Opto-Electronics and Photonics

Lecture 10 : Total Internal Reflection

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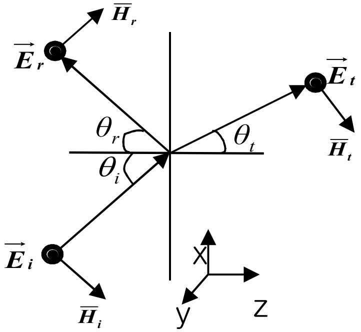
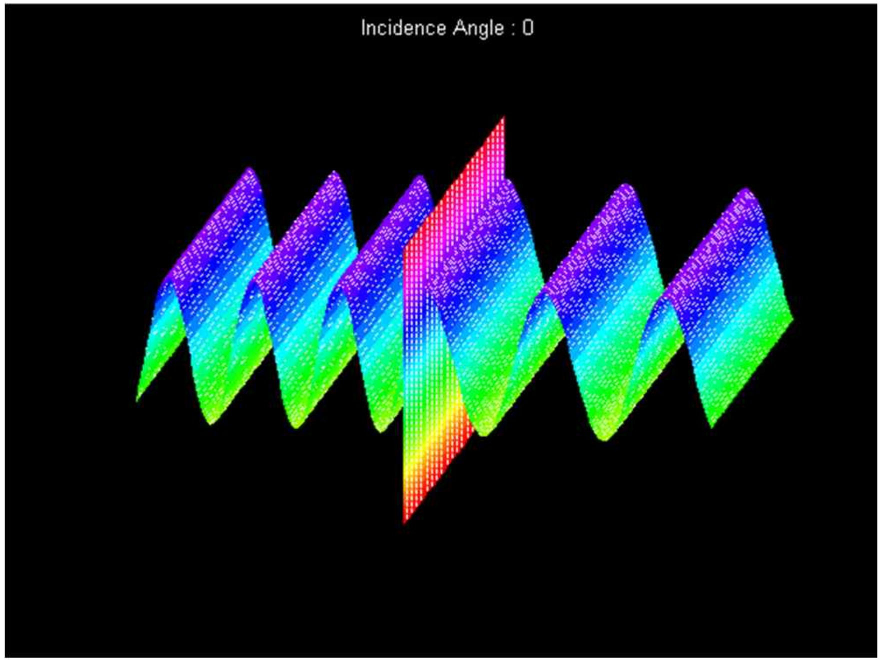
Lecture 10: Total Internal Reflection

(Cheng, 8-10.1)

$n_1 = 1.4$



$n_2 = 1$



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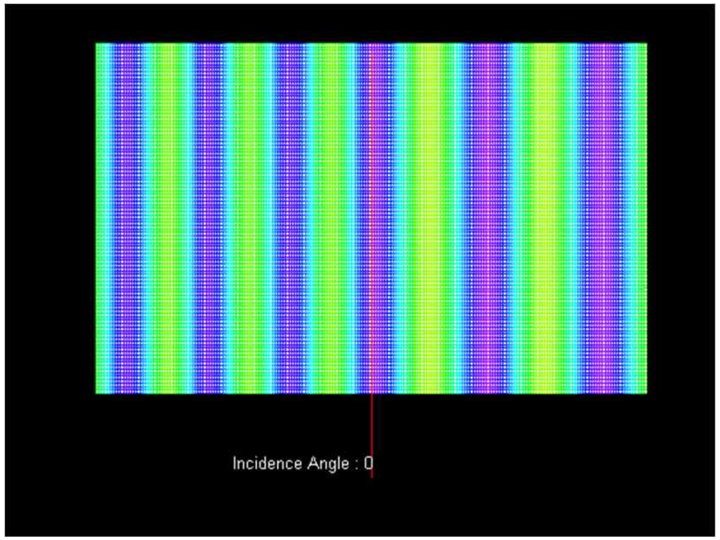
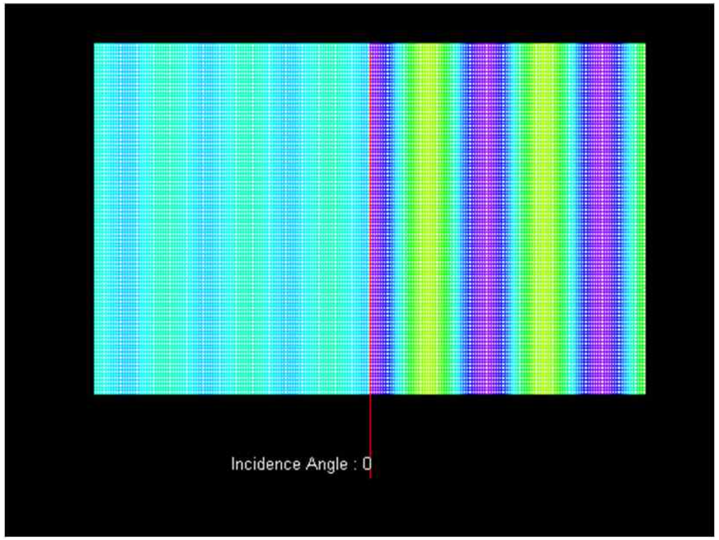
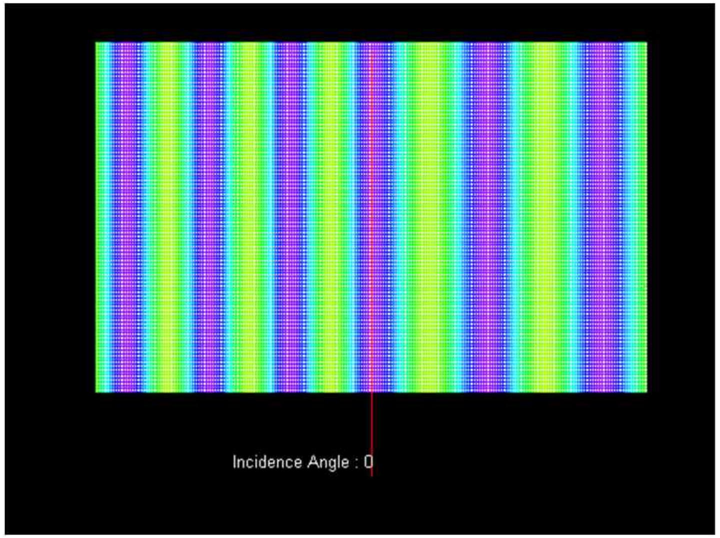
Incident and transmitted



Reflected and transmitted



Total



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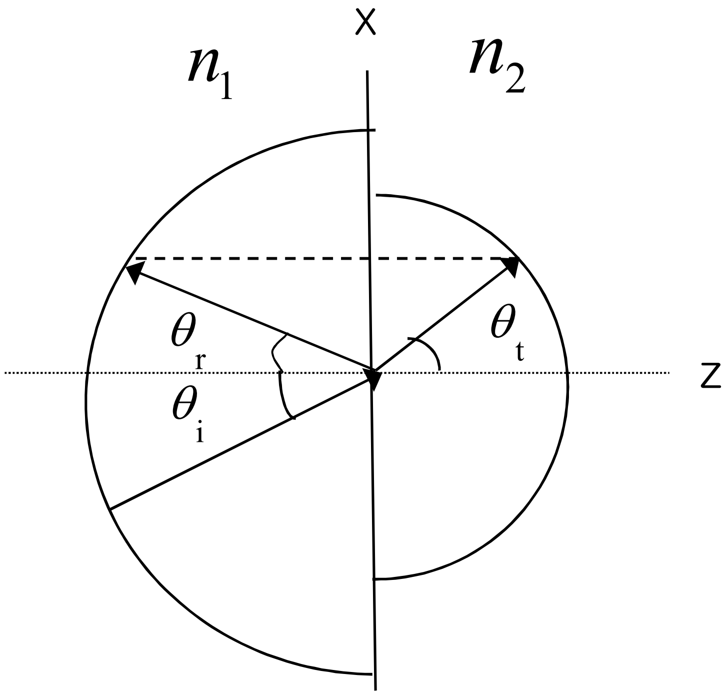
Snell's Law: $n_1 \sin\theta_i = n_2 \sin\theta_t$

If $n_1 > n_2$, $\theta_i < \theta_t$ $\quad \theta_t = 90°$ when $\sin\theta_i = \frac{n_2}{n_1}$ (Critical angle, θ_c)

What happens when $\theta_i > \theta_c$? Total Internal Reflection



From B.C.'s, $\theta_i = \theta_r$ $\quad \beta_{1x} = \beta_{2x}$

When $\theta_i > \theta_c$, $\beta_{2x} > n_2 k_0$

Since $\beta_{2x}{}^2 + \beta_{2z}{}^2 = (n_2 k_0)^2$

$$\beta_{2z}{}^2 < 0$$

Let $\beta_{2z} = -j\alpha$, $\quad \beta_{2x}{}^2 - \alpha^2 = (n_2 k_0)^2$

$$\alpha = \sqrt{\beta_{2x}{}^2 - (n_2 k_0)^2} = [(n_1 k_0 \sin\theta_i)^2 - (n_2 k_0)^2]^{\frac{1}{2}}$$

$$E_t = \tau e^{-j\beta_{2x}x} e^{-j\beta_{2z}z} = \tau e^{-j\beta_{2x}x} e^{-\alpha z}$$

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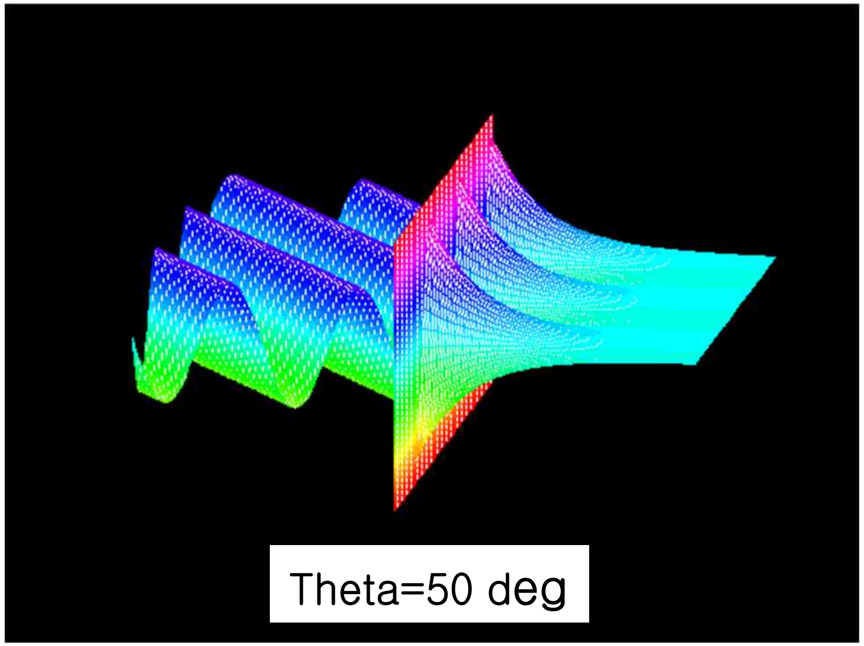
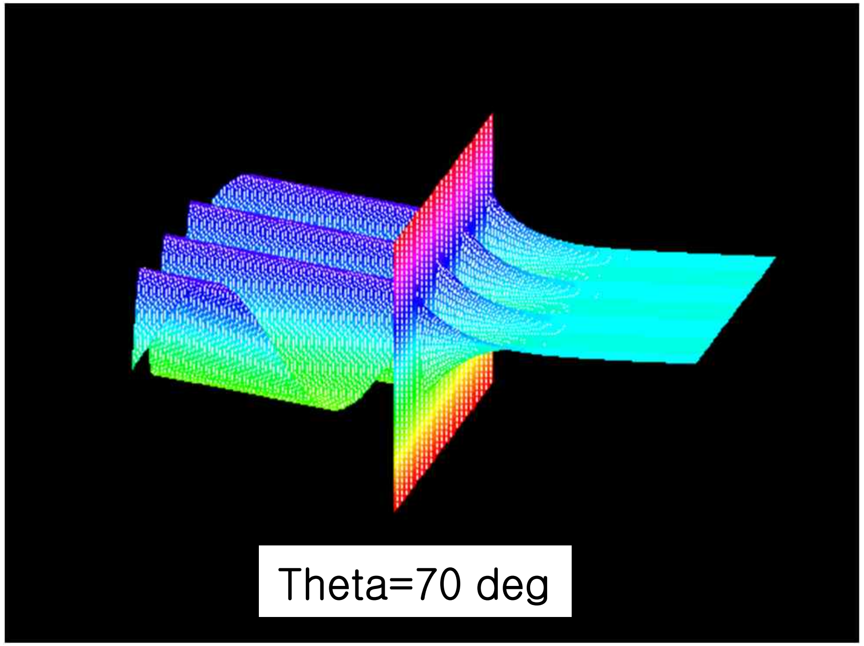
$$E_t \sim e^{-j\beta_{2x}x} e^{-\alpha z} \qquad n_1 = \sqrt{2},\ n_2 = 1 \qquad \theta_c = \sin^{-1}\left(1/\sqrt{2} \right) = 45^\circ$$

$$\alpha = [(n_1 k_0 \sin\theta_i)^2 - (n_2 k_0)^2]^{\frac{1}{2}}$$





Evanescent wave, Surface wave

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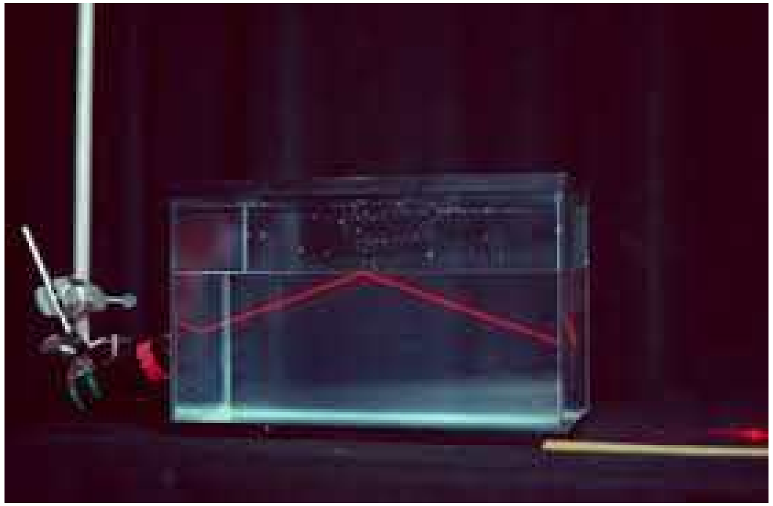

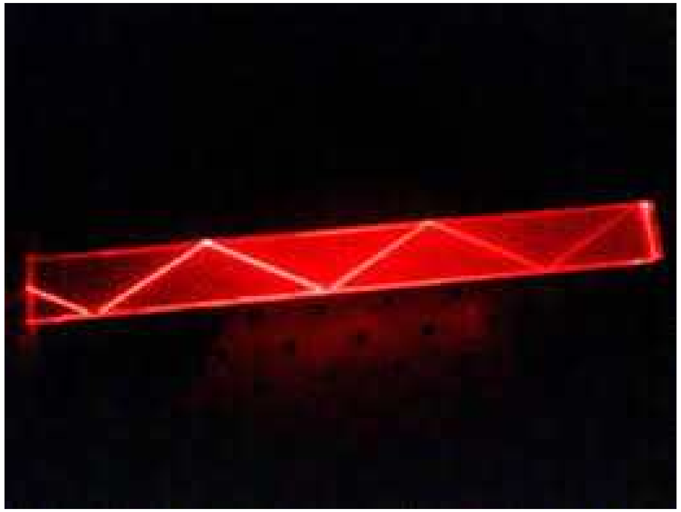

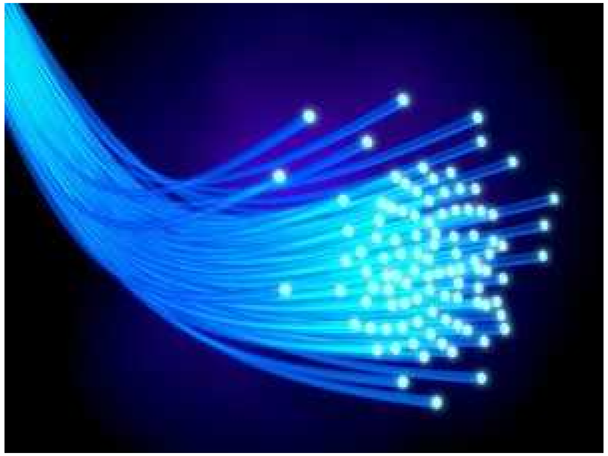

Optical fiber
for optical communication

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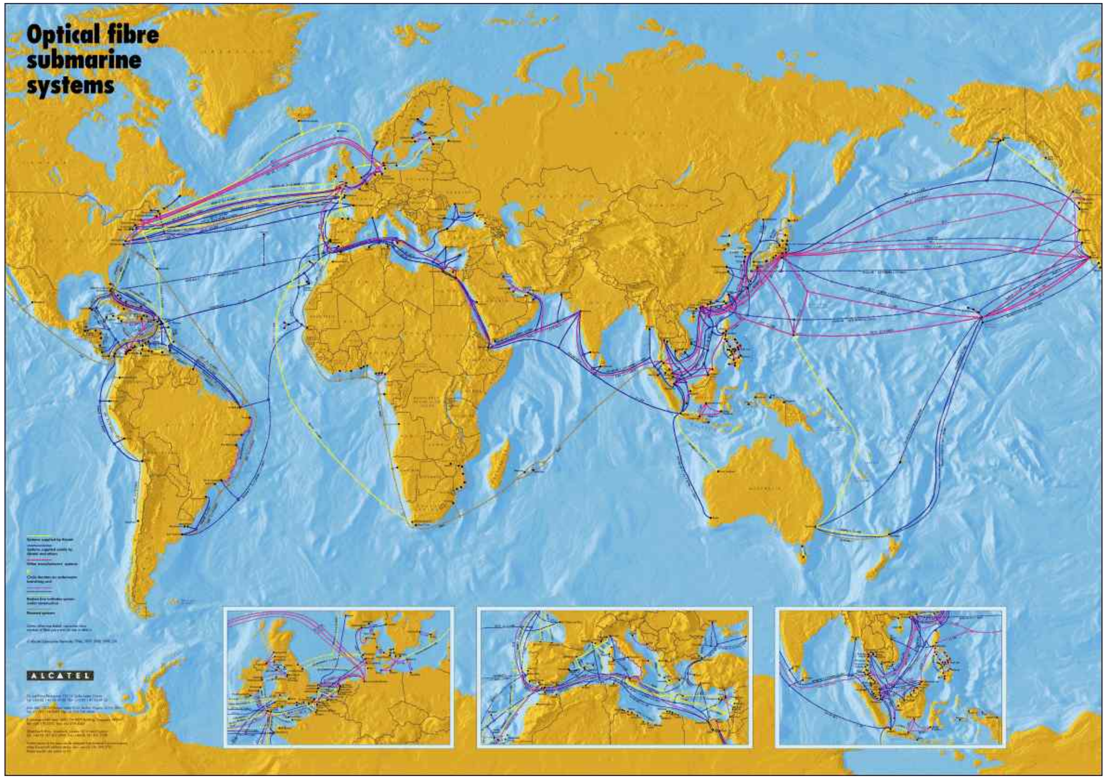
Optical Communication Networks



Total undersea fiber length: 0.4 billion km (628 round trips between earth and moon)

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What is Γ for TIR?

$$\Gamma_{\perp} = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} \qquad \cos^2\theta_t = 1-\sin^2\theta_t = 1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i$$

From Snell's Law: $n_1\sin\theta_i = n_2\sin\theta_t$: $\qquad \therefore \cos\theta_t = \left(1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i\right)^{1/2}$

$$\Gamma_{\perp} = \frac{n_1\cos\theta_i - n_2\left(1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i\right)^{1/2}}{n_1\cos\theta_i + n_2\left(1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_i\right)^{1/2}} = \frac{\cos\theta_i - \left(\frac{n_2^2}{n_1^2}-\sin^2\theta_i\right)^{1/2}}{\cos\theta_i + \left(\frac{n_2^2}{n_1^2}-\sin^2\theta_i\right)^{1/2}}$$

$$= \frac{\cos\theta_i - \left[n^2-\sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2-\sin^2\theta_i\right]^{1/2}} \quad \text{with } n = \frac{n_2}{n_1}$$

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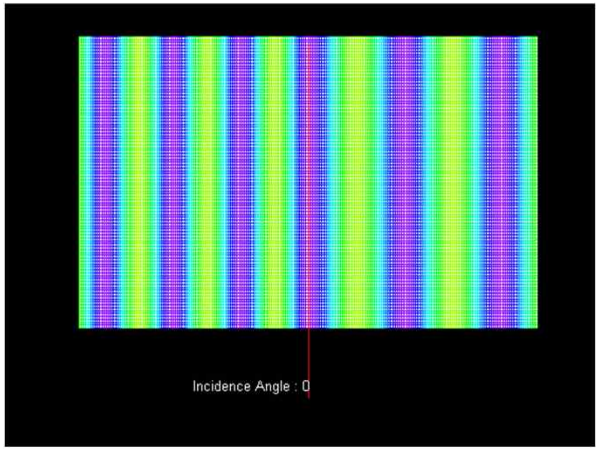
$$\Gamma_{\perp} = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}} \quad \text{with } n = \frac{n_2}{n_1} \qquad \sin\theta_i > \sin\theta_c \text{ for TIR} \qquad \sin\theta_c = \frac{n_2}{n_1} = n$$

$$\text{Let } \left[n^2 - \sin^2\theta_i\right]^{1/2} = -j\left[\sin^2\theta_i - n^2\right]^{1/2} \qquad \Gamma_{\perp} = \frac{\cos\theta_i + j\left[\sin^2\theta_i - n^2\right]^{1/2}}{\cos\theta_i - j\left[\sin^2\theta_i - n^2\right]^{1/2}} = |\Gamma_{\perp}|e^{j\phi_{\perp}}$$

$$|\Gamma_{\perp}| = 1, \qquad \phi_{\perp} = \tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i}\right) - \left(-\tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i}\right)\right) = 2\tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i}\right)$$

For TIR, only phase shift between incident and reflected waves



It can be shown for parallel polarization,

$$\Gamma_{\parallel} = e^{j\phi_{\parallel}}, \quad \phi_{\parallel} = \pi + 2\tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{n^2\cos\theta_i}\right)$$

$\tau_{\perp}, \tau_{\parallel}$ can be determined from $\Gamma_{\perp}, \Gamma_{\parallel}$

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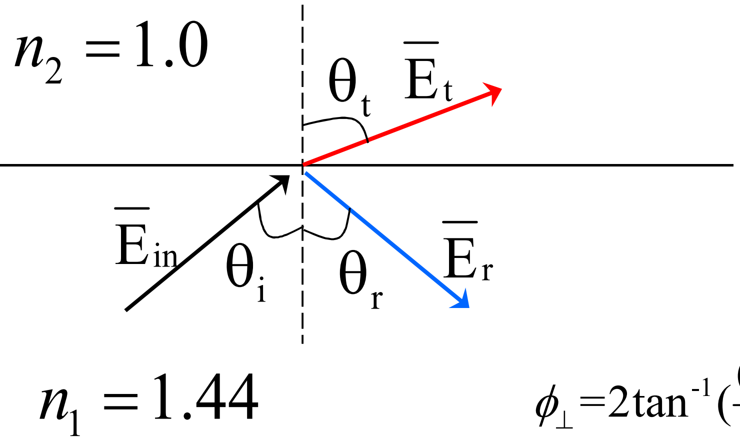
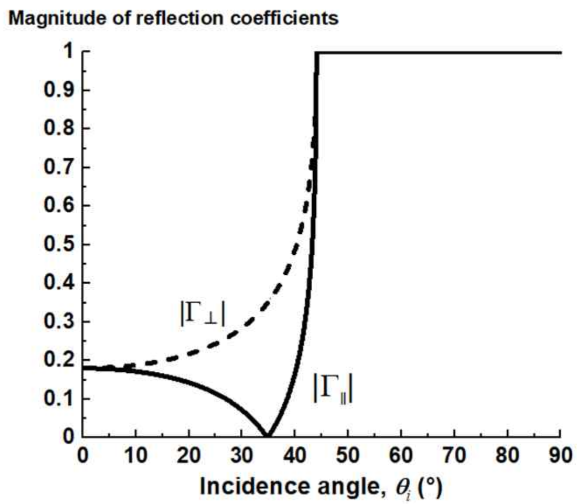


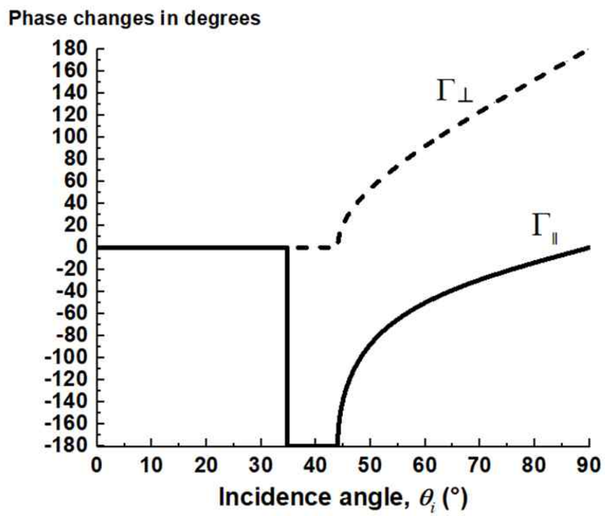
$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) \sim 34.8^\circ$$

$$\theta_C = \sin^{-1}\left(\frac{n_2}{n_1}\right) \sim 44^\circ$$

$$\phi_\perp = 2\tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i}\right) \qquad \phi_\parallel = \pi + 2\tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{n^2\cos\theta_i}\right)$$

$$\left(n = \frac{n_2}{n_1}\right)$$





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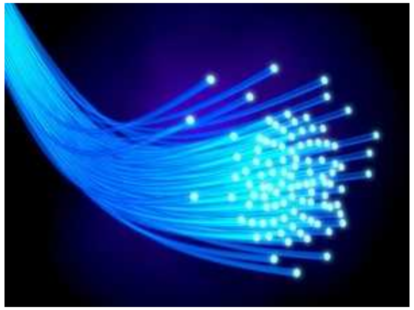
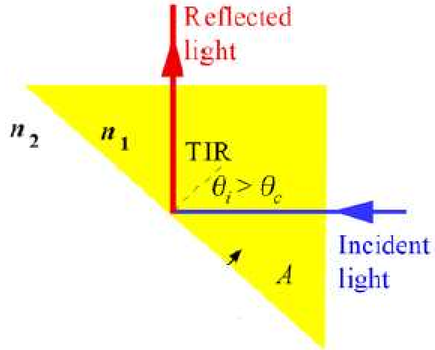
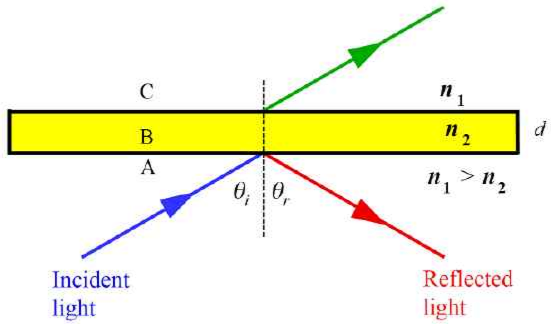
Applications of TIR



Dielectric waveguides (Fiber)



Dielectric Mirror



Optical tunneling ➔ Beam splitter

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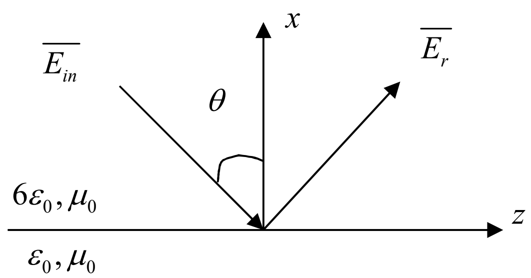
Homework (Due 10/11)

An EM wave whose E-field given as $\underline{E}$ = ($\underline{x}$ sinΘ + $\underline{y}$ + $\underline{z}$ E_z) exp($jk_x x$) exp($-jk_z z$) is obliquely incident on the dielectric interface as shown below.



For the incident angle of 30 degrees, what is the the reflected E-field?