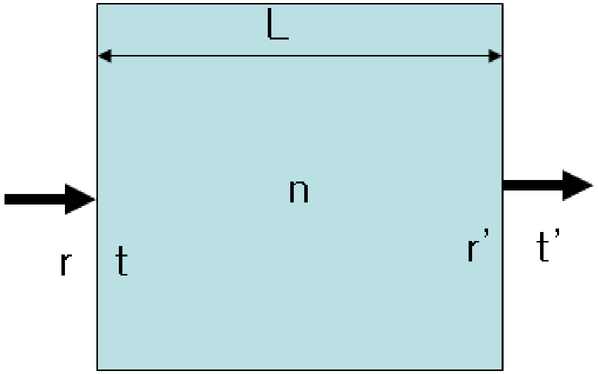
Opto-Electronics and Photonics

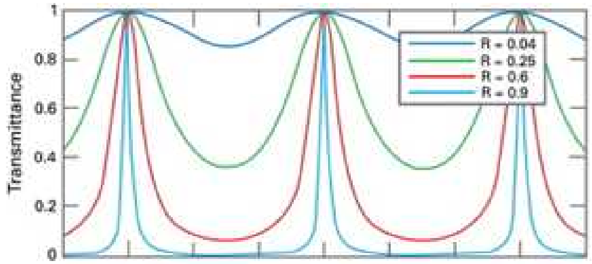
Lecture 12 : Interferometers

Woo-Young Choi

Dept. of Electrical and Electronic Engineering
Yonsei University

Lecture 12: Interferometers





$$T = \frac{I_{out}}{I_{in}} = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)} \quad (R = r'^2)$$

Fabry-Perot Interferometer

Charles Fabry
(1867-1945)

Alfred Perot
(1863-1925)

FP Interferometer: two parallel reflectors

Accurate determination of L or λ

➔ Precision instrumentation, Laser cavity

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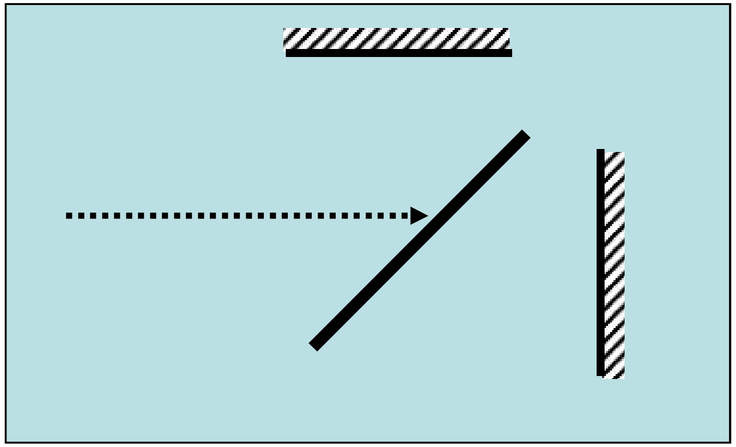
Michelson Interferometer

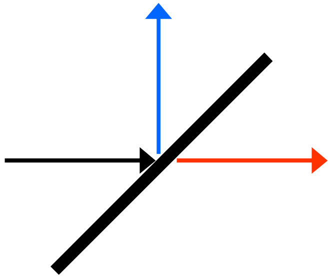


Two mirrors and one beam splitter

Albert Michelson
(1852~1931)
Nobel Prize in Physics in 1907

Beam Splitter:

Partially reflecting/transmitting mirror



3-dB beam splitter

$$R=\frac{1}{2},\quad T=\frac{1}{2}$$

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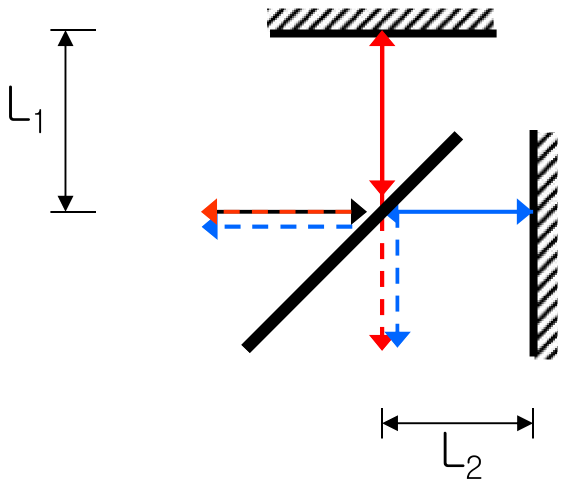
Michelson Interferometer:



Four outputs:

Side	1:	r	r
	2:	t	t
Bottom	1:	r	t
	2:	t	r

Assuming $E_{in} = 1$

$$E_{out,side} = re^{-jkl_1}(-1)e^{-jkl_1}r + te^{-jkl_2}(-1)e^{-jkl_2}t = -r^2e^{-j2kl_1} - t^2e^{-j2kl_2}$$

$$E_{out,bottom} = re^{-jkl_1}(-1)e^{-jkl_1}t + te^{-jkl_2}(-1)e^{-jkl_2}r = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

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$$E_{out,side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2} \qquad E_{out,bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

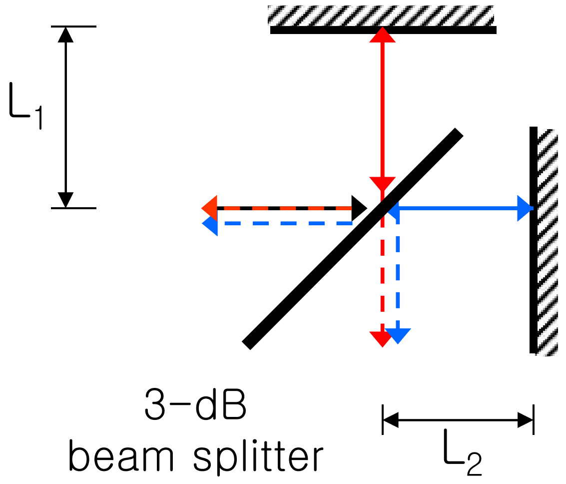
With $r = t = \dfrac{1}{\sqrt{2}}$,

$$E_{out,side} = -\frac{1}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right) = -\frac{1}{2}e^{-jk(l_1+l_2)}\left(\mathrm{e}^{-jk(l_1-l_2)} + \mathrm{e}^{jk(l_1-l_2)}\right)$$

$$I_{out,side} = \left|E_{out,side}\right|^2 = \cos^2[k(l_1 - l_2)]$$

$$E_{out,bottom} = -\frac{1}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right)$$
$$= -\frac{1}{2}e^{-jk(l_1+l_2)}\left(\mathrm{e}^{-jk(l_1-l_2)} + \mathrm{e}^{jk(l_1-l_2)}\right)$$

$$I_{out,bottom} = \left|E_{out,bottom}\right|^2 = \cos^2[k(l_1 - l_2)]$$



Against energy conservation!

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With $r = \frac{1}{\sqrt{2}},\ t = j\frac{1}{\sqrt{2}}$

(r and t should have $\pi/2$ phase difference in order to satisfy energy conservation)

$$E_{out,\,side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2}$$

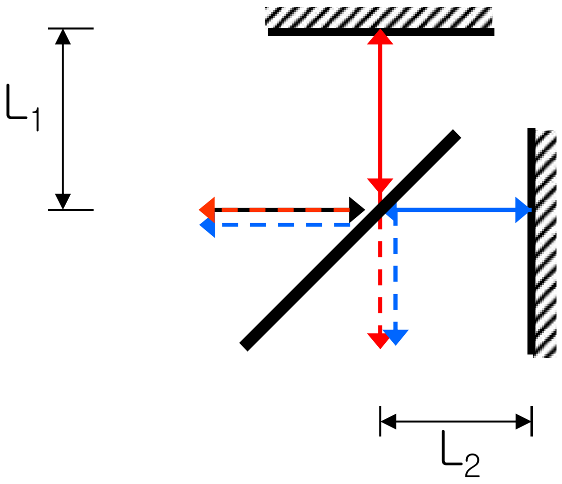
$$= -\frac{1}{2}e^{-j2kl_1} + \frac{1}{2}e^{-j2kl_2}$$

$$I_{out,\,side} = \left|E_{out,\,side}\right|^2 = \sin^2[k(l_1 - l_2)]$$

$$E_{out,\,bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

$$= -\frac{j}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right)$$

$$I_{out,\,bottom} = \left|E_{out,\,bottom}\right|^2 = \cos^2[k(l_1 - l_2)]$$



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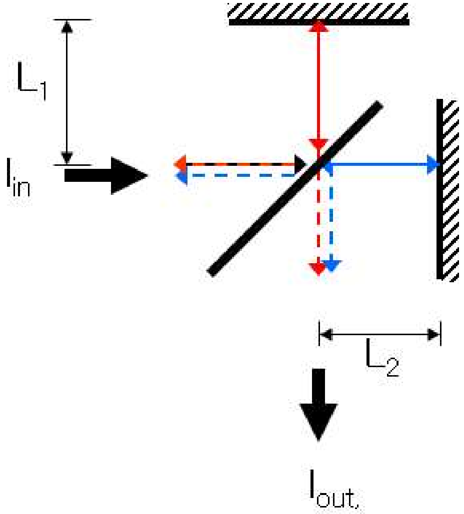
Michelson Interferometer

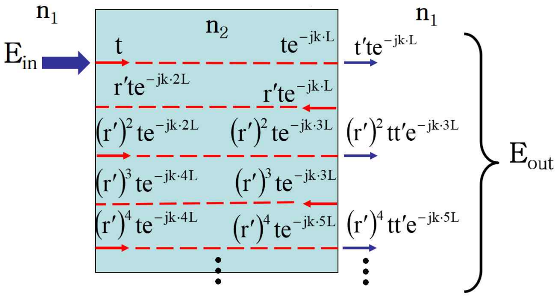


$$\frac{I_{out,\,bottom}}{I_{in}} = \cos^2[k(l_1 - l_2)]$$

FSR: $\Delta k(l_1 - l_2) = \pi$

FWHM: $\Delta k(l_1 - l_2) = \dfrac{\pi}{2}$

Finesse: 2



Finesse: $\dfrac{\pi\sqrt{R}}{(1-R)}$

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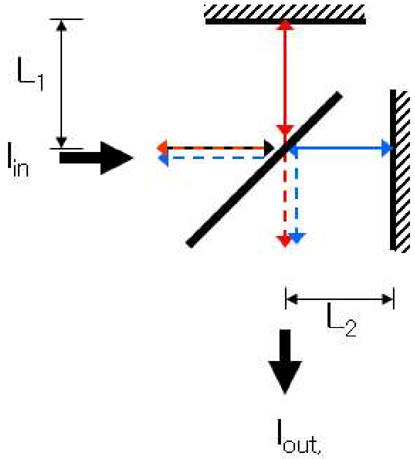
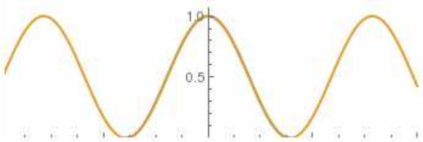
Michelson Interferometer



$$\frac{I_{out,bottom}}{I_{in}} = \cos^2[k(l_1 - l_2)]$$

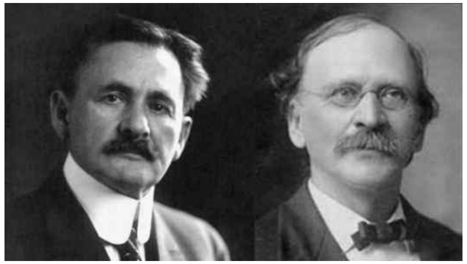
$k(l_1 - l_2)$ can be measured very precisely

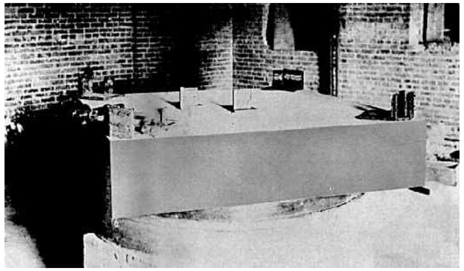
Assume $l_1 = l_2$ initially, l_2 changes Δl



Resolution in Δl: $k\Delta l = \pi$ $\quad \Delta l = \frac{\lambda}{2}$

Michelson-Morley experiment (1887)





Looking for aether

Determined speed of light
always the same

➔ Einstein's special relativity

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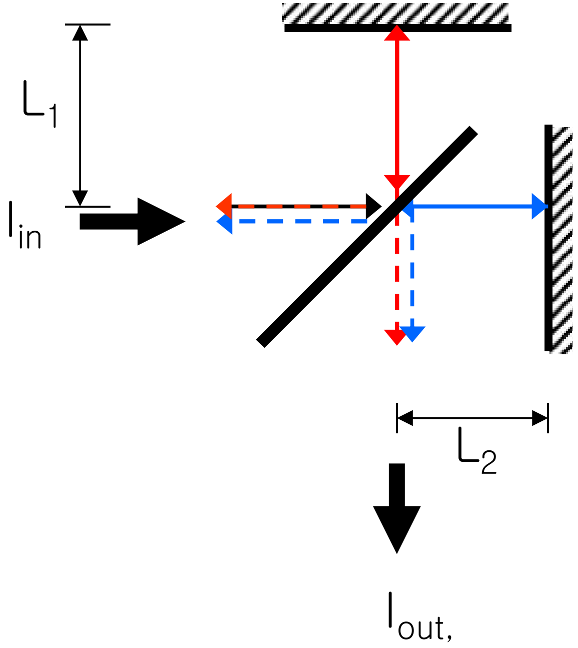
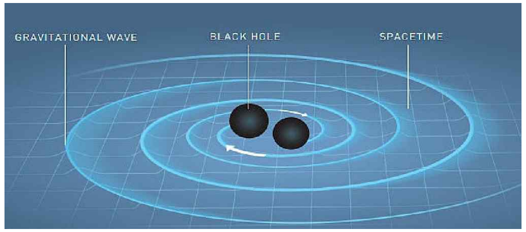
Michelson Interferometer

$$\frac{I_{out,\,bottom}}{I_{in}} = \cos^2[k(l_1 - l_2)]$$

$k(l_1 - l_2)$ can be measured very precisely





LIGO : Laser Interferometer Gravitational-wave Observatory

Einstein's General Relativity: Gravity changes space

2017 Nobel Prize in Physics

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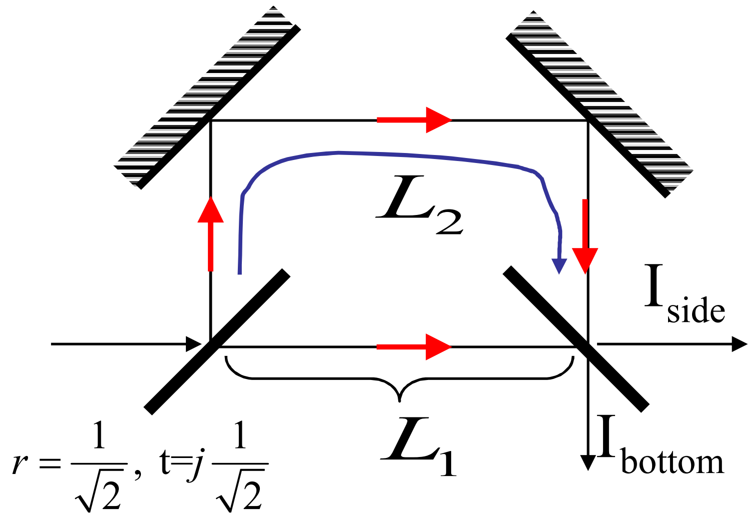
Mach-Zehnder Interferometer:



Ludwig Mach (1879~1951):
Austrian, Son of Ernst Mach

Ludwig Zehnder (1854~1949):
Swiss, Student of Röntgen

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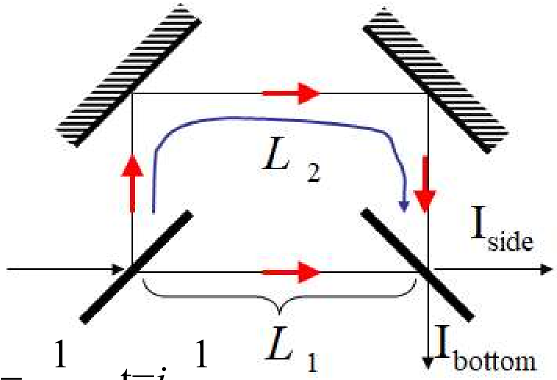
Mach-Zehnder Interferometer



$r=\frac{1}{\sqrt{2}}$, t$=j\frac{1}{\sqrt{2}}$

Assuming $E_{in}=1$,

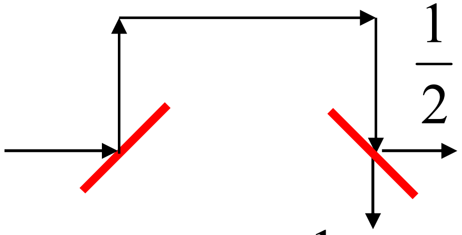
$$E_{out,\,side}=\frac{1}{2}\left(e^{-jkl_2}-e^{-jkl_1}\right)=\frac{1}{2}e^{-jk\frac{l_2+l_1}{2}}\left(\mathrm{e}^{-jk\frac{l_2-l_1}{2}}-\mathrm{e}^{jk\frac{l_2-l_1}{2}}\right)$$

$$I_{out,\,side}=\sin^2\left(k\frac{l_1-l_2}{2}\right)$$

$$E_{out,\,bottom}=\frac{j}{2}\left(e^{-jkl_1}+e^{-jkl_2}\right)=\frac{j}{2}e^{-jk\frac{l_1+l_2}{2}}\left(e^{-jk\frac{l_1-l_2}{2}}+e^{jk\frac{l_1-l_2}{2}}\right)$$

$$I_{out,\,bottom}=\cos^2\left(k\frac{l_1-l_2}{2}\right)$$

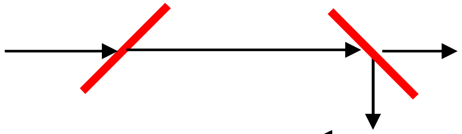
Case#1



$\frac{1}{2}\mathrm{e}^{-\mathrm{jkl}_2}$

$\mathrm{j}\frac{1}{2}\mathrm{e}^{-\mathrm{jkl}_2}$

Case#2



$-\frac{1}{2}\mathrm{e}^{-\mathrm{jkl}_1}$

$j\frac{1}{2}e^{-jkl_1}$

➔ MZ Modulator

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Can any EM wave have interference?

➔ Only EM waves having clear phase relationship experience interference: coherent

How large (l_1-l_2) can be?

➔ As long as two separated waves are coherent or within coherent length
Separated waves become incoherent due to intrinsic phase noises

Lecture 12: Interferometers

Homework (Due on 10/19)
Determine I_{out}/I_{in} for an interferometer shown below. It is made of two beam splitters, two mirrors and a block of material having refractive index n and length l_1 placed in one arm. Assume there is no reflection from this block of material, both beam splitters have r = 1/sqrt(2), t= j 1/sqrt(2), and both mirrors have r = −1.