Lect. 10: Diffraction Gratings
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The diagram illustrates the relationship between the slit shape (y) and the far-field (k_y) through Fourier Transform:

\[ \text{slit shape (y)} \leftrightarrow \text{far-field (k}_y\text{)} \]

\[ \text{Fourier Transform} \]

\[ \sin\left(\frac{k_y a}{2}\right) \]

\[ \frac{k_y a}{2} \]

\[ y = \sin \theta \]

\[ a \]

\[ R \]

\[ \theta \]
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(Assume infinitely many for simplicity)

Diffracted light from periodic slits (Diffraction Grating)

$\Rightarrow$ Far-field only for discrete $k_y$'s

$k_y = k \sin \theta = m \frac{2\pi}{d}$
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Grating equation

\[ k_y = m \frac{2\pi}{d} \]

Bragg Condition

\[ \sin \theta \frac{2\pi}{\lambda} = m \frac{2\pi}{d} \]

\[ d \sin \theta = m \lambda \]

William Henry Bragg (1862-1942)
Nobel Prize in Physics (1915)
The only father-son joint Nobel winner

William Lawrence Bragg (1890-1971)

W. L. Bragg is the youngest Nobel Physics winner
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$$d \sin \theta = m \lambda$$

Width for each diffracted beam?

$m = 0$ Zero-order
$m = 1$ First-order
$m = -1$ First-order
$m = 2$ Second-order
$m = -2$ Second-order
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Input with tilted angle

\[ E_{total} = \int_{-\infty}^{\infty} \frac{A}{R} e^{jky\sin \theta} \, dy \quad \Rightarrow \quad E_{total} = \int_{-\infty}^{\infty} \frac{A}{R} e^{jky(\sin \theta - \sin \theta_i)} \, dy \]
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Tilted incidence on grating

\[ \lambda = \theta_m \sin \theta \]

(a) Transmission grating

Incident light wave

Zero-order

First-order

First-order

\[ E_{total} = \int_{-\infty}^{\infty} \frac{A}{R} e^{j k y (\sin \theta - \sin \theta_i)} dy \]

\[ d \sin \theta = m \lambda \]

\[ d (\sin \theta - \sin \theta_i) = m \cdot \lambda \]
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Reflection-type grating

Grating is often realized with periodic shaping of reflection surface

Same diffraction equation applies

\[ d \left( \sin \theta - \sin \theta_i \right) = m \cdot \lambda \]
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\[ d \left( \sin \theta_{out} - \sin \theta_{in} \right) = m \cdot \lambda \]

\[ \frac{2\pi}{\lambda} \left( \sin \theta_{out} - \sin \theta_{in} \right) = m \cdot \frac{2\pi}{d} \]

\[ \frac{2\pi}{\lambda} \sin \theta_{out} = \frac{2\pi}{\lambda} \sin \theta_{in} + m \cdot \frac{2\pi}{d} \]

K-vector perspective?

\[ k_{x, out} = k_{x, in} + m \cdot \frac{2\pi}{d} \]

Grating shifts \( k_x \) by the integer multiples of \( 2\pi/d \)
Grating imposes BC on the incident wave

\[ E_r(x, y = 0) = E_{in}(x, y = 0) \times f(x) \]

\[ f(x) = \sum_m m \frac{2\pi}{d} \]

Far-Field Diffraction: F.T. of \( E_{in}(x, y = 0) \times f(x) \)

\[ E(k_x)_{out} = E(k_x)_{in} \ast \]

Spatial modulation ↔ Sidebands formation
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2-D Diffraction Grating

Grating 1 + Grating 2

Take x and y separately
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Si Photonic Integrated Circuit

How do we get light in and out of PIC?

Good coupling possible with spot-size conversion

Requires
- Precise alignment
- Polishing fiber facet
- AR coating
- Dicing of a chip

No dicing required

Wafer-level optical testing possible

Light source (laser) cannot be integrated
→ Externally supplied

Si Photonics (2018/2)
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Condition for vertical coupling out?

\[
\theta_i = 90^\circ \quad \theta = 0^\circ
\]

\[
d = \lambda \quad \text{With } m=-1
\]

But with \( d=\lambda \)

\[
m=-2, \quad \theta = -90^\circ
\]

Also satisfied → Not desired
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\[ \theta_i = 90^\circ \]

\[ d \left( \sin \theta - 1 \right) = m \cdot \lambda \]

If \( d = \frac{\lambda}{1 - \sin \theta} \) (\( m = -1 \))

No other diffracted output
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Snell’s Law

\[
\frac{2\pi}{\lambda} \sin \theta_c = \frac{2\pi}{\lambda} + m \cdot \frac{2\pi}{d}
\]

For \( m = -1 \)

\[
d = \frac{\lambda}{n_{\text{eff}} - n_c \sin \theta_c} = \frac{\lambda}{n_{\text{eff}} - \sin \theta_{\text{air}}}
\]

Diffraction at 180° - \( \theta_c \) is also allowed
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Design Exercise: Grating coupler