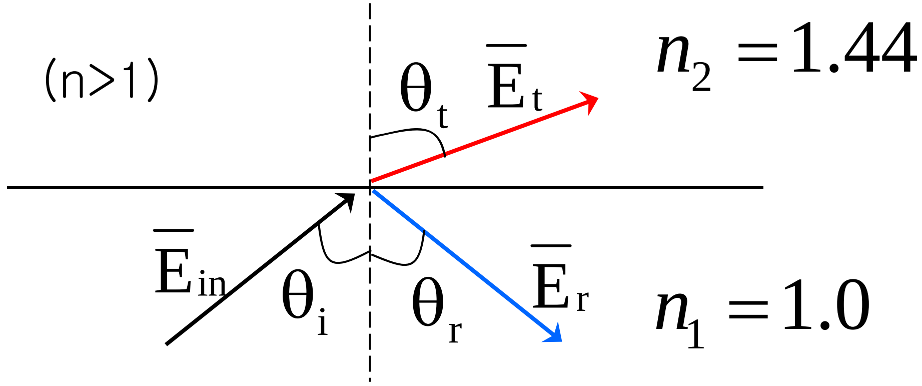
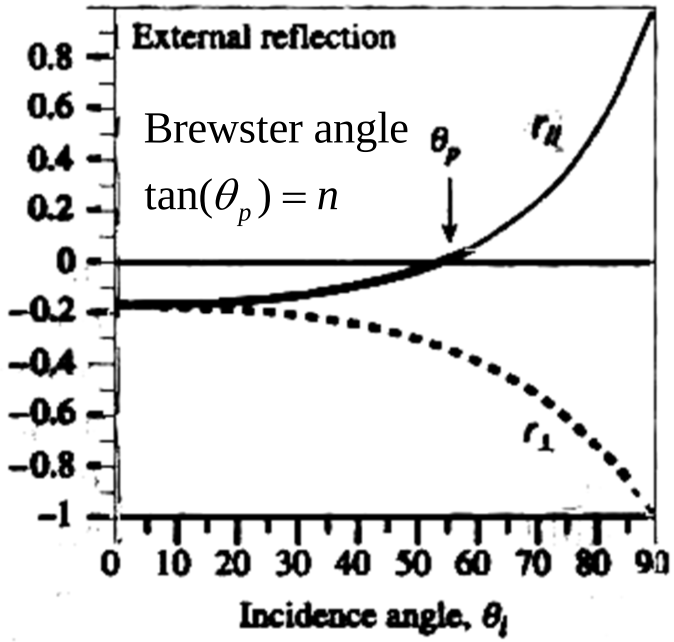
Lect. 7: Total Internal Reflection





Γ ==> r, τ ==> t

$$r_{\perp} = \frac{E_r}{E_i} = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$r_{\parallel} = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

$$t_{\perp} = \frac{E_t}{E_i} = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}} \quad \left(n = \frac{n_2}{n_1}\right)$$

$$t_{\parallel} = \frac{2n\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

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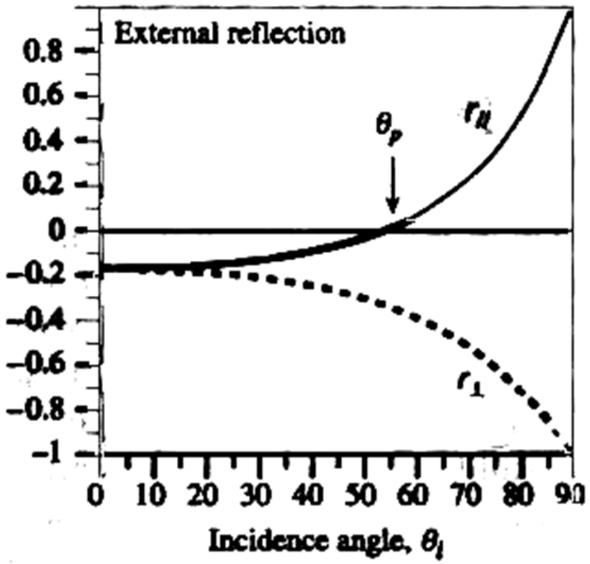
Brewster angle

$\tan(\theta_p) = n$



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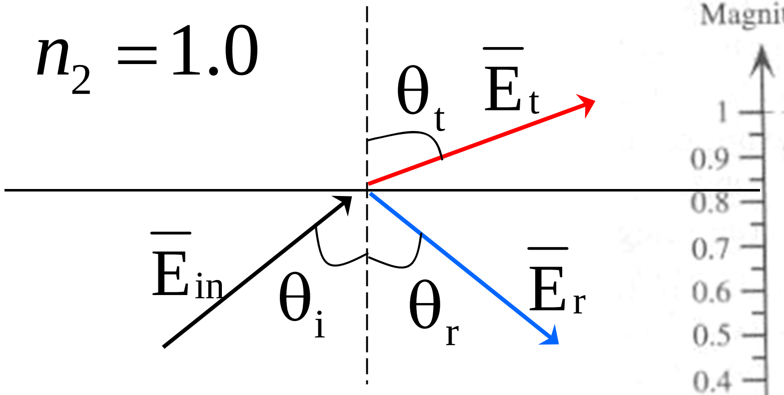
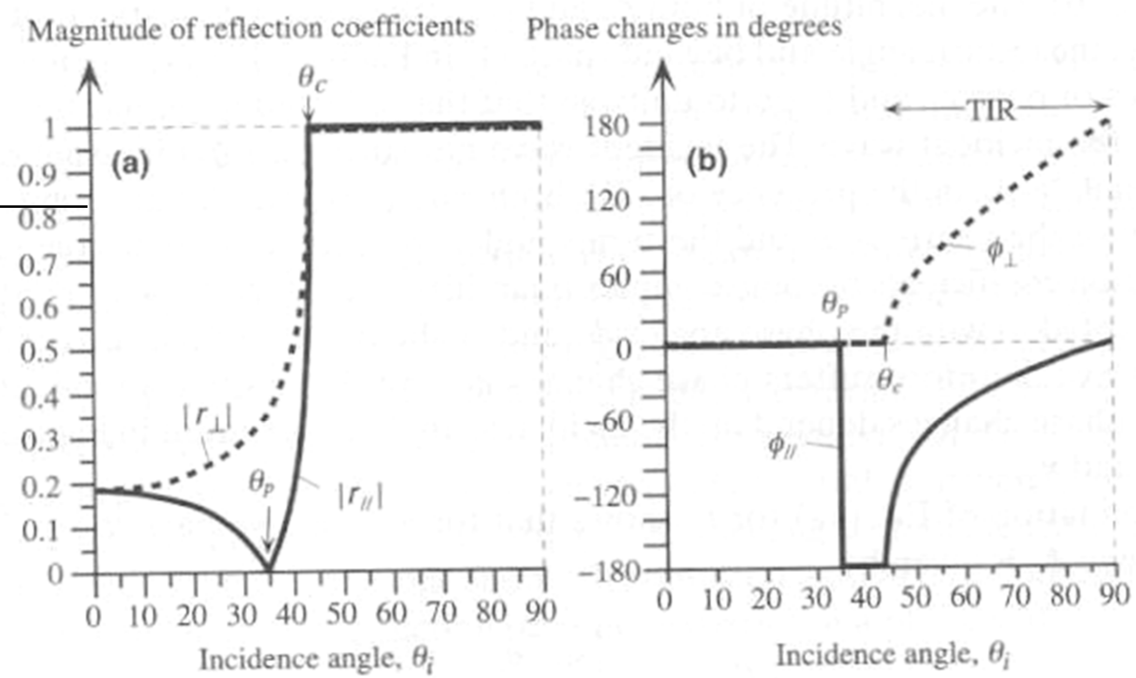


$n_1 = 1.44$ (n < 1)



What is θ_c (critical angle)?

What happens when $\theta_i > \theta_c$?

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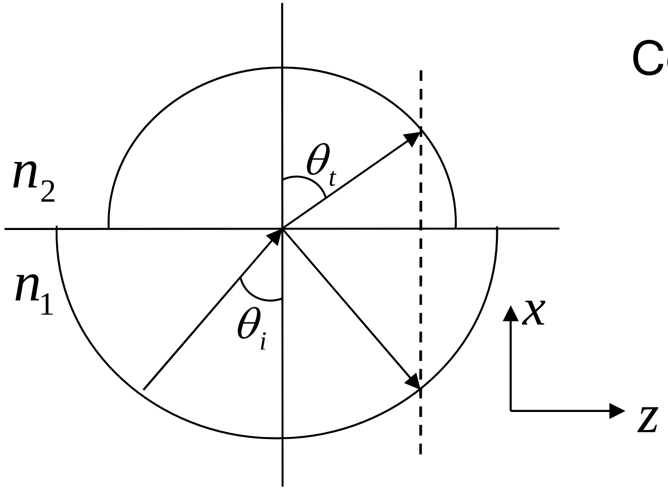

Consider k-vector diagram:

- Direction: Direction of wave propagation
- Magnitude: $\omega\sqrt{\mu\varepsilon}$
- As θ_i changes, k_i, k_r, k_t traces on a circle

From Lect. 4, $k_{i,z} = k_{r,z} = k_{t,z}$

$$n_1 k_0 \sin\theta_i = n_2 k_0 \sin\theta_t \quad \rightarrow \quad n_1 \sin\theta_i = n_2 \sin\theta_t$$

As θ_i increases, θ_t increases

θ_t reaches 90 deg before θ_i

$$n_1 \sin\theta_i = n_2 \qquad \theta_c \text{ (critical angle)} = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}(n)$$

What if θ_i > θ_t ?

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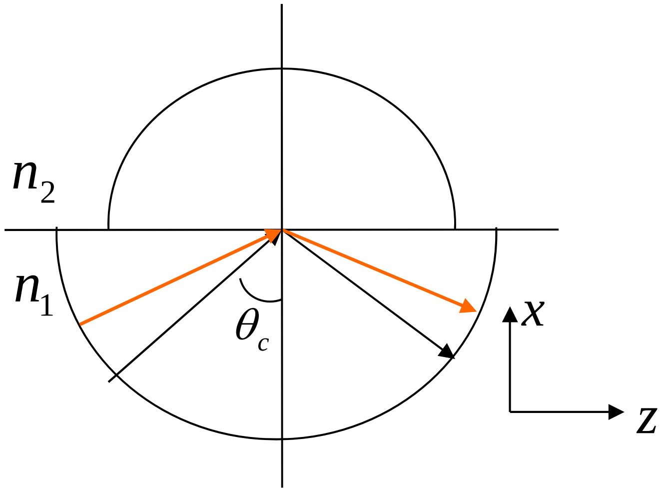


What happens if $\theta_i > \theta_c$?

$$k_t^{\ 2} = k_{t,z}^2 + k_{t,x}^2$$

$$k_{t,x}^2 = k_t^{\ 2} - k_{t,z}^2 \ = (n_2 k_0)^2 - (n_1 k_0 \sin\theta_i)^2 \ < 0$$

$$(\because k_{t,z} = k_{i,z} = n_1 k_0 \sin\theta_i)$$

$$\therefore k_{t,x} = -j\alpha$$

Attenuation in x-direction!

$$\therefore E_t = tE_i e^{-jk_{tx}\cdot x} e^{-jk_{tz}\cdot z} = tE_i e^{(-\alpha x)} e^{(-jk_{t,z} z)}$$

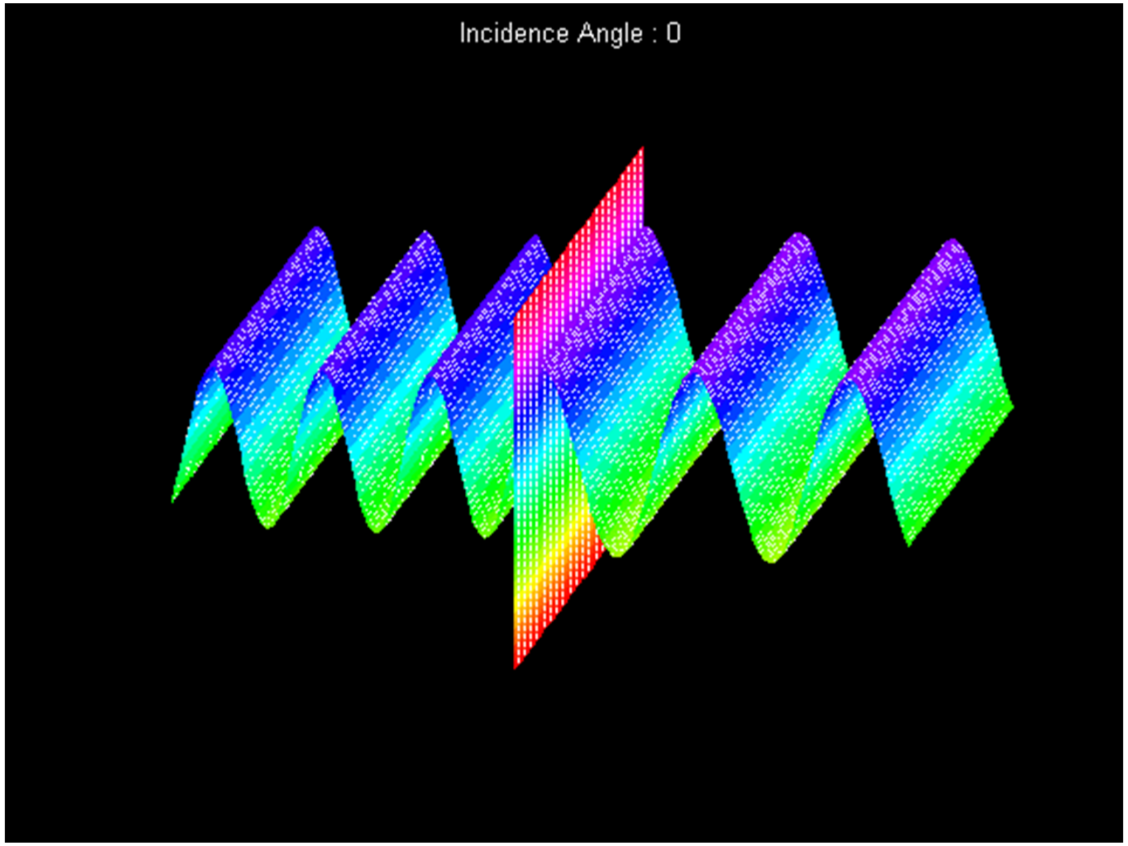
➔ Total internal reflection

$\alpha = ?$ From $k_t^{\ 2} = k_{t,z}^{\ 2} + k_{t,x}^{\ 2}$, $(n_2 k_0)^2 = (n_1 k_0 \sin\theta_i)^2 - \alpha^2$

$$\alpha^2 = (n_1 k_0 \sin\theta_i)^2 - (n_2 k_0)^2 \quad \therefore \alpha = k_0[(n_1 \sin\theta_i)^2 - n_2^{\ 2}]^{\frac{1}{2}}$$

$\varepsilon_1 = 2\varepsilon_2, \ \mu_1 = \mu_2$ and θ_i: from 0° to 90°

Incident and Transmitted Waves for perpendicular polarization



Lect. 7: Total Internal Reflection

What is $r_\perp$, $t_\perp$? for total internal reflection

From $r_\perp = \dfrac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$

$\sin\theta_i > n$ ($\because$ $\sin\theta_i > \sin\theta_c = n$) Let $\left[n^2 - \sin^2\theta_i\right]^{1/2} = -j\left[\sin^2\theta_i - n^2\right]^{1/2}$

$$r_\perp = \frac{\cos\theta_i + j\left[\sin^2\theta_i - n^2\right]^{1/2}}{\cos\theta_i - j\left[\sin^2\theta_i - n^2\right]^{1/2}} = \left|r_\perp\right| e^{j\phi_\perp}$$

$\left|r_\perp\right| = 1$ $\phi_\perp = \tan^{-1}\left(\dfrac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i}\right) - \left(-\tan^{-1}\left(\dfrac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i}\right)\right) = 2\tan^{-1}\left(\dfrac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i}\right)$

For $t_\perp$, use $E_i + E_r = E_t$ or $1 + r_\perp = t_\perp$

=> $t_\perp = 1 + r_\perp$

Lect. 7: Total Internal Reflection

What is $r_{\parallel}$, $t_{\parallel}$? for total internal reflection

From $r_{\parallel} = \dfrac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$

$\sin\theta_i > n \ (\because \ \sin\theta_i > \sin\theta_c = n)$ Let $\left[n^2 - \sin^2\theta_i\right]^{1/2} = -j\left[\sin^2\theta_i - n^2\right]^{1/2}$

$$r_{\parallel} = \frac{-j\left[\sin^2\theta_i - n^2\right]^{1/2} - n^2\cos\theta_i}{-j\left[\sin^2\theta_i - n^2\right]^{1/2} + n^2\cos\theta_i} = \left|r_{\parallel}\right| e^{j\phi_{\parallel}}$$

$$\left|r_{\parallel}\right| = 1 \text{ and } \phi_{\parallel} = -\pi + \tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{n^2\cos\theta_i}\right) - \left(-\tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{n^2\cos\theta_i}\right)\right) = -\pi + 2\tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{n^2\cos\theta_i}\right)$$

From $H_i + H_r = H_t \Rightarrow 1 + \dfrac{H_r}{H_i} = \dfrac{H_t}{H_i}$ Since $r_{\parallel} = -\dfrac{H_r}{H_i}$ and $t_{\parallel} = \dfrac{\eta_1 H_t}{\eta_2 H_i}$

$1 - r_{\parallel} = \dfrac{\eta_2}{\eta_1} t_{\parallel} = n t_{\parallel}$ $\therefore 1 - r_{\parallel} = n t_{\parallel}$

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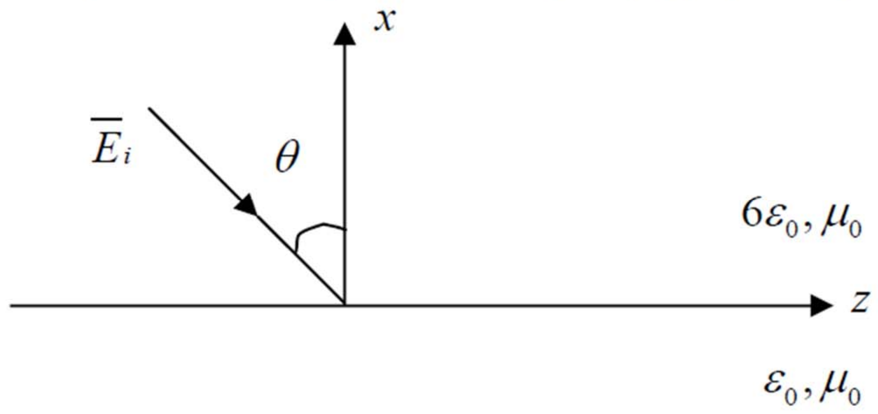
Homework:

A plane wave is incident at an angle Θ=30 degrees on a dielectric boundary as shown below. Its electric field is given as

$$\bar{E}_i = E_0 (\bar{x}\sin\Theta + \bar{y} + \bar{z}\cos\Theta)\exp(jk_x x - jk_z z).$$



(a) What is the polarization of the incident wave?

(b) It is observed that the polarization of the reflected wave polarization is circular. Explain how this can happen.