High-Speed Serial Interface Circuits and Systems

Lect. 4 – Phase-Locked Loop (PLL)

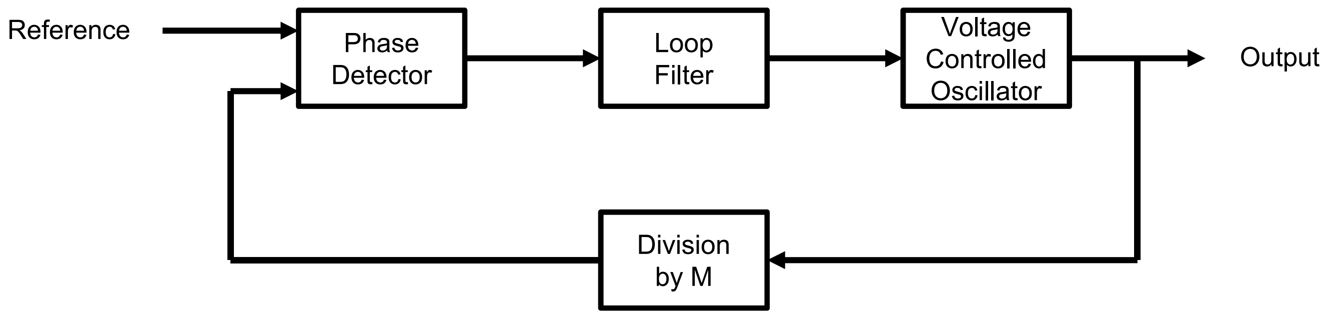
Type 1

(Chap. 8 in Razavi)

PLL

- Phase locked loop

  A (*negative-feedback*) control system that generates an output signal whose phase (*and frequency*) is related to the phase (*and frequency*) of an input reference signal – *wikipedia* –



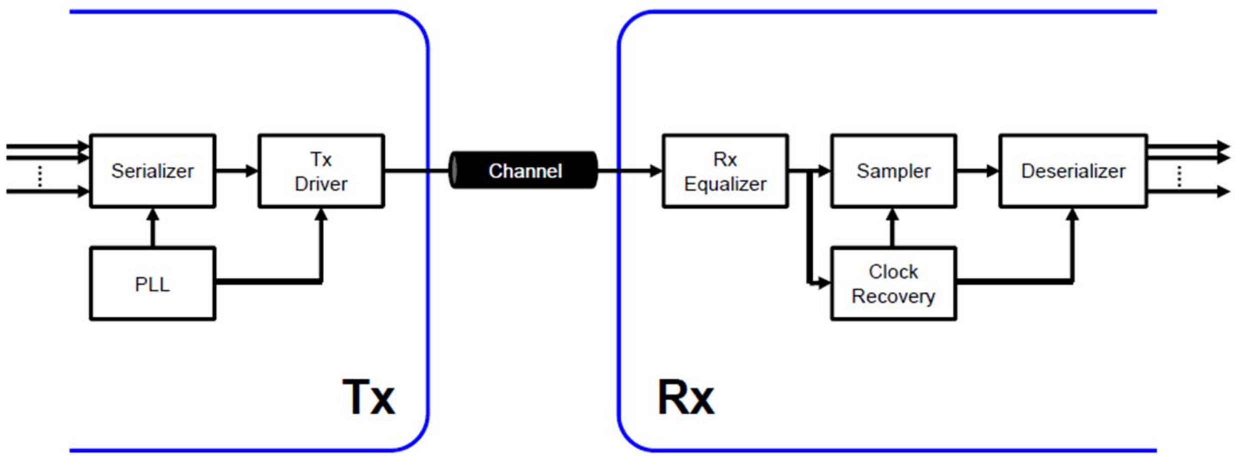
- Output signal is phase-locked to reference signal:

  ➔ Constant phase relationship

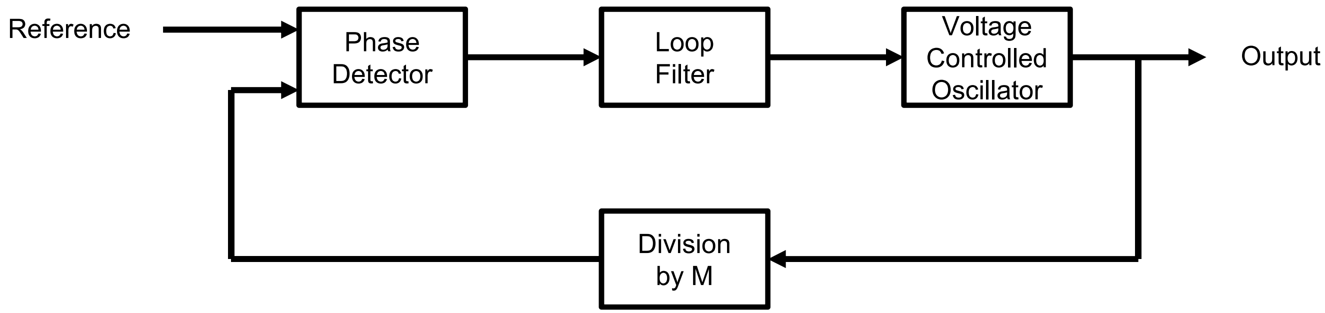
  ➔ $f_{out} = M \times f_{Reference}$

PLL

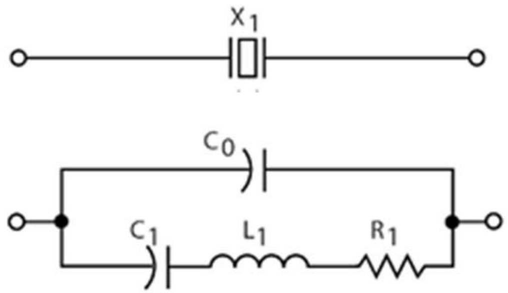
- Applications
  - Frequency Synthesis
    - Clocks for digital systems
    - LO in RF systems
  - Clock recovery
  - Modulation/Demodulaton



PLL Block Diagram



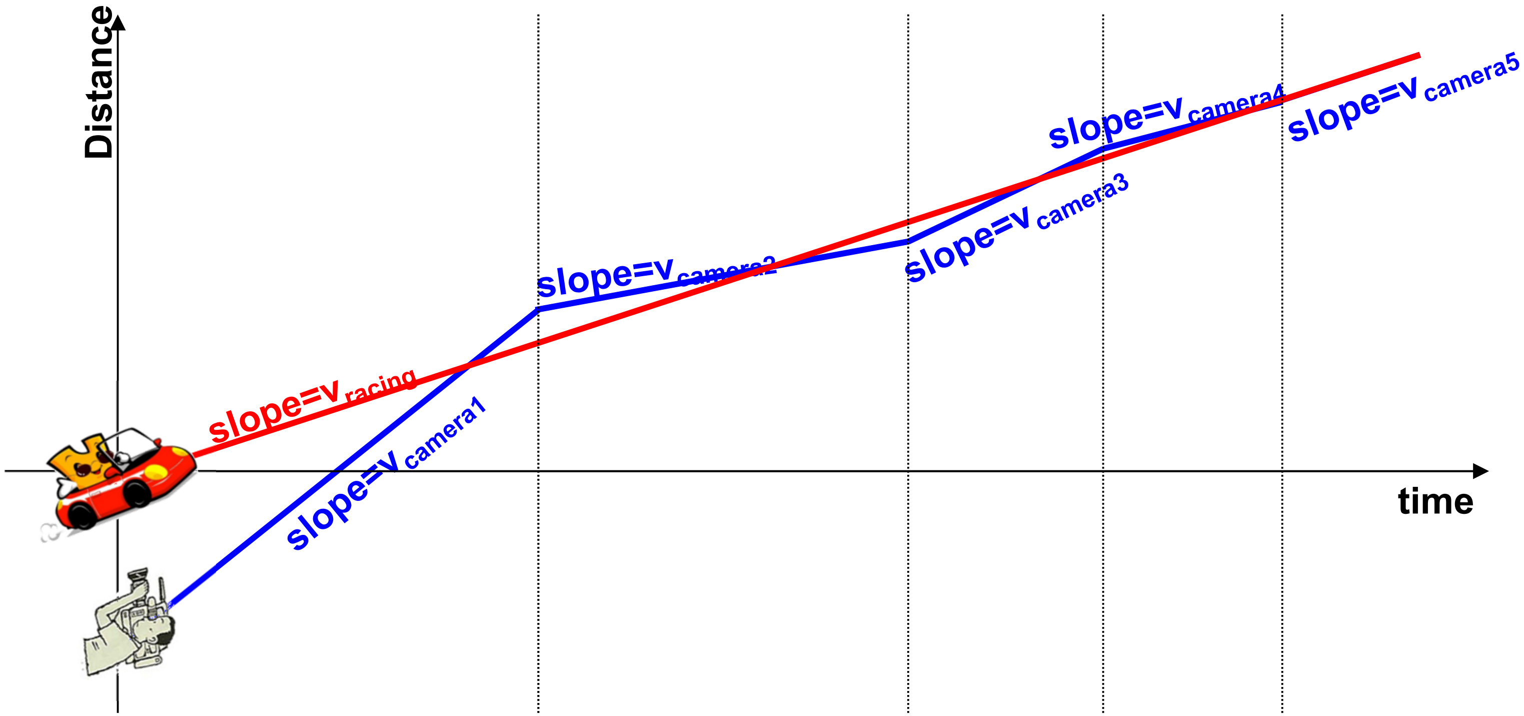
- Reference?



Equivalently, an LC oscillator

Oscillation frequency: kHz ~ a few hundred MHz

Phase-tracking by PLL

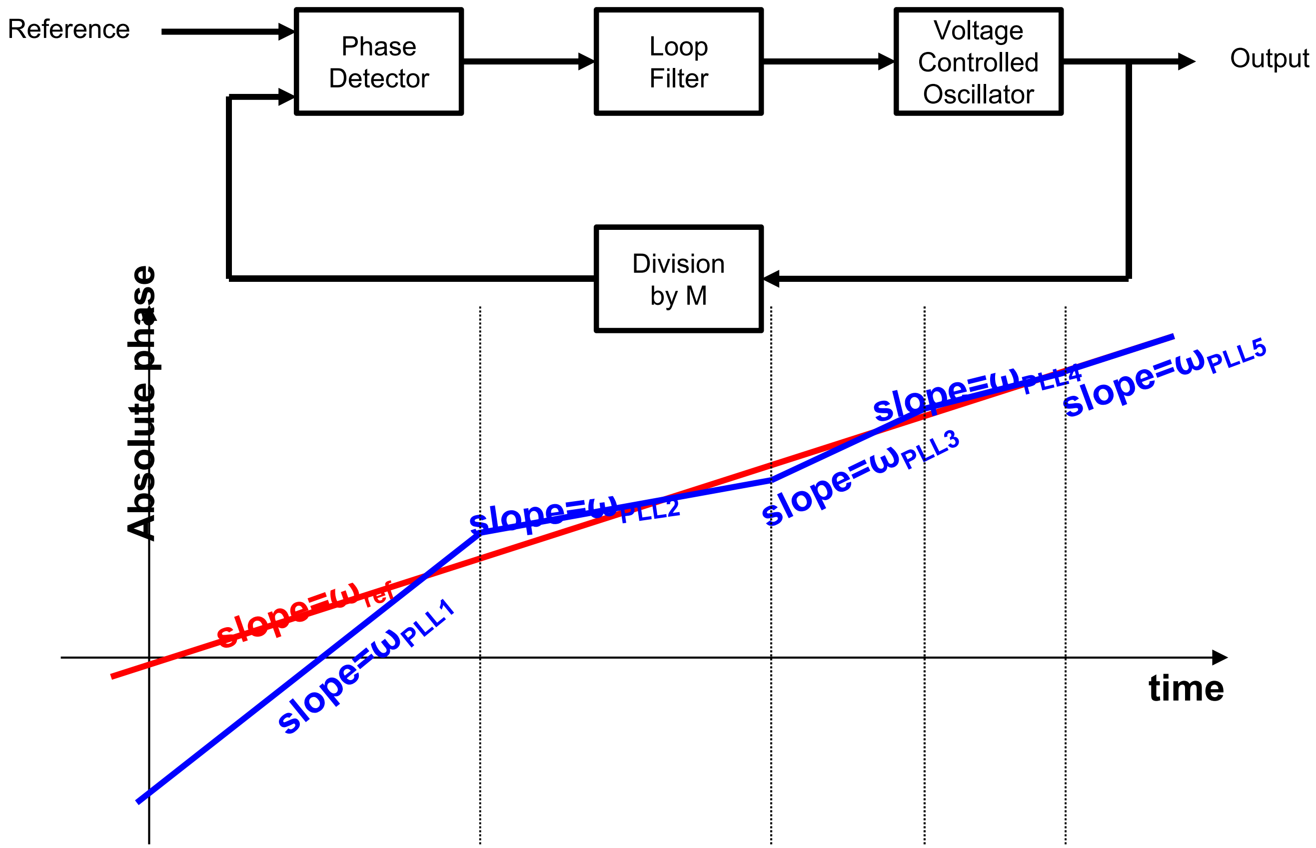


- Cameraman can only control the accelerator (velocity)
- Distance: Integral of velocity

➔Control of velocity in order to lock the distance

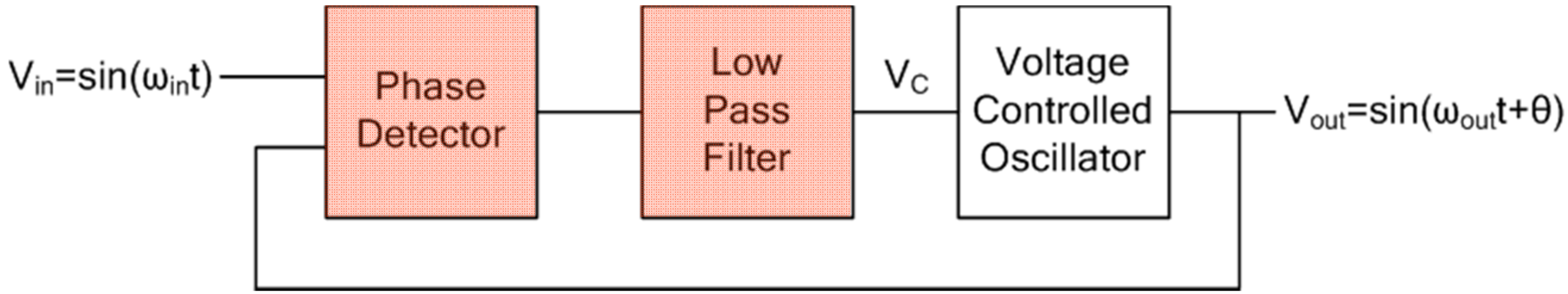
➔After locking, distance and velocity should be the same

Phase-tracking by PLL



PLL achieves phase-locking by changing VCO frequency

Basic PLL Operation



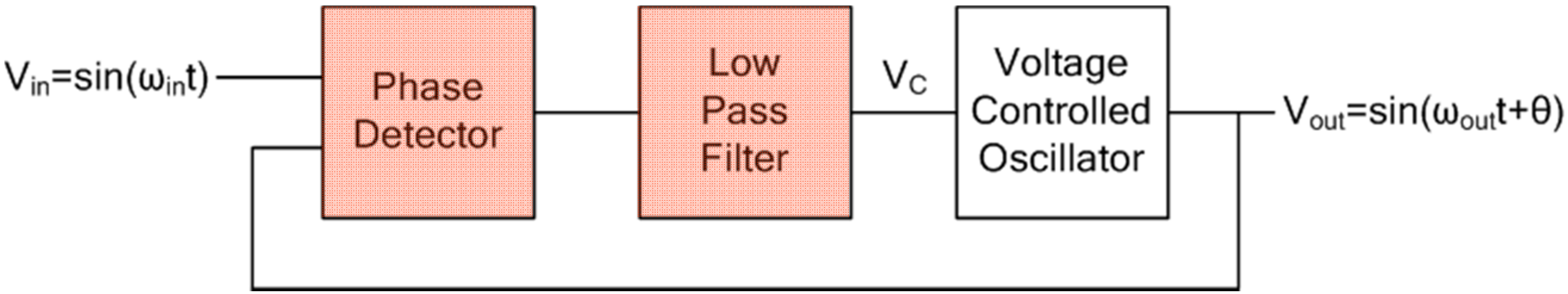
PD (Phase Detector): Compares phases of input and output signal and converts the phase difference to voltage signal

LPF (Low Pass Filter): Takes an average level of PD's output voltage signal

PD can be realized with a multiplier

$$\sin(\omega_{in}t)\sin(\omega_{out}t+\theta) = \frac{1}{2}\left\{\cos[(\omega_{in}-\omega_{out})t-\theta]-\cos[(\omega_{in}+\omega_{out})t+\theta]\right\}$$

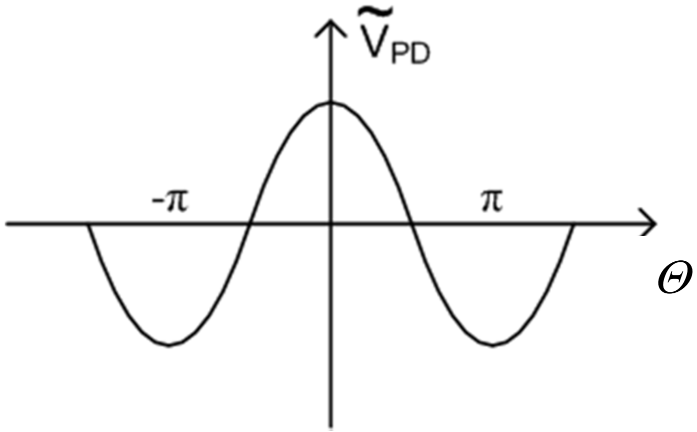
Basic PLL Operation



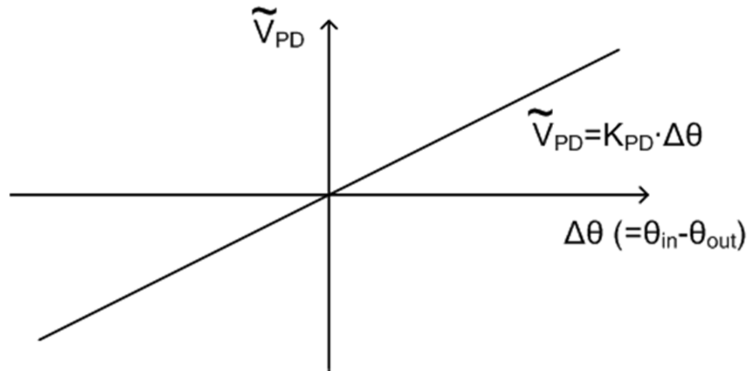
$$\sin(\omega_{in}t)\sin(\omega_{out}t+\theta) = \frac{1}{2}\left\{\cos[(\omega_{in}-\omega_{out})t-\theta]-\cos[(\omega_{in}+\omega_{out})t+\theta]\right\}$$

Filtered out by LPF

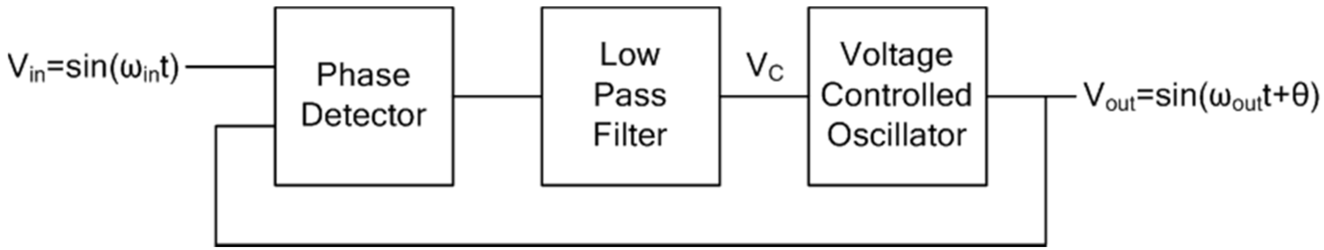
Assuming $\omega_{in}=\omega_{out}$



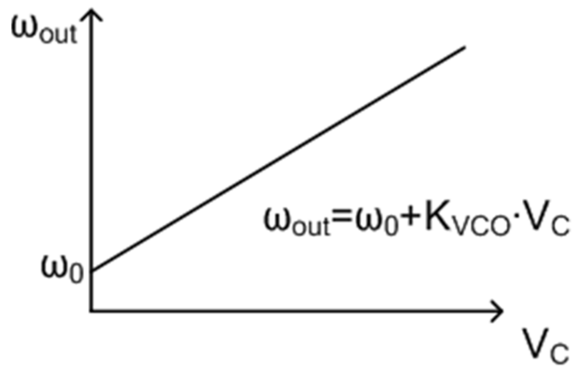
Approximately linear for $\theta = -\pi/2$



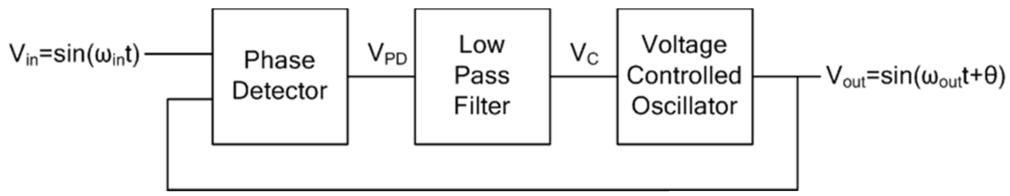
Basic PLL Operation



VCO (Voltage Controlled Oscillator): Frequency-tunable oscillator
➔ Output frequency is a function of control voltage (V_C)

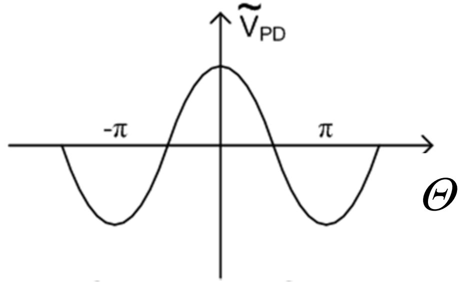


Basic PLL Operation



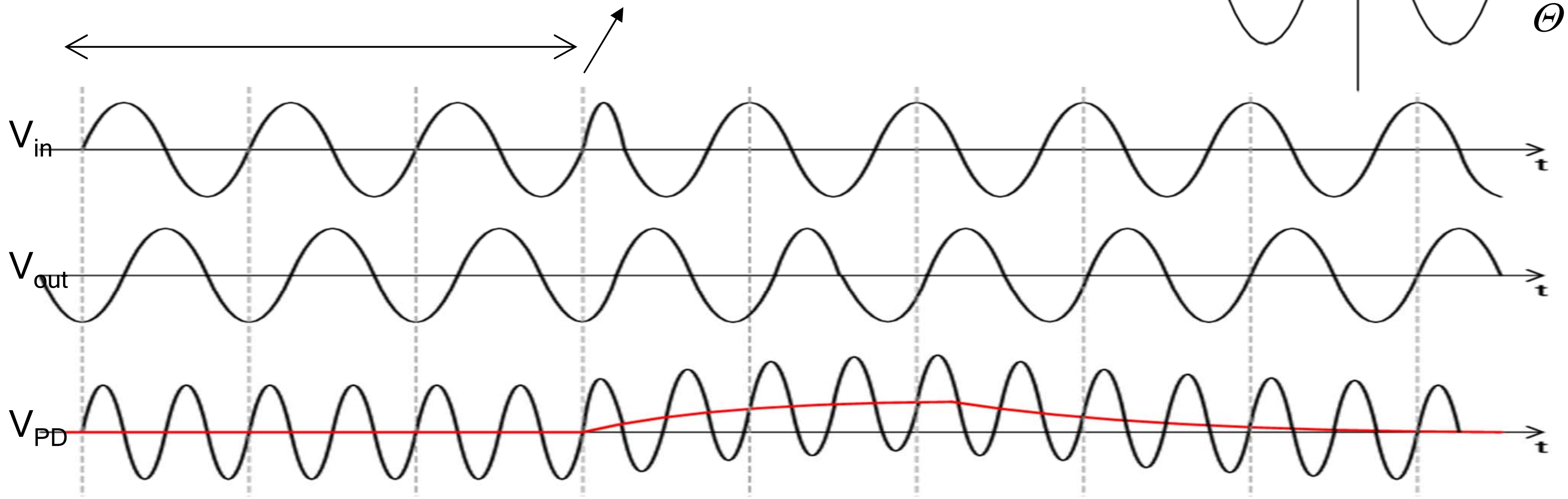
Phase tracking of PLL

$\omega_{in}=\omega_{out}$



Initially, V_{out} is locked to V_{in}
With $\theta = -\pi/2$

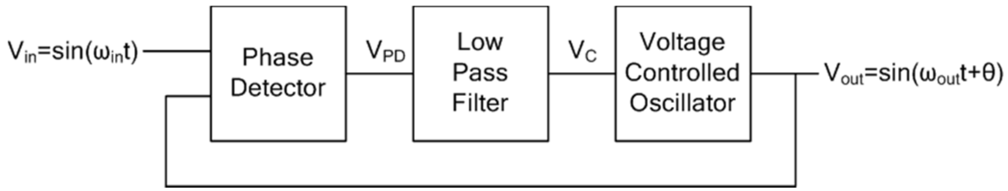
Phase jump in V_{in} here



Basic PLL Operation

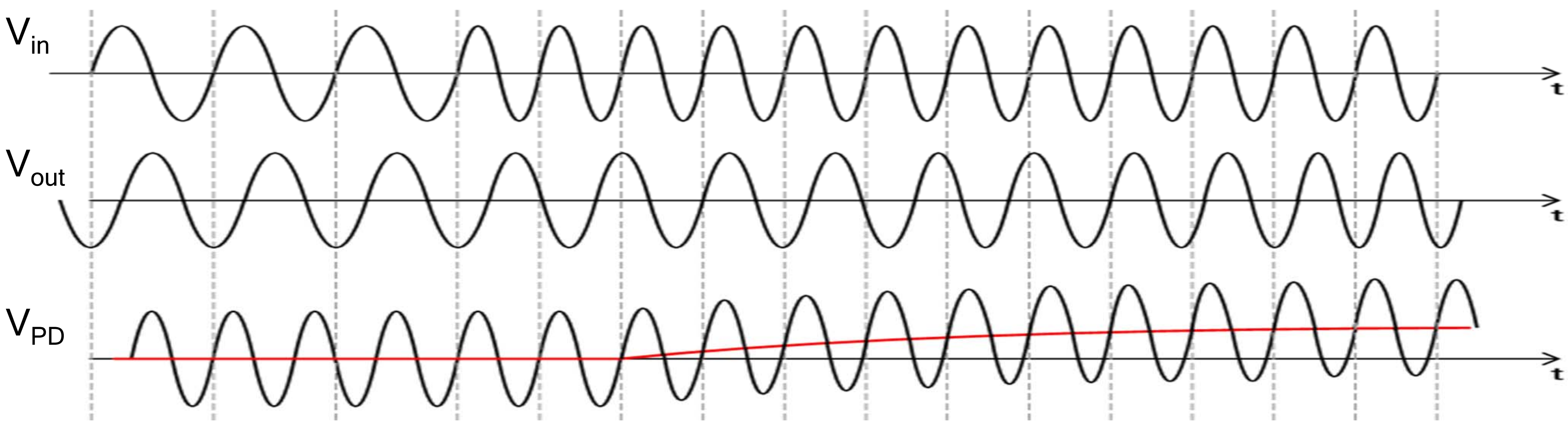
Frequency tracking of PLL

$\omega_{in} \neq \omega_{out}$



V_{out} is locked to V_{in}
With $\theta = -\pi/2$

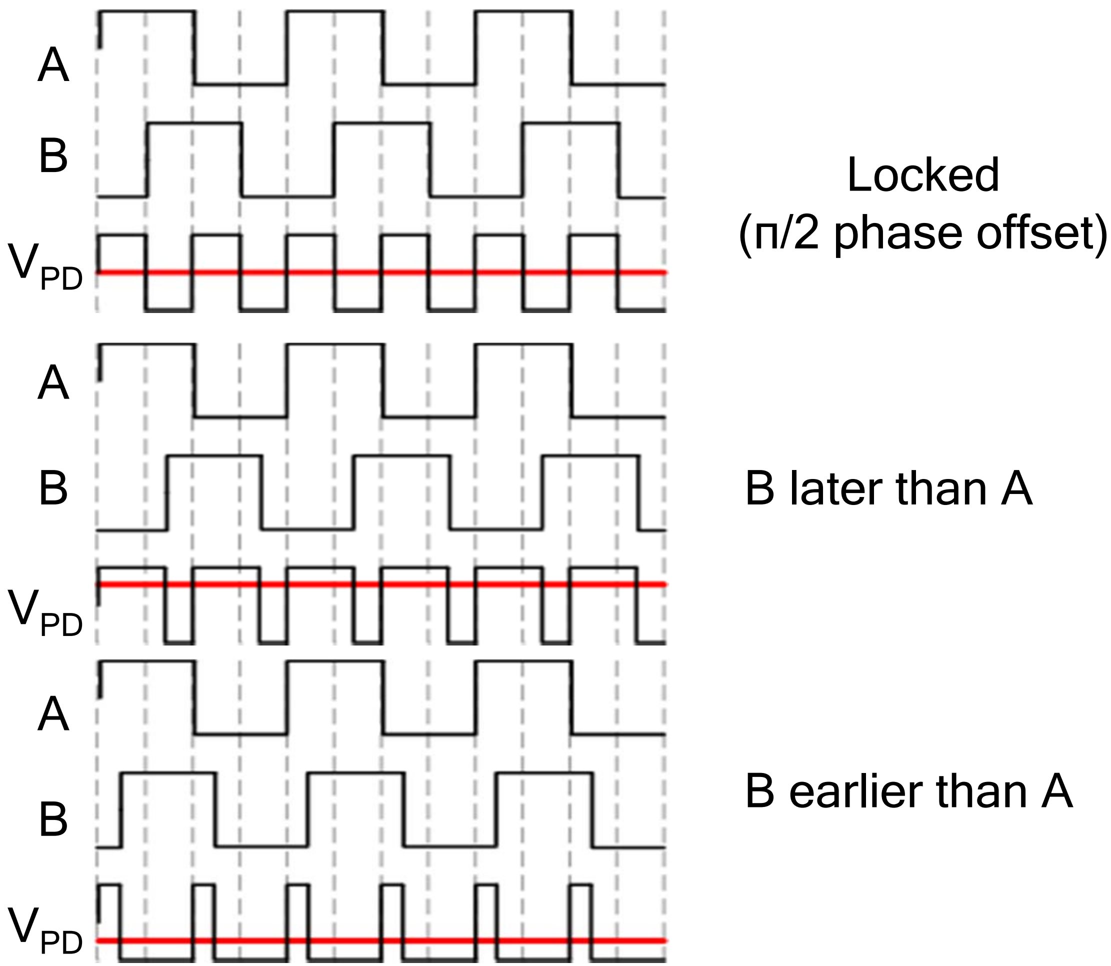
Frequency jump in V_{in} here



Basic PLL Operation

XOR gate can be used as PD for digital signals

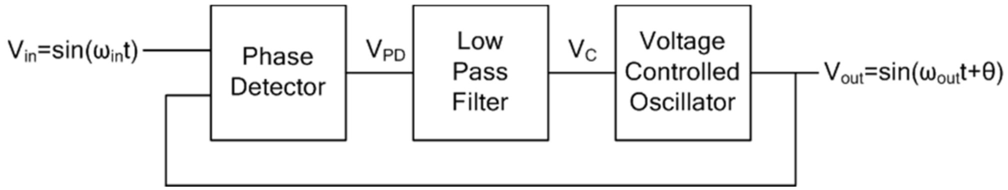
A	B	V_{PD}
0	0	0
0	1	1
1	0	1
1	1	0

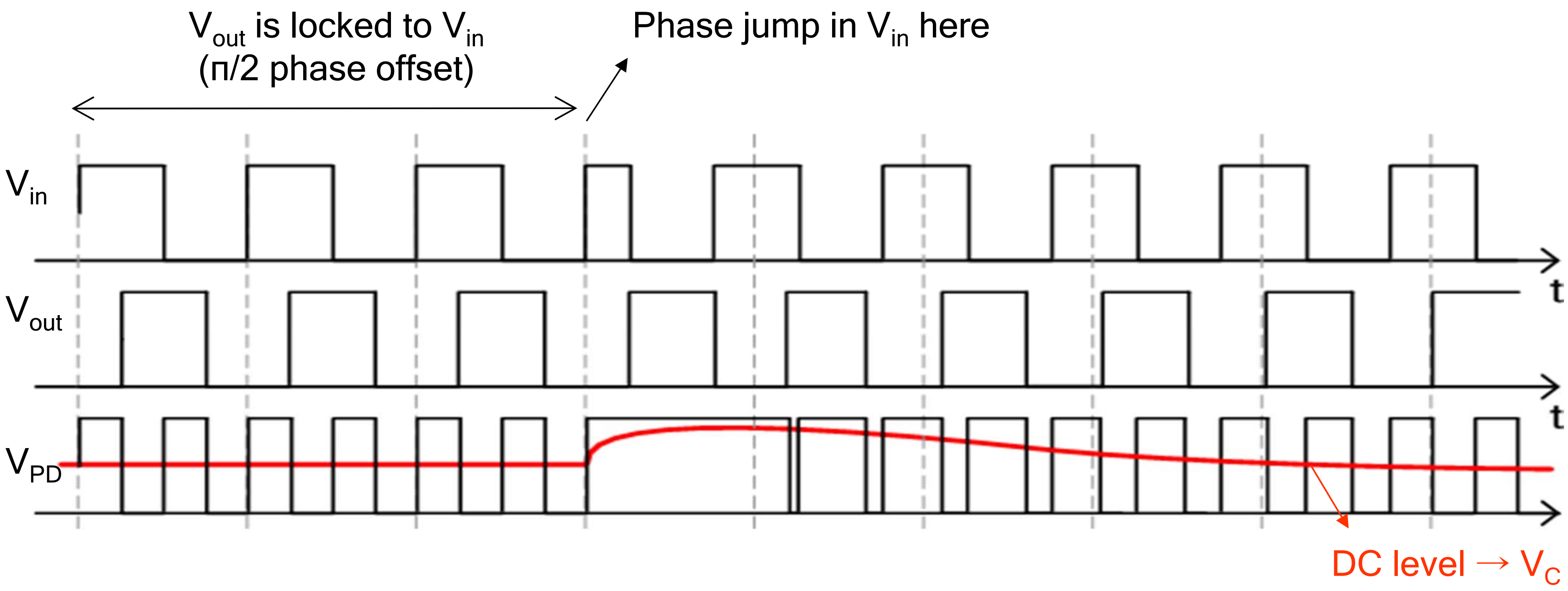


Basic PLL Operation

Phase tracking of PLL

$\omega_{in}=\omega_{out}$

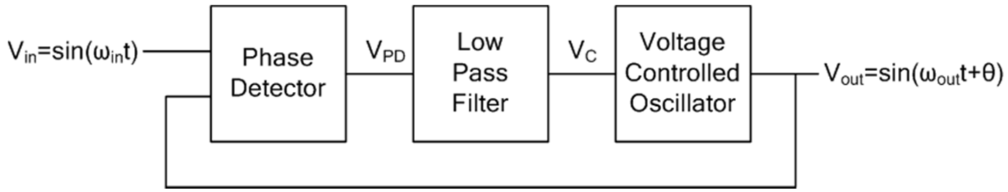


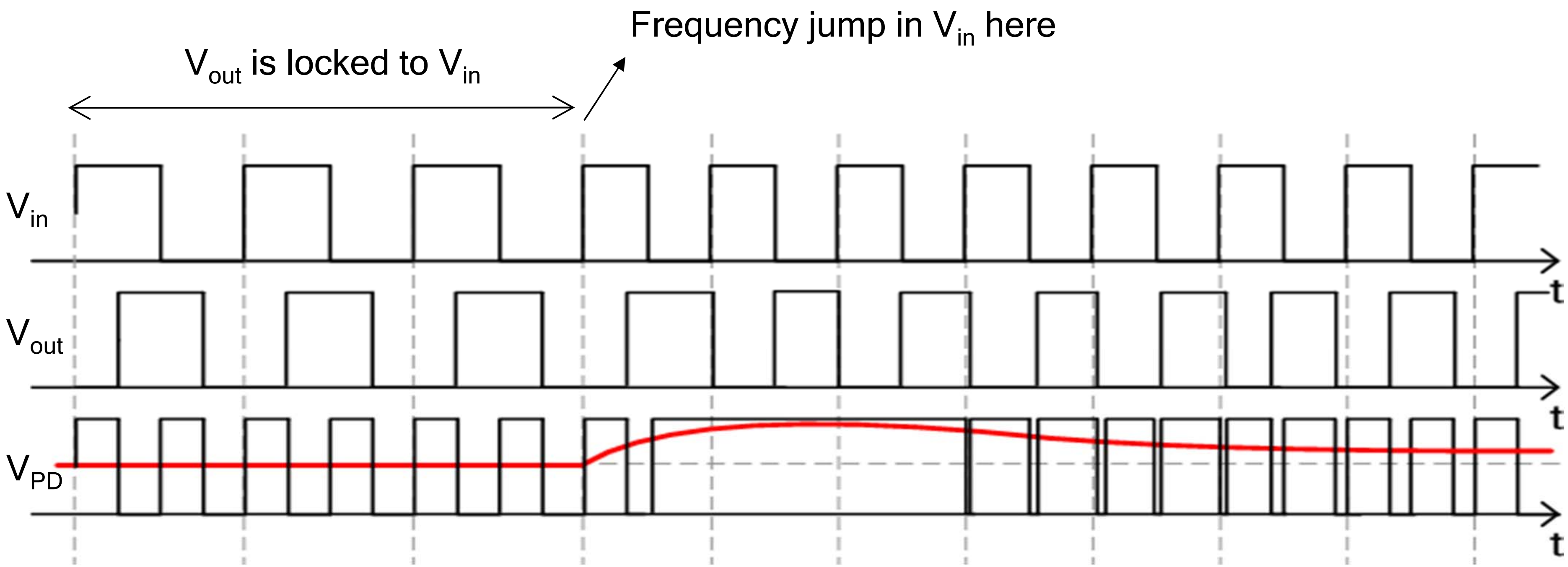


Basic PLL Operation

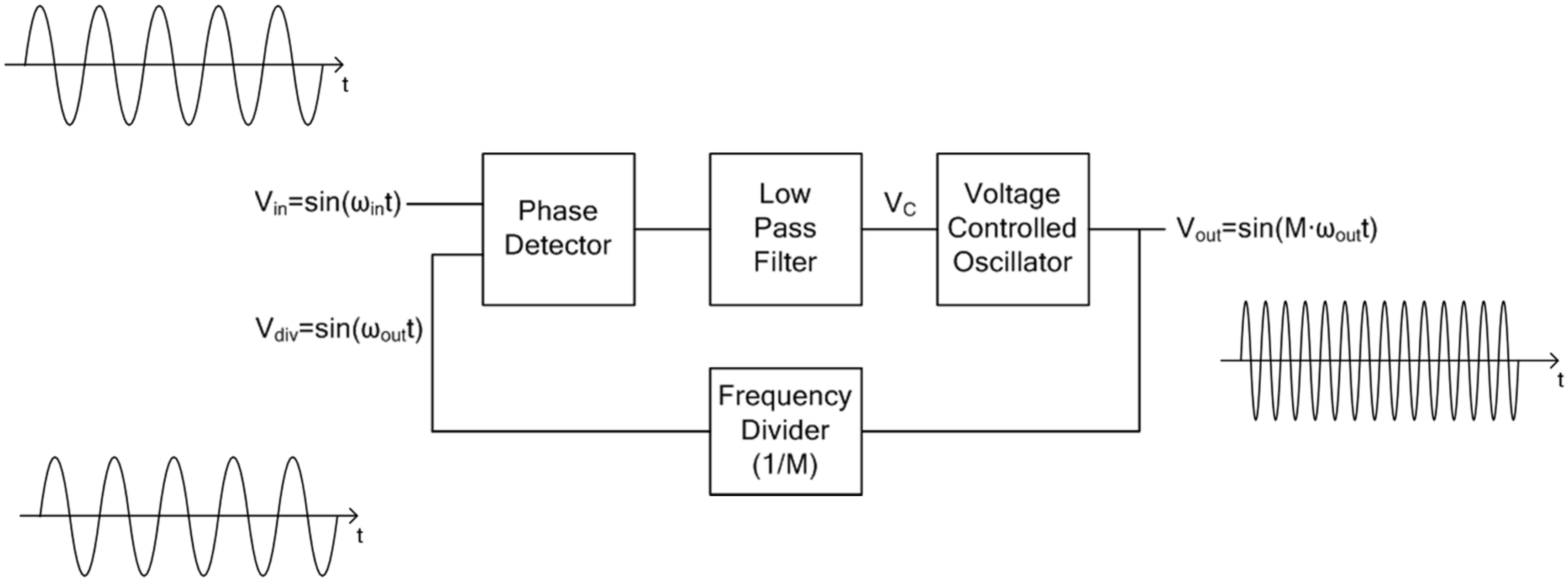
Frequency tracking of PLL

$\omega_{in} \neq \omega_{out}$

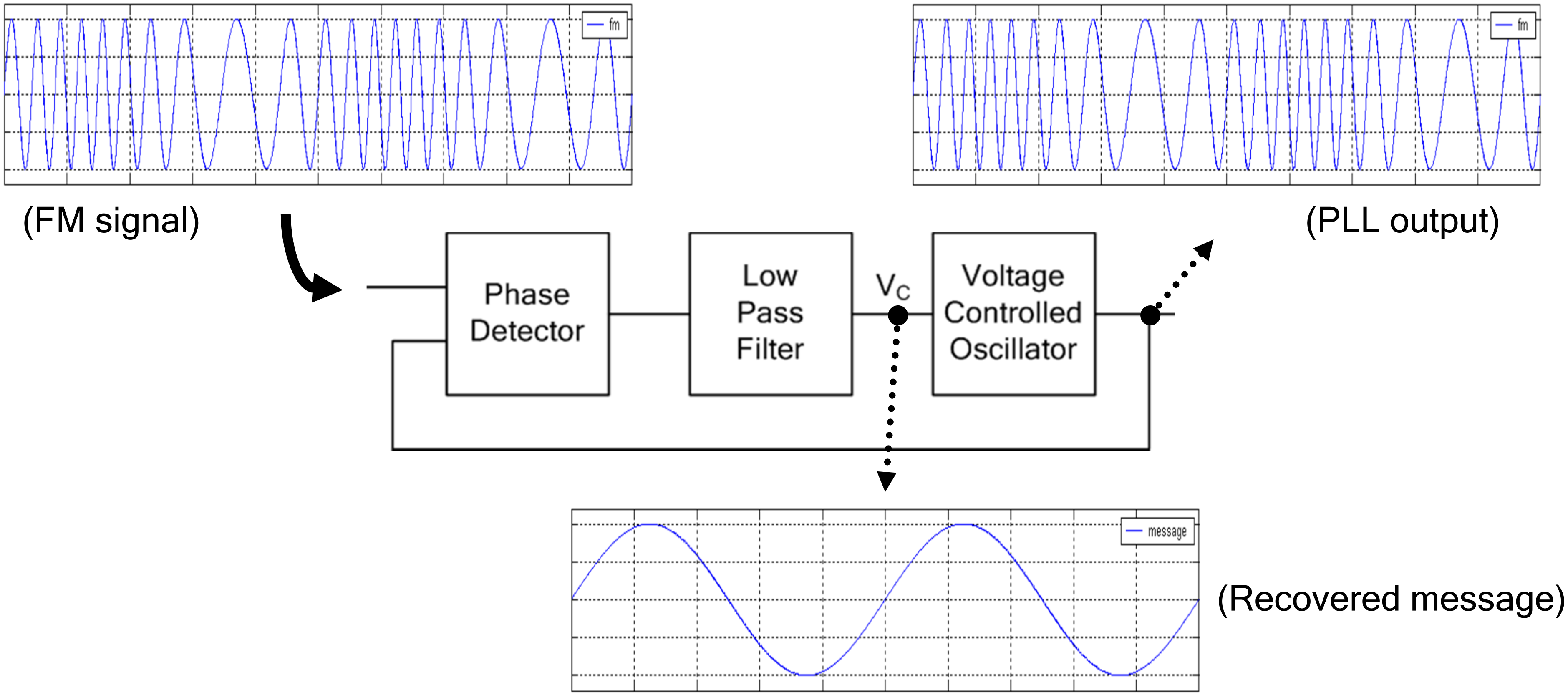




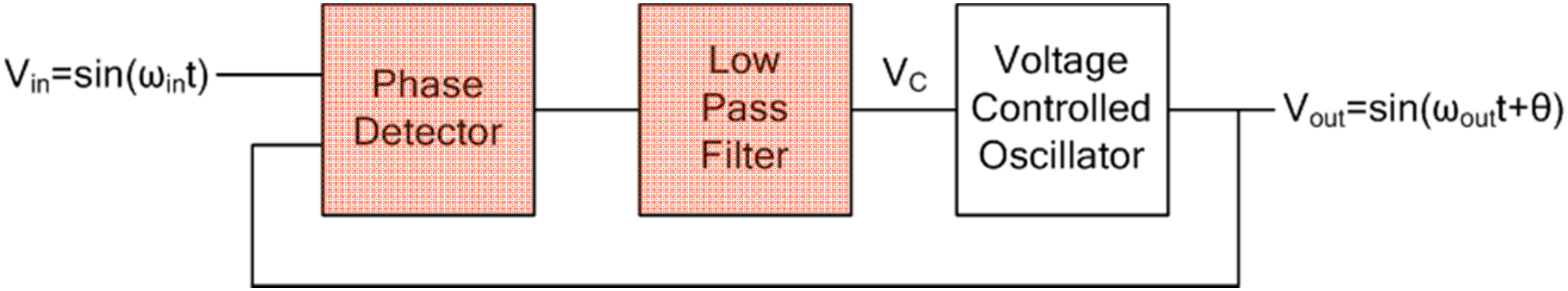
Frequency Synthesis with PLL



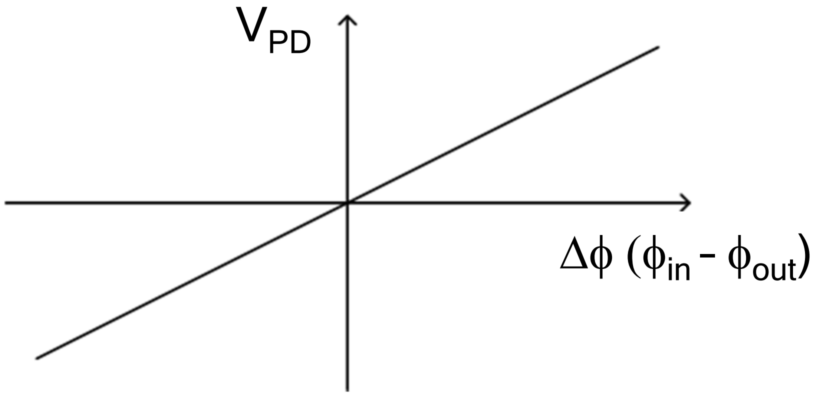
Frequency Demodulation with PLL



PLL Dynamics: Linear Model



Linear approximation for PD characteristics

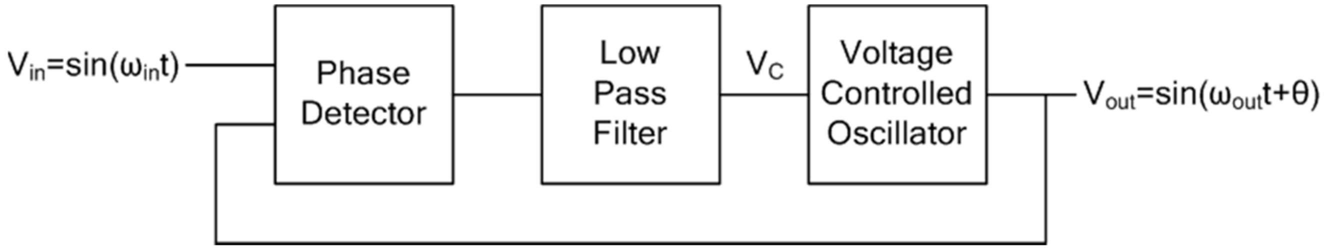


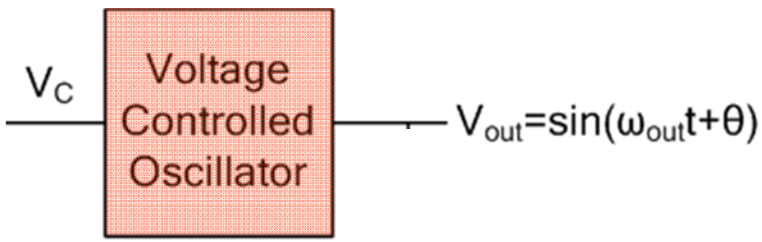
$V_{PD}(s) = K_{PD} \times \Delta\phi(s)$

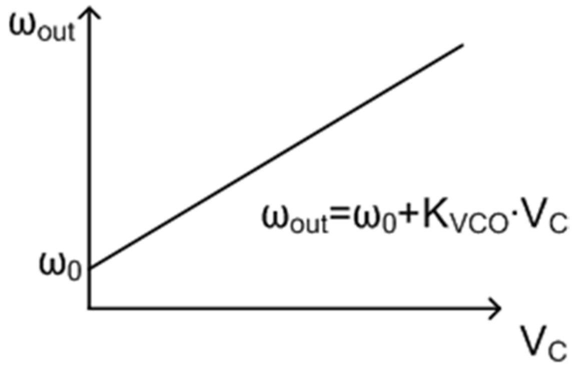
LPF: First order with pole at s = - ω_p

$$T(s) = \frac{V_C(s)}{V_{PD}(s)} = \frac{\omega_p}{s + \omega_p}$$

PLL Dynamics: Linear Model







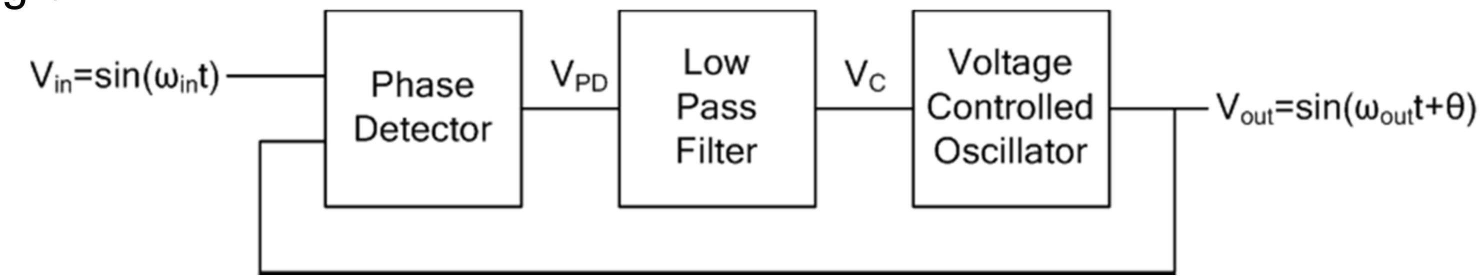
For simplicity let $\omega_0=0$

Since $d\phi/dt = \omega$

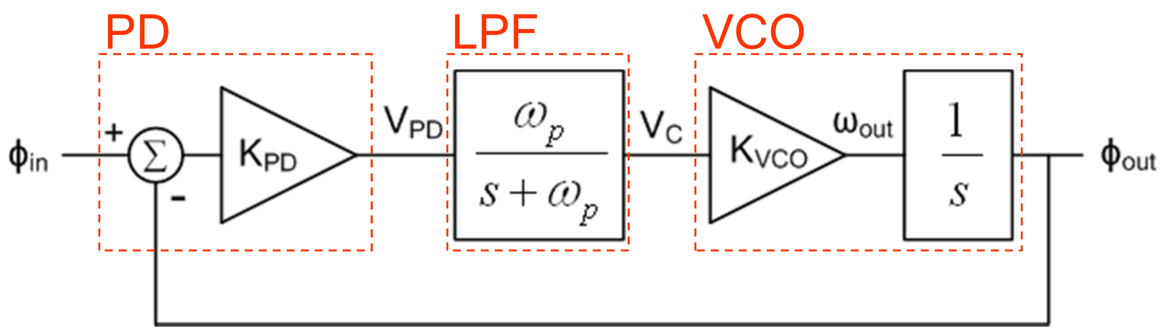
$\phi_{out}(s) = (1/s)\, K_{VCO} \times V_C(s)$

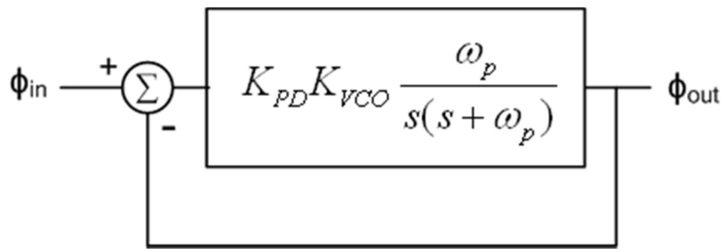
PLL Dynamics: Linear Model

PLL Block Diagram

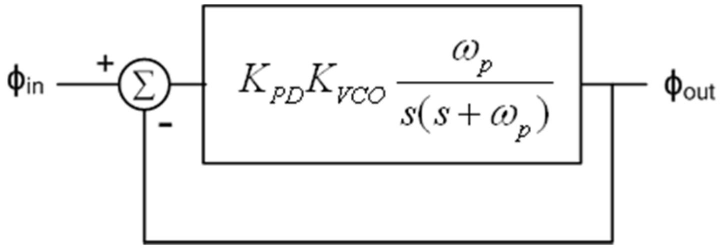


Linear Model for $\phi_{out}(s) / \phi_{in}(s)$





PLL Dynamics: Linear Model



Open loop gain:

$$G(s) = K_{PD}K_{VCO}\frac{\omega_p}{s(s+\omega_p)}$$

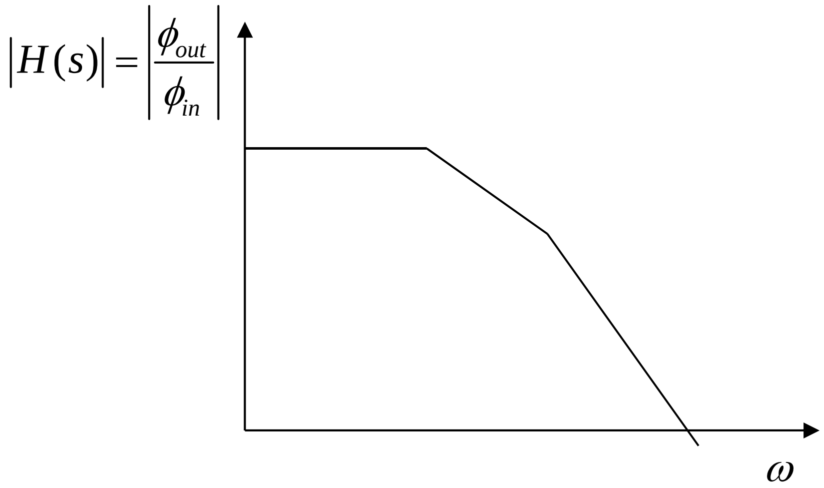
Closed loop gain

$$H(s) = \frac{\phi_{out}(s)}{\phi_{in}(s)} = \frac{G(s)}{1+G(s)} = \frac{K_{PD}K_{VCO}\frac{\omega_p}{s(s+\omega_p)}}{1+K_{PD}K_{VCO}\frac{\omega_p}{s(s+\omega_p)}} = \frac{K_{PD}K_{VCO}\omega_p}{s^2+\omega_p s+K_{PD}K_{VCO}\omega_p}$$

➔ 2nd order LPF!

PLL Dynamics: Linear Model

$$H(s) = \frac{\phi_{out}(s)}{\phi_{in}(s)} = \frac{K_{PD}K_{VCO}\omega_p}{s^2 + \omega_p s + K_{PD}K_{VCO}\omega_p}$$

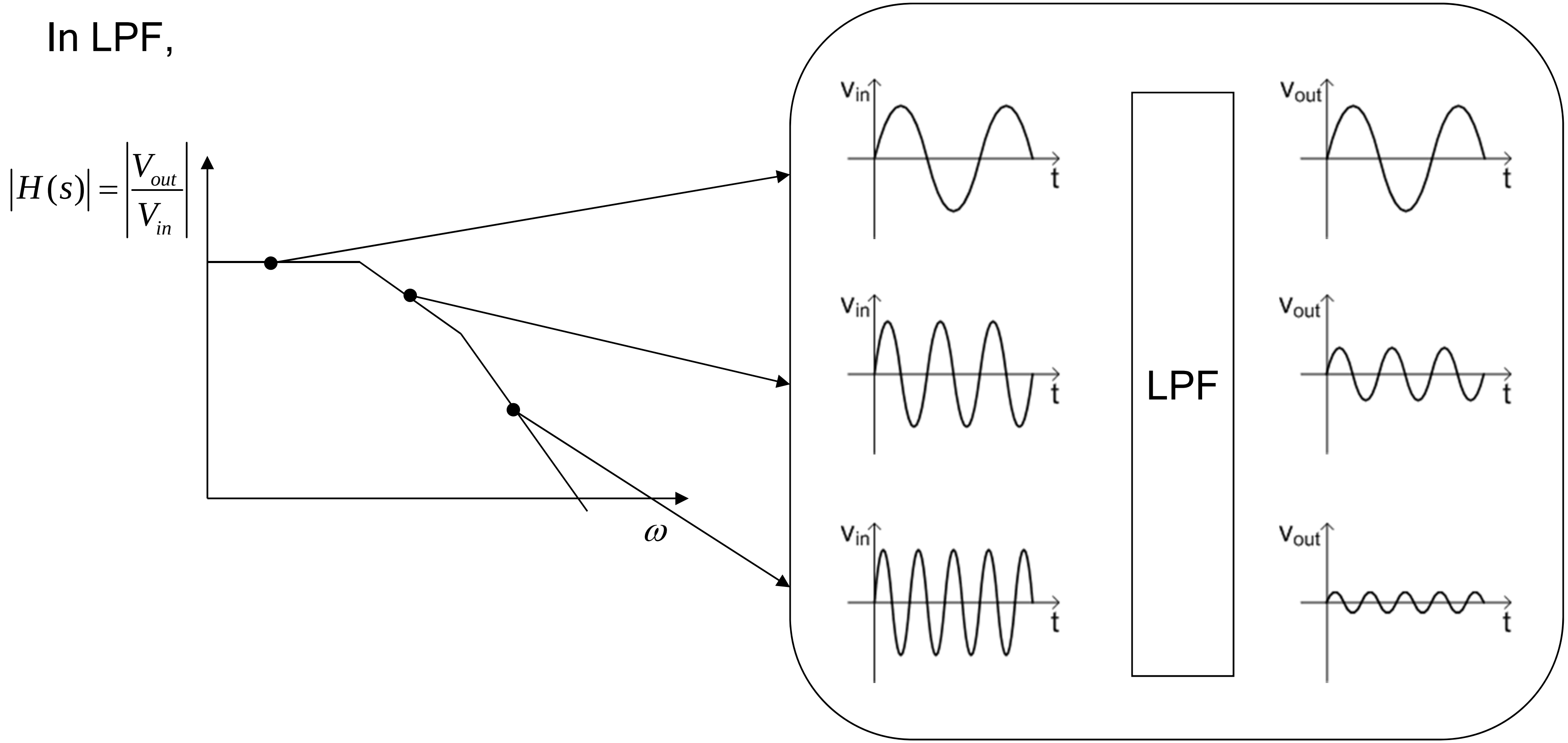


Note that input and output are *'phase'*

(Assuming two real poles)

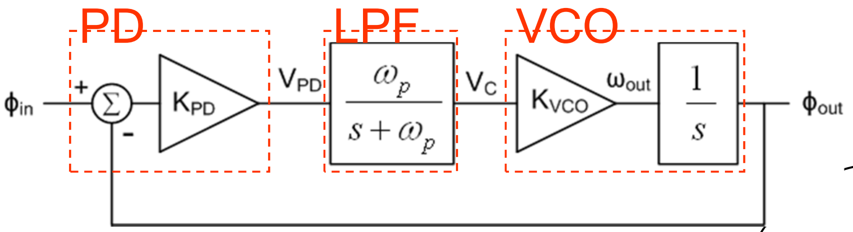
Magnitude Response vs Phase Response

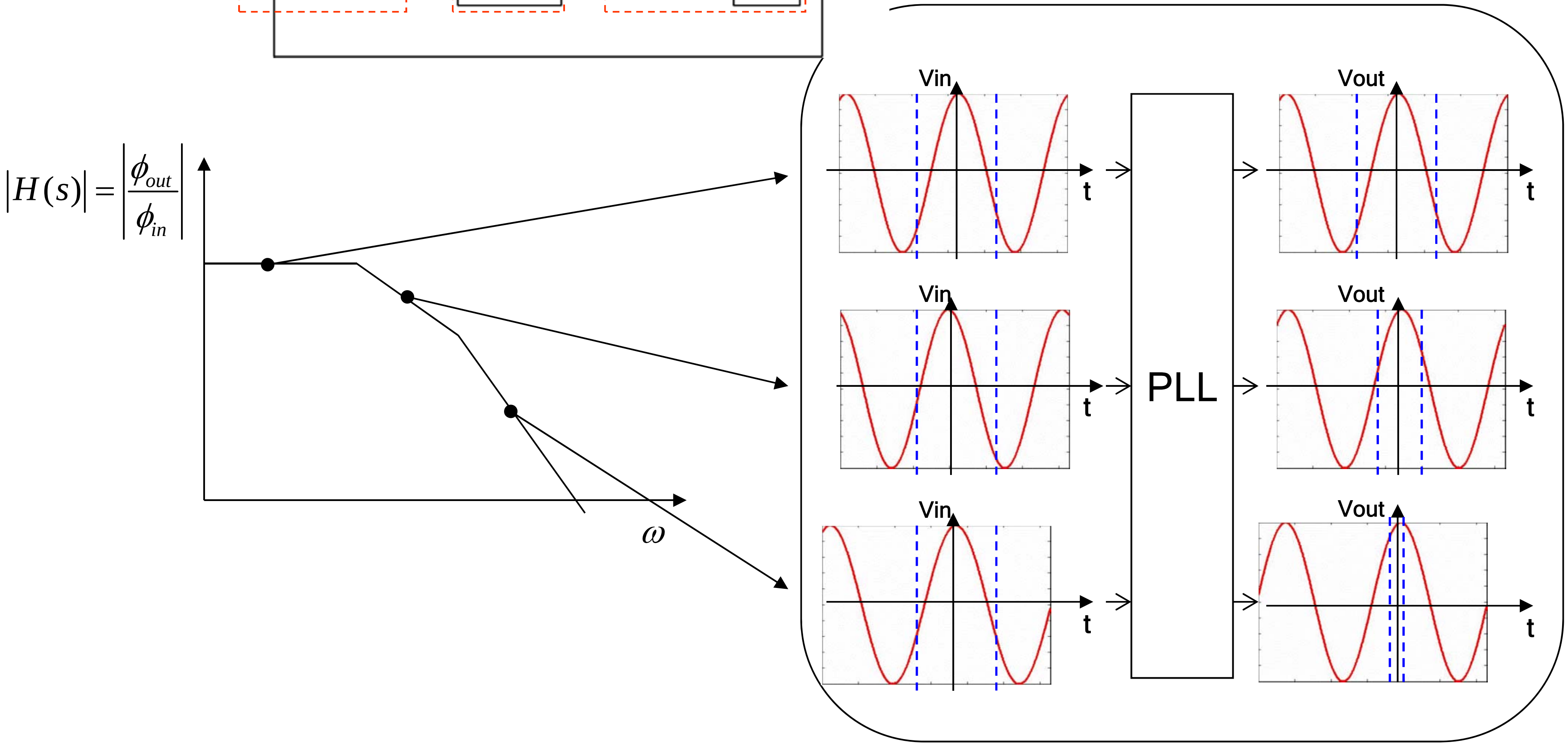
In LPF,



Magnitude Response vs Phase Response

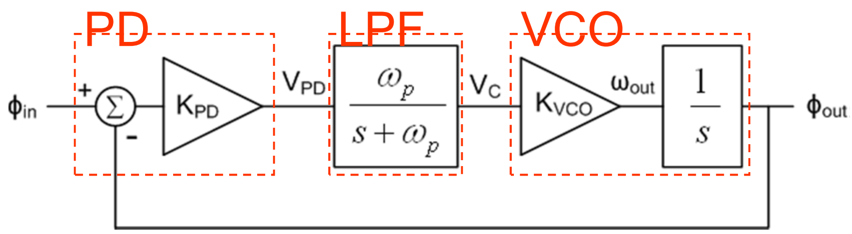
In PLL,





Second-Order System

$$H(s)=\frac{\phi_{out}(s)}{\phi_{in}(s)}=\frac{K_{PD}K_{VCO}\omega_p}{s^2+\omega_p s+K_{PD}K_{VCO}\omega_p}$$



2nd order system

$$H(s)=\frac{\omega_n^2}{s^2+(\omega_n/Q)s+\omega_n^2}$$

$$H(s)=\frac{\omega_n^2}{s^2+2\varsigma\omega_n s+\omega_n^2}$$

$$\omega_n=\sqrt{K_{PD}K_{VCO}\omega_P}$$

$$Q=\sqrt{\frac{K_{PD}K_{VCO}}{\omega_P}}$$

$$\varsigma=\frac{1}{2Q}=\frac{1}{2}\sqrt{\frac{\omega_P}{K_{PD}K_{VCO}}}$$

Where are the poles?

$$(-\zeta\pm\sqrt{\zeta^2-1})\omega_n.$$

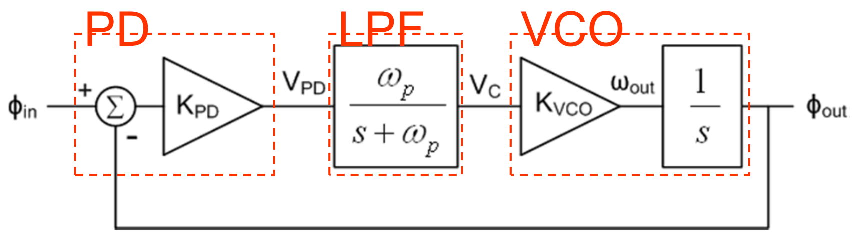
$\zeta > 1$: Over damped

$\zeta < 1$: Under damped

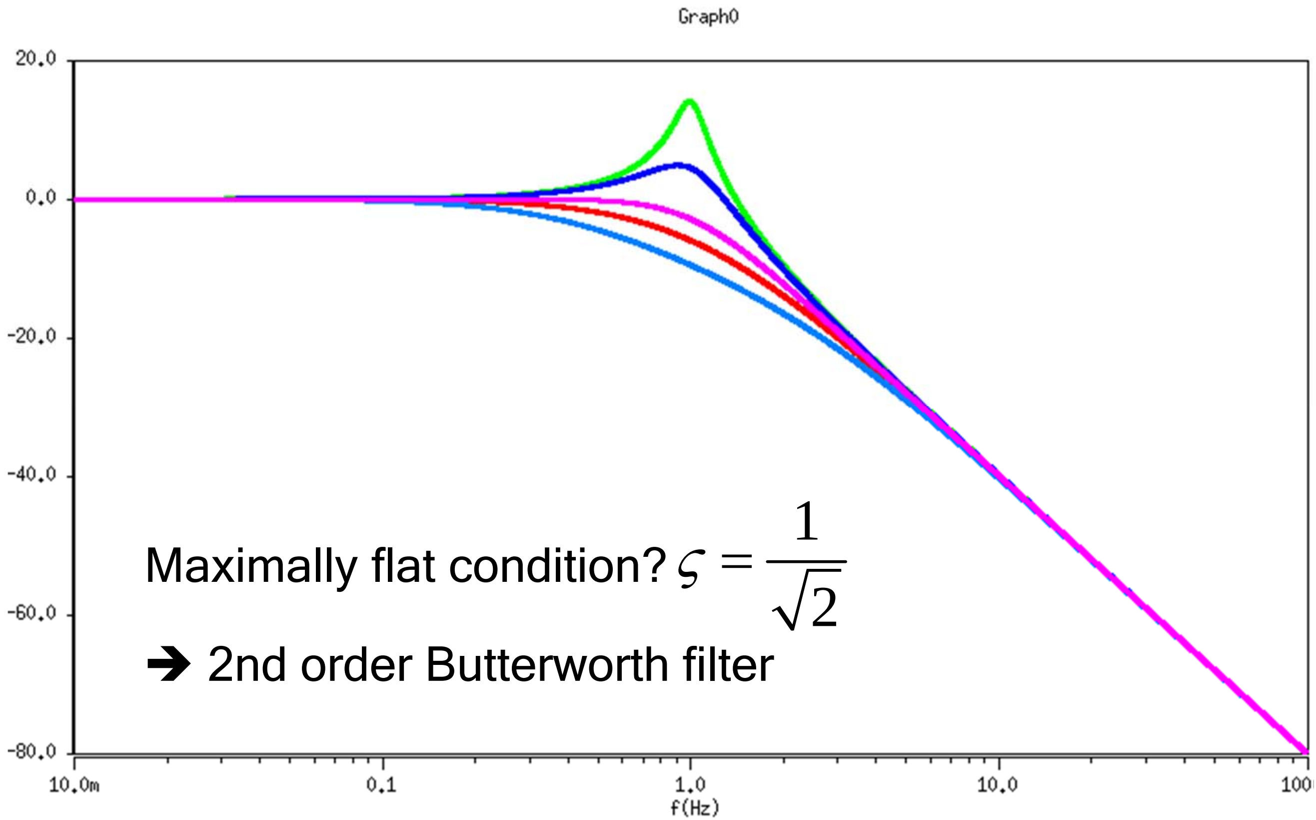
$\zeta = 1$: Critically damped

Second-Order System

$$H(s) = \frac{\phi_{out}(s)}{\phi_{in}(s)} = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$



Frequency Response



Maximally flat condition? $\varsigma = \frac{1}{\sqrt{2}}$

➔ 2nd order Butterworth filter

$\omega_n = 2\pi$

$\xi = 0.1$

$\xi = 0.3$

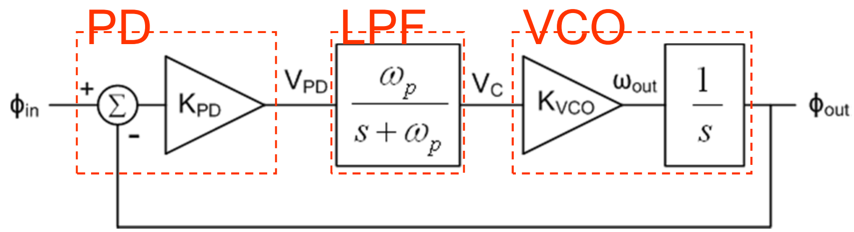
$\xi = 0.7$

$\xi = 1$

$\xi = 1.5$

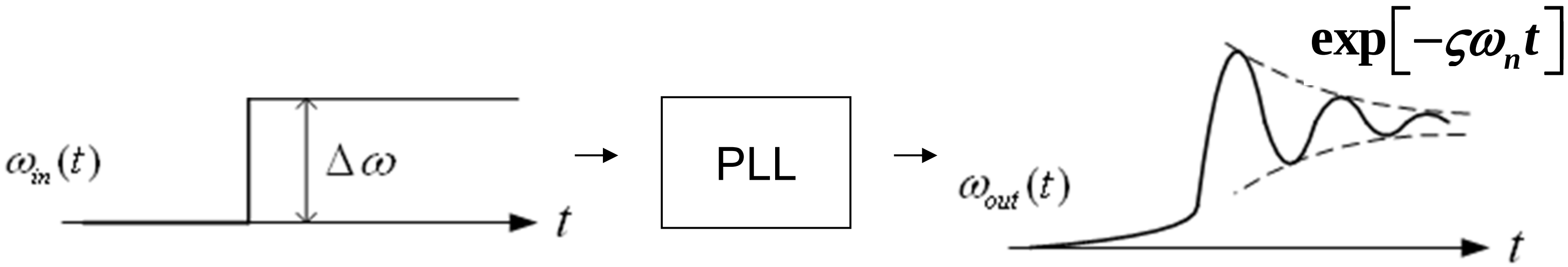
Second-Order System

$$H(s) = \frac{\phi_{out}(s)}{\phi_{in}(s)} = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$



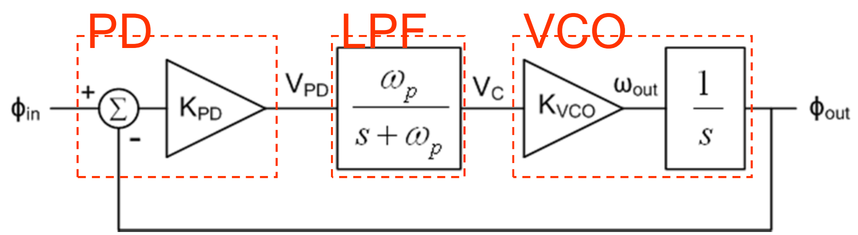
$$\frac{\omega_{out}(s)}{\omega_{in}(s)} = \frac{s\phi_{out}(s)}{s\phi_{in}(s)} = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

Step response: $\omega_{in}(t) = \Delta\omega \cdot u(t)$

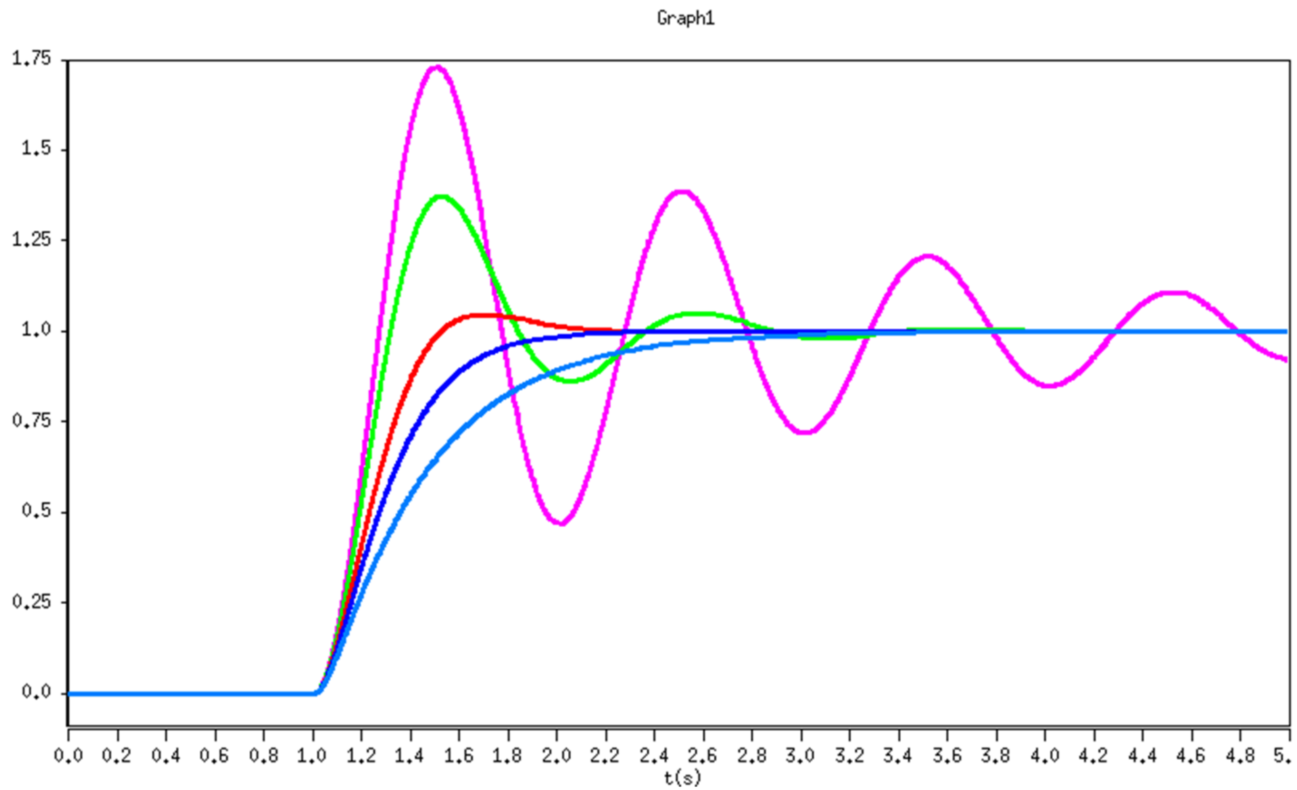


Second-Order System

$$\frac{\omega_{out}(s)}{\omega_{in}(s)} == \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$



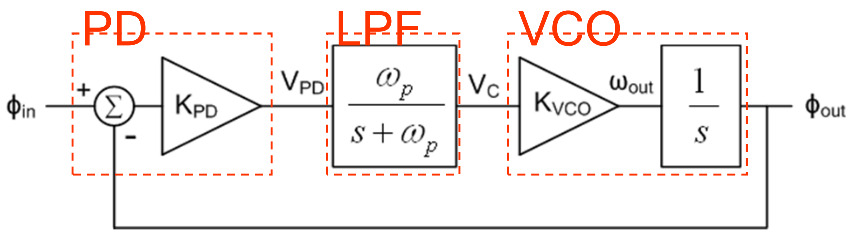
Step response: $\omega_{in}(t) = \Delta\omega \cdot u(t)$



$\omega_n = 2\pi$

$\xi = 0.1$
$\xi = 0.3$
$\xi = 0.7$
$\xi = 1$
$\xi = 1.5$

Second-Order System



$$H(s) = \frac{\phi_{out}(s)}{\phi_{in}(s)} = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

- Transfer function for phase error?

$$H_e(s) = 1 - H(s) = \frac{s^2 + 2\varsigma\omega_n s}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

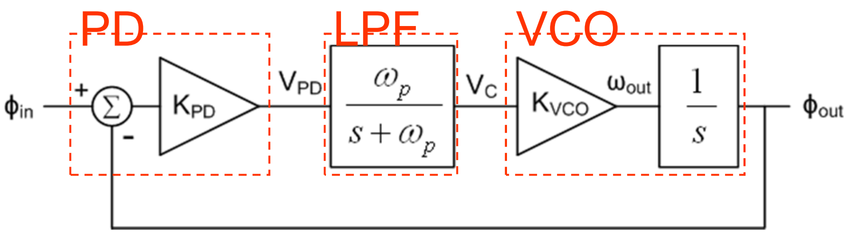
- Phase error due to phase step?

$$\phi_{in}(t) = \Delta\phi \cdot u(t) \qquad \phi_{in}(s) = \frac{\Delta\phi}{s}$$

$$\phi_e(t = \infty) = \lim_{s \to 0}\left[sH_e(s)\frac{\Delta\phi}{s} \right]$$

- No steady-state phase error

Second-Order System



$$H(s) = \frac{\phi_{out}(s)}{\phi_{in}(s)} = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$H_e(s) = \frac{s^2 + 2\varsigma\omega_n s}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

- Phase error due to frequency step?

$$\omega_{in}(t) = \Delta\omega \cdot u(t)$$

$$\omega_{in}(s) = \frac{\Delta\omega}{s}$$

$$\phi_{in}(s) = \frac{\Delta\omega}{s^2}$$

$$\phi_e(t=\infty) = \lim_{s\to 0}\left[sH_e(s)\frac{\Delta\omega}{s^2}\right]$$

$$= \frac{2\varsigma}{\omega_n}\Delta\omega$$

$$= \frac{\Delta\omega}{K_{PD}K_{VCO}}$$