Opto-Electronics and Photonics

Lecture 11 : Interference

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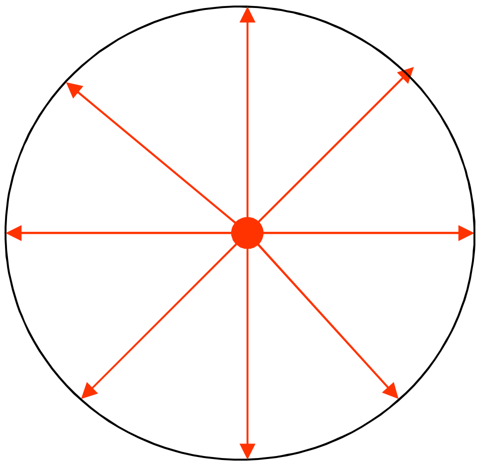
Lecture 11: Interference

Consider isotropic EM wave radiation by a point source.

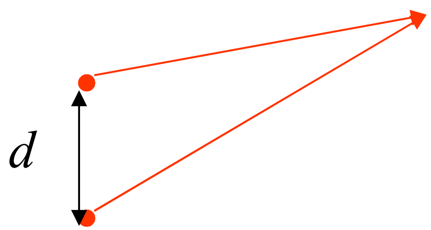


$$E \sim \frac{1}{R} e^{-jkR} \text{ (Spherical wave)}$$

Why $\frac{1}{R}$ dependence?

Because $\int |E|^2 R^2 \sin\theta d\theta d\phi$ should be constant.

Two point sources separated by d



$$E_{\text{total}} = E_1 + E_2$$

(Assuming same E-field direction)

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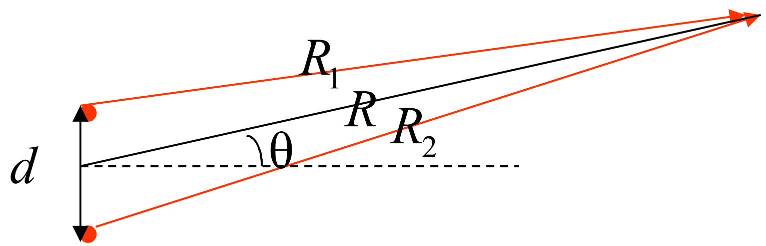

$$E(R,\theta) = E_1 + E_2 = \frac{A}{R_1}e^{-jkR_1} + \frac{A}{R_2}e^{-jkR_2}$$

Assume $R >> d$ $\quad R_1 \approx R - \frac{d}{2}\sin\theta \qquad R_2 \approx R + \frac{d}{2}\sin\theta$

$$\therefore E(R,\theta) \approx \frac{A}{R}e^{-jk(R-\frac{d}{2}\sin\theta)} + \frac{A}{R}e^{-jk(R+\frac{d}{2}\sin\theta)}$$

$$= \frac{A}{R}e^{-jkR}(e^{jk\frac{d}{2}\sin\theta} + e^{-jk\frac{d}{2}\sin\theta}) = \frac{2A}{R}e^{-jkR}\cos(k\frac{d}{2}\sin\theta)$$

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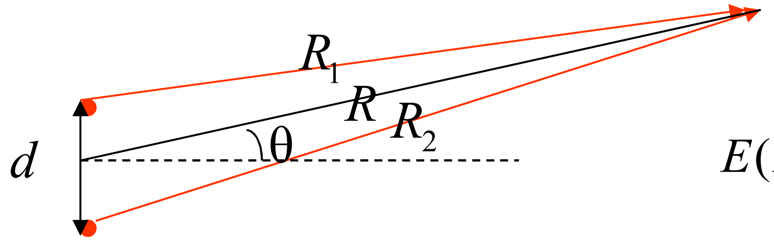

$$E(R,\theta) = \frac{2A}{R} e^{-jkR} \cos(k\frac{d}{2}\sin\theta)$$

$$|E|^2 = 4(\frac{A}{R})^2 \cos^2(k\frac{d}{2}\sin\theta)$$ (Intensity: I) => Max. and min. intensity conditions

For max., $k\frac{d}{2}\sin\theta = m\pi$ $\quad kd\sin\theta = 2m\pi$

For min., $k\frac{d}{2}\sin\theta = (m+\frac{1}{2})\pi$ $\quad kd\sin\theta = (2m+1)\pi$

$kd\sin\theta$: phase difference

In phase

➔ Constructive interference

Out of phase

➔ Destructive interference

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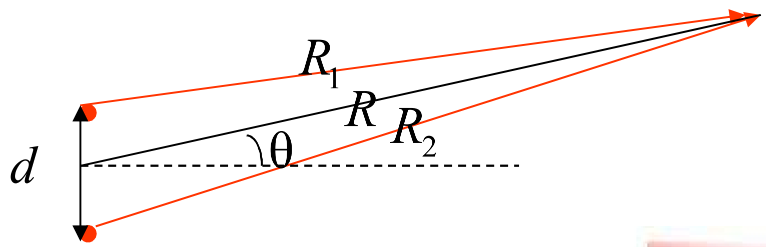
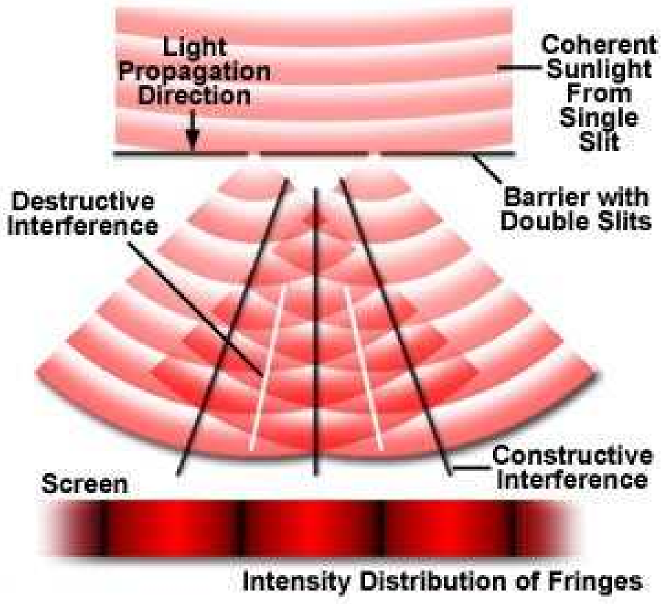
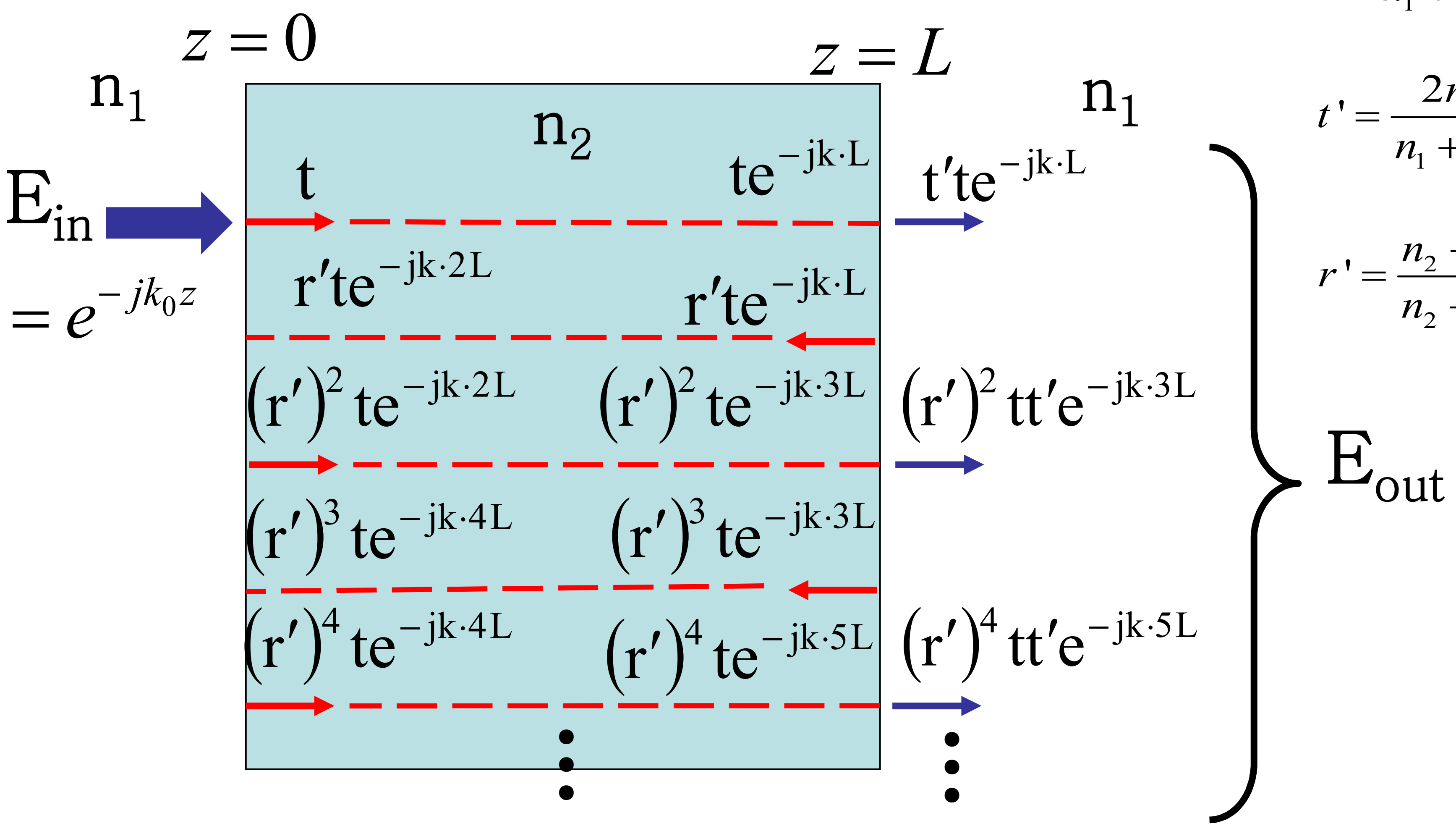


$$I(\text{Intensity}) = |E|^2 = 4\left(\frac{A}{R}\right)^2 \cos^2\left(k\frac{d}{2}\sin\theta\right)$$



Double Slit interference

Interference in a dielectric slab



$$t = \frac{2n_1}{n_1 + n_2}$$

$$t' = \frac{2n_2}{n_1 + n_2}$$

$$r' = \frac{n_2 - n_1}{n_2 + n_1}$$

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$$E_{out} = tt'e^{-jk\cdot L} + \left(r'\right)^2 tt'e^{-jk\cdot 3L} + \left(r'\right)^4 tt'e^{-jk\cdot 5L} + \bullet\bullet\bullet = \frac{tt'e^{-jk\cdot L}}{1-\left(r'\right)^2 e^{-j2kL}}$$

$$T = \frac{\left|E_t\right|^2}{\left|E_i\right|^2} = \frac{(tt')^2}{[1-\left(r'\right)^2 e^{-j2kL}][1-\left(r'\right)^2 e^{j2kL}]} = \frac{(tt')^2}{[1-\left(r'\right)^2]^2 + 4r'^2 \sin^2(kL)}$$

$$[1-(r')^2 e^{-j2kL}]\,[1-(r')^2 e^{j2kL}] = 1-(r')^2 e^{j2kL} - (r')^2 e^{-j2kL} + (r')^4$$

$$= 1-2(r')^2 \cos(2kL) + (r')^4$$

$$= 1-2(r')^2(1-2\sin^2 kL) + (r')^4$$

$$= [1-(r')^2]^2 + 4(r')^2 \sin^2(kL)$$

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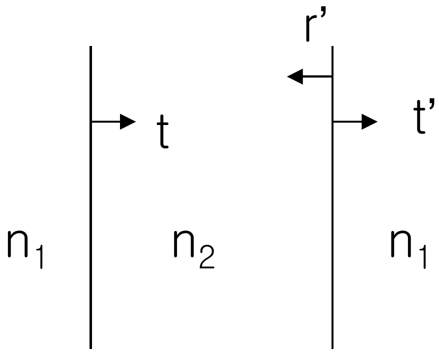
$$T = \frac{|E_t|^2}{|E_i|^2} = \frac{(tt')^2}{[1-(r')^2]^2 + 4r'^2 \sin^2(kL)}$$

$$t = \frac{2n_1}{n_1+n_2}, \quad r' = \frac{n_2-n_1}{n_2+n_1}, \quad t' = \frac{2n_2}{n_1+n_2}$$



$$tt' = \frac{4n_1n_2}{(n_1+n_2)^2}, \quad 1-r'^2 = \frac{4n_1n_2}{(n_2+n_1)^2}$$

Let $R = r'^2$ $\quad T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$

Reflected Intensity Ratio: $\frac{4R\sin^2(kL)}{(1-R)^2 + 4R\sin^2(kL)}$

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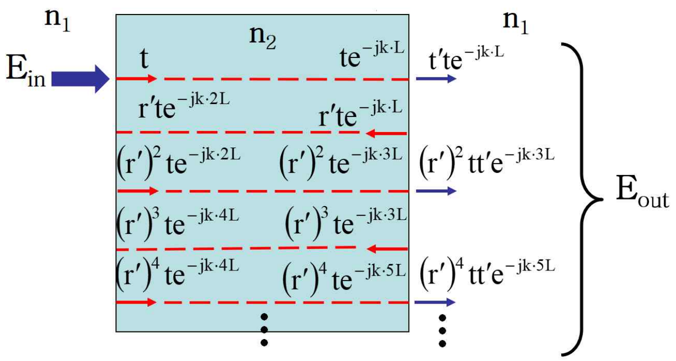
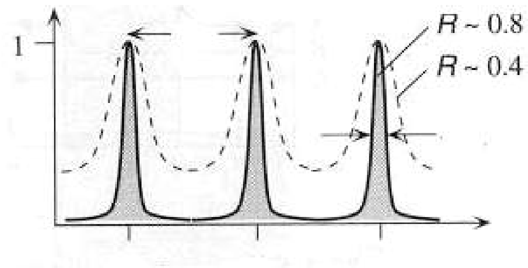


$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$



Filter

Max. Transmission: $\sin(kL) = 0$ => $T = 1$ (resonance)

$$kL = m\pi;\ n_2 \frac{2\pi}{\lambda} L = m\pi \ \Rightarrow L = m\frac{\lambda}{2n_2}$$ (half wavelength condition)

Min. Transmission: $\sin(kL) = 1$

$$kL = (m+\frac{1}{2})\pi;\ n_2 \frac{2\pi}{\lambda} L = (m+\frac{1}{2})\pi \ \Rightarrow L = \frac{\lambda}{2n_2}(m+\frac{1}{2}) = \frac{\lambda}{n_2}(\frac{2m+1}{4})$$

(quarter wavelength condition)

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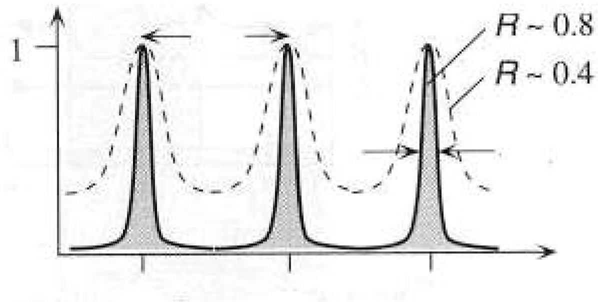
$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$



Period? ➔ Free Spectral Range

$\Delta kL = \pi$ 2π phase shift for round trip => $\Delta k = \dfrac{\pi}{L}$

$\Delta\omega = ?$ Since $k = n_2 \dfrac{\omega}{c}$, $\Delta\omega = \dfrac{c}{n_2}\Delta k = \dfrac{c}{n_2}\dfrac{\pi}{L}$ $\Delta f = \dfrac{c}{2n_2 L} = \dfrac{1}{T}$;

$T = \dfrac{2L}{c/n_2}$; round-trip time

$\Delta\lambda = ?$ $\lambda = n_2 \dfrac{2\pi}{k}$ $\Delta\lambda \sim \dfrac{d\lambda}{dk}\Delta k = -n_2 \dfrac{2\pi}{k^2}\Delta k = -\dfrac{\lambda^2}{2n_2 L}$

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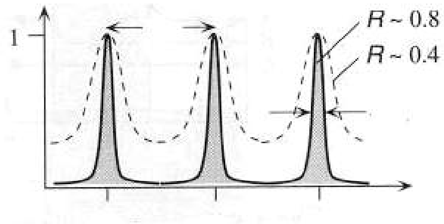
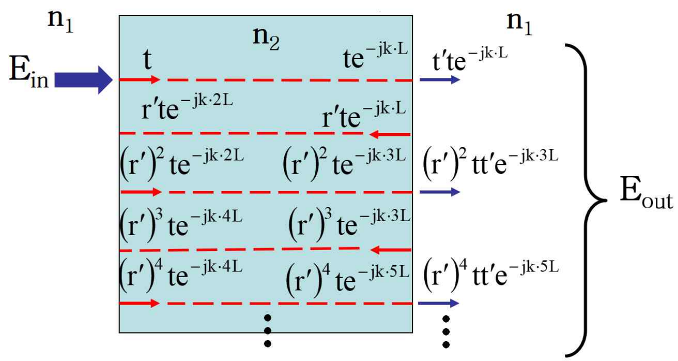
$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$





Sharpness?

Determine peak width for T=0.5

FWHM (Full Width at Half Maximum):

$$\frac{1}{2} = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(\Delta kL)} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(\Delta kL)} \qquad \frac{4R}{(1-R)^2}\sin^2(\Delta kL) = 1$$

$$\Delta kL = \sin^{-1}\sqrt{\frac{(1-R)^2}{4R}} = \sin^{-1}\frac{(1-R)}{2\sqrt{R}}$$

FWHM $\quad 2\sin^{-1}\frac{(1-R)}{2\sqrt{R}} \sim \frac{(1-R)}{\sqrt{R}}$

Usually R is close to 1 for practical applications

As R increases, FWHM decreases => Larger Q-factor

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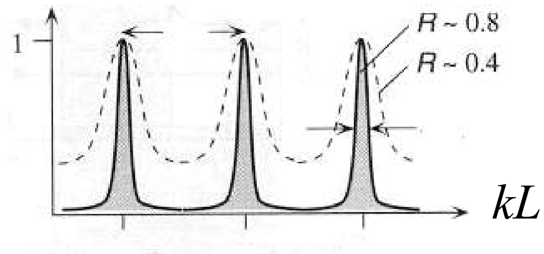
$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$



Finesse: FSR / FWHM

FSR: π

FWHM: $\dfrac{(1-R)}{\sqrt{R}}$

Finesse: $\dfrac{\pi\sqrt{R}}{(1-R)}$

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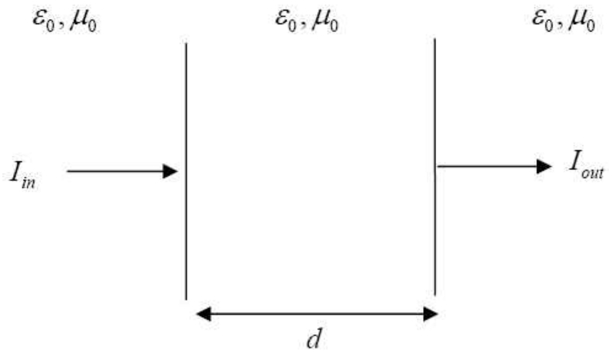
Homework
(Due on 10/11)

Consider a Fabry-Perot interferometer made of two identical partially reflecting/transmitting mirrors as shown below. The mirrors transmit and reflect half of the incident power. (Or it has r = 1/sqrt(2) and t = j/sqrt(2)).



(a) What is I_{out}/I_{in}? Give your answer as a function of sin(kd), where k is the wavenumber in the vacuum and d is the distance between two mirrors.

(b) If the output power is plotted as a function of the frequency of the input light, what is the frequency separation between two adjacent peaks? Express your answer interms of c, speed of light, d, and other fundamental parameters if required.

(c) What is the finesse of this interferometer? Give a numerical answer.

[Finesse = (Free Spectral Range)/FWHM]