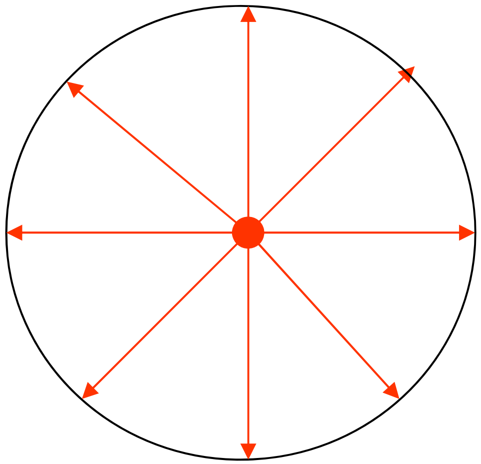
Lect. 9: Interference

Consider isotropic EM wave radiation by a point source.

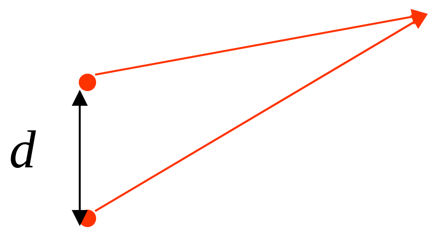


$$E \sim \frac{1}{R} e^{-jkR} \text{ (Spherical wave)}$$

Why $\frac{1}{R}$ dependence?

because $\int |E|^2 R^2 \sin\theta d\theta d\phi$ should be constant.

Two point sources separated by d



$$E_{\text{total}} = E_1 + E_2$$

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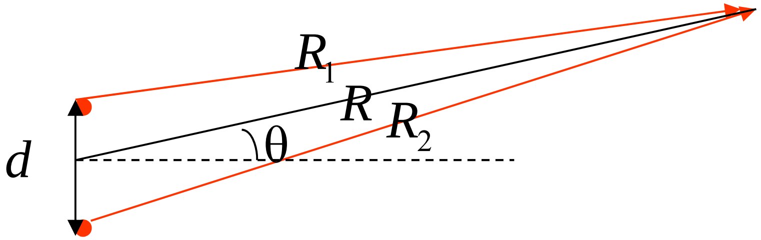

$$E(R,\theta,\phi) = E_1 + E_2 = \frac{A}{R_1}e^{-jkR_1} + \frac{A}{R_2}e^{-jkR_2}$$

Assume $R >> d$ $\quad R_1 \approx R - \frac{d}{2}\sin\theta \qquad R_2 \approx R + \frac{d}{2}\sin\theta$

$$\therefore E(R,\theta) \approx \frac{A}{R}e^{-jk(R-\frac{d}{2}\sin\theta)} + \frac{A}{R}e^{-jk(R+\frac{d}{2}\sin\theta)}$$

$$= \frac{A}{R}e^{-jkR}\left(e^{jk\frac{d}{2}\sin\theta} + e^{-jk\frac{d}{2}\sin\theta}\right) = \frac{2A}{R}e^{-jkR}\cos\left(k\frac{d}{2}\sin\theta\right)$$

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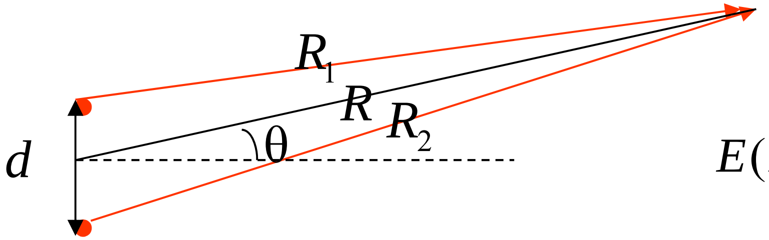

For max., $k\frac{d}{2}\sin\theta = m$

$$E(R,\theta) = \frac{2A}{R} e^{-jkR} \cos(k\frac{d}{2}\sin\theta)$$

$I(\text{Intensity}) : |E|^2 = 4(\frac{A}{R})^2 \cos^2(k\frac{d}{2}\sin\theta)$ => There exist max. and min. intensity conditions

For max., $k\frac{d}{2}\sin\theta = m\pi$ $\quad kd\sin\theta = 2m\pi$

phase difference: $2m\pi$

length difference: $m\lambda$

For min., $k\frac{d}{2}\sin\theta = (m+\frac{1}{2})\pi$

phase difference = $(2m+1)\pi$

length difference: $(m+\frac{1}{2})\lambda$

➔ Constructive interference ➔ Destructive interference

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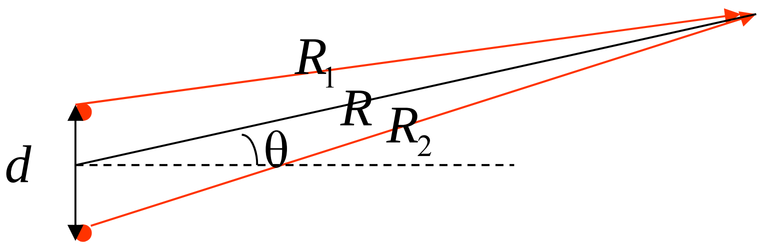
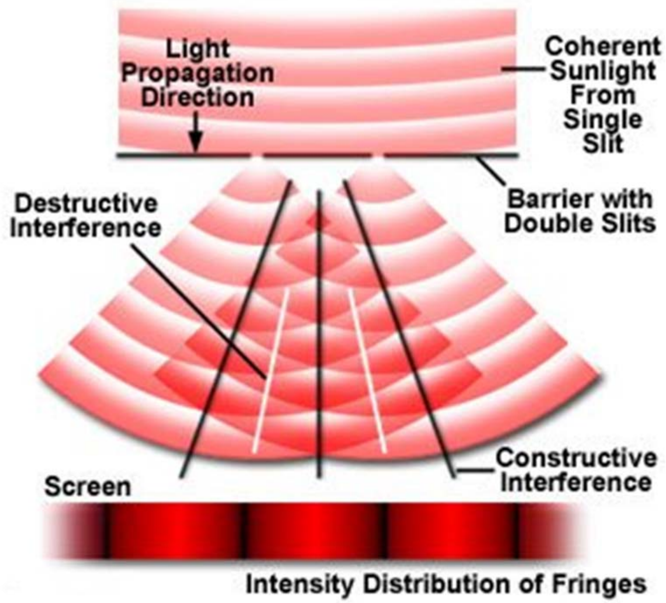
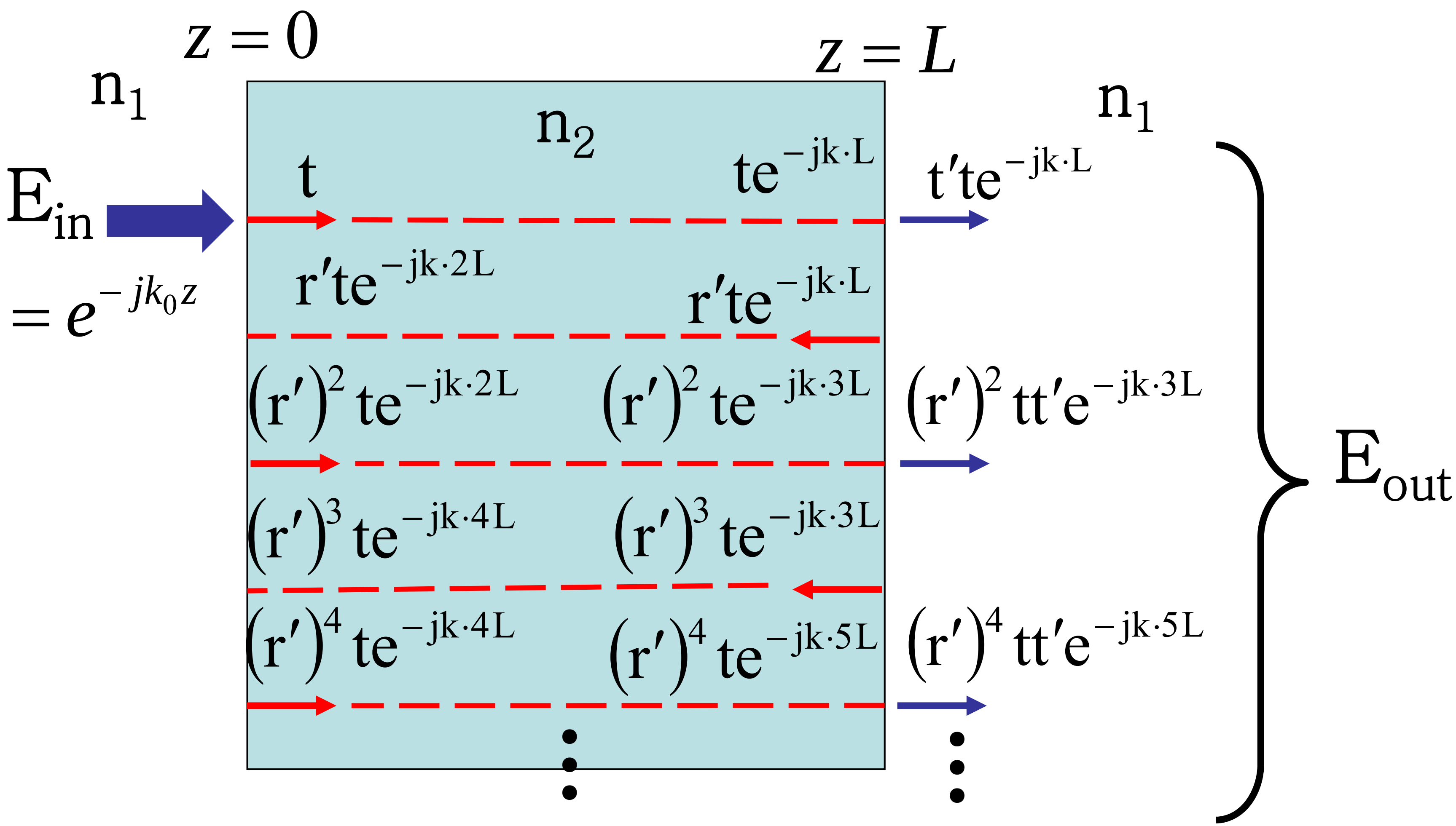


$$I\,(\text{Intensity}) = \left|E\right|^2 = 4(\frac{A}{R})^2 \cos^2(k\frac{d}{2}\sin\theta)$$

Double Slit Interference



Interference in a dielectric slab



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$$E_{out} = E_{t,total} = tt'e^{-jk\cdot L} + (r')^2 tt'e^{-jk\cdot 3L} + (r')^4 tt'e^{-jk\cdot 5L} + \bullet\bullet\bullet = \frac{tt'e^{-jk\cdot L}}{1-(r')^2 e^{-j2kL}}$$

$$T = \frac{|E_t|^2}{|E_i|^2} = \frac{(tt')^2}{[1-(r')^2 e^{-j2kL}][1-(r')^2 e^{j2kL}]} = \frac{(tt')^2}{[1-(r')^2]^2 + 4r'^2 \sin^2(kL)}$$

$$[1-(r')^2 e^{-j2kL}][1-(r')^2 e^{j2kL}] = 1-(r')^2 e^{j2kL} - (r')^2 e^{-j2kL} + (r')^4$$

$$= 1-2(r')^2\cos(2kL) + (r')^4$$

$$= 1-2(r')^2(1-2\sin^2 kL) + (r')^4$$

$$= [1-(r')^2]^2 + 4(r')^2\sin^2(kL)$$

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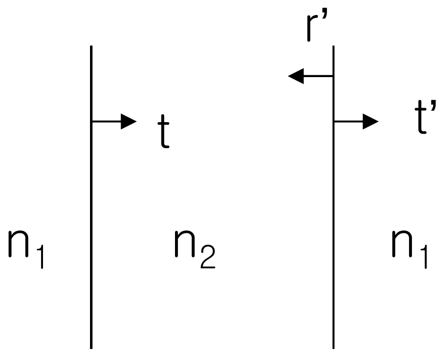
$$T = \frac{|E_t|^2}{|E_i|^2} = \frac{(tt')^2}{[1-(r')^2]^2 + 4r'^2 \sin^2(kL)}$$



$$t = \frac{2n_1}{n_1+n_2}, \quad r' = \frac{n_2-n_1}{n_2+n_1}, \quad t' = \frac{2n_2}{n_1+n_2}$$

$$\therefore tt' = \frac{4n_1 n_2}{(n_1+n_2)^2}, \qquad 1 - r'^2 = \frac{4n_1 n_2}{(n_2+n_1)^2}$$

$$\text{Let } R = r'^2$$

$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)}$$

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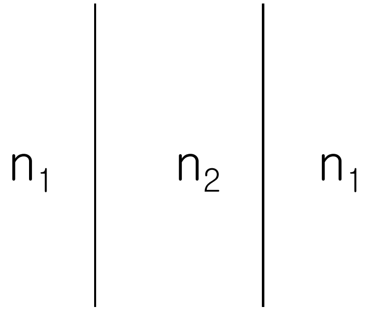
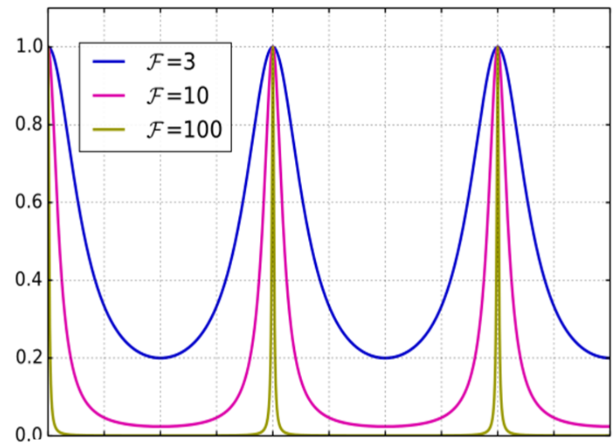


$$T = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)}$$

Fabry-Perot Interference



Max. Transmission: $\sin(kL) = 0 \;\; => T = 1$

$$kL = m\pi; \;\; n_2 \frac{2\pi}{\lambda} L = m\pi \;\; => L = m\frac{\lambda}{2n_2} \quad \text{(half wavelength)}$$

Min. Transmission: $\sin(kL) = 1$

$$kL = (m + \frac{1}{2})\pi; \;\; n_2 \frac{2\pi}{\lambda} L = (m + \frac{1}{2})\pi \;\; => L = \frac{\lambda}{2n_2}(m + \frac{1}{2}) \quad \text{(quarter wavelength)}$$

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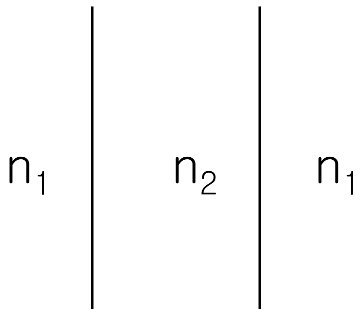
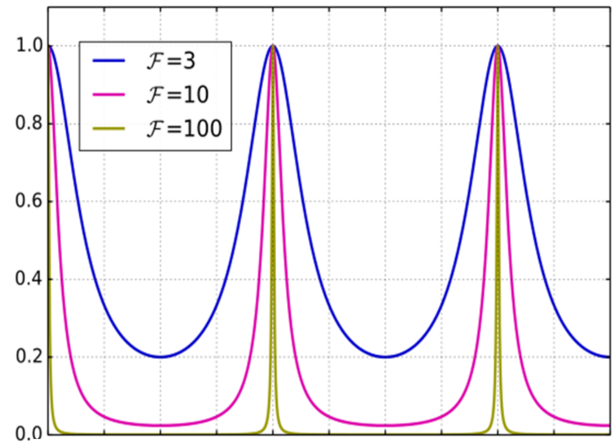


$$T = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)}$$



Period? ➔ Free Spectral Range

$$\Delta kL = \pi \qquad \Delta k = \frac{\pi}{L}$$

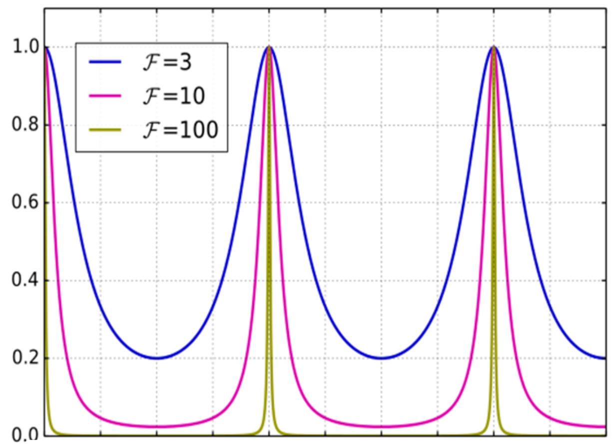
$$\Delta\omega = ? \qquad k = n_2\frac{\omega}{c} \qquad \Delta\omega = \frac{c}{n_2}\Delta k = \frac{c}{n_2}\frac{\pi}{L}$$

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{c}{2n_2L} = \frac{1}{T} \qquad T = \frac{2L}{c/n_2}$$

(round-trip time)

$$\Delta\lambda = ? \qquad \lambda = n_2\frac{2\pi}{k} \qquad \Delta\lambda = \frac{d\lambda}{dk}\Delta k = -n_2\frac{2\pi}{k^2}\Delta k = -\frac{\lambda^2}{2n_2L}$$

$$T = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)}$$



Sharpness (Linewidth)?

Determine k where $T = 0.5$

$$\frac{1}{2} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(2m\pi + \Delta kL)} \qquad \frac{4R}{(1-R)^2}\sin^2(\Delta kL) = 1$$

$$\Delta kL = \sin^{-1}\sqrt{\frac{(1-R)^2}{4R}} = \sin^{-1}\frac{(1-R)}{2\sqrt{R}} \sim \frac{(1-R)}{\sqrt{R}} \quad \text{(If FWHM<<1)}$$

FWHM (Full Width at Half Maximum): $2\Delta kL = \frac{(1-R)}{\sqrt{R}}$

As R increases, FWHM decreases => sharper response

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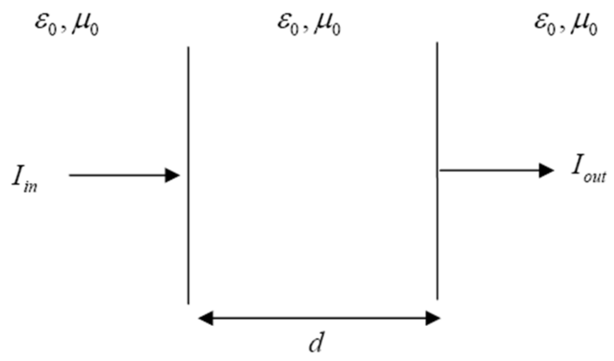
Homework:

Consider a Fabry-Perot interferometer made of two identical partially reflecting/transmitting mirrors as shown below. The mirrors transmit and reflect half of the incident power. (Or it has r = 1/sqrt(2) and t = j/sqrt(2)).



(a) What is I_{out}/I_{in}? Give your answer as a function of sin(kd), where k is the wavenumber in the vacuum and d is the distance between two mirrors.

(b) If the output power is plotted as a function of the frequency of the input light, what is the frequency separation between two adjacent peaks? Express your answer interms of c, speed of light, d, and other fundamental parameters if required.

(c) What is the finesse of this interferometer? Give a numerical answer.

[Finesse = (Free Spectral Range)/FWHM]