Opto-Electronics and Photonics

Lecture 13 : Diffraction

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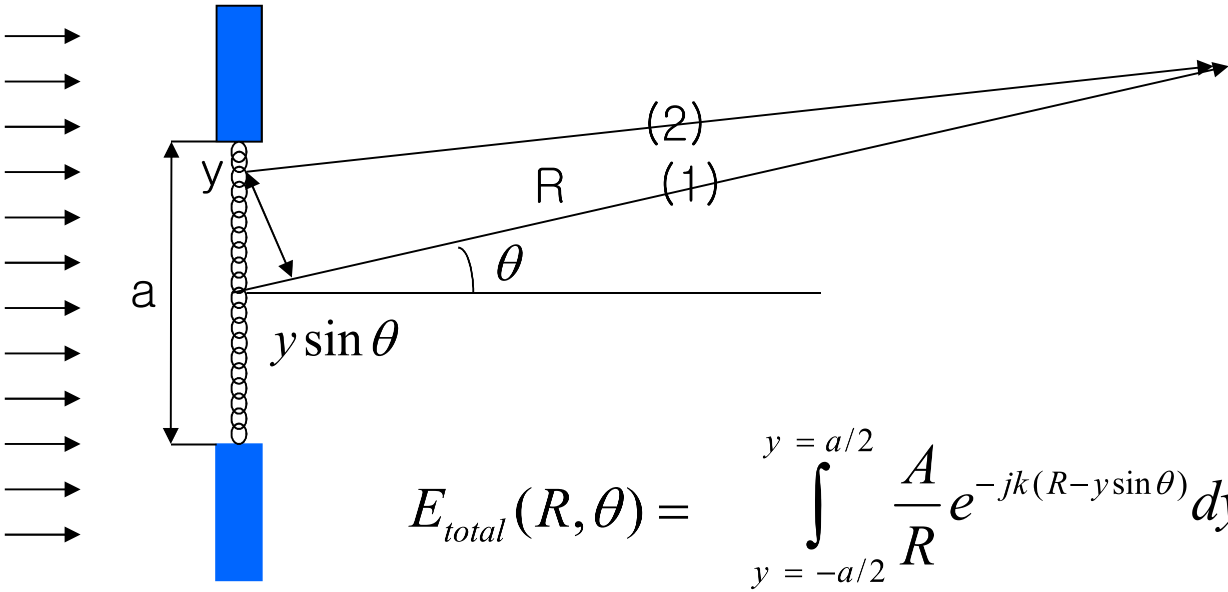
Lecture 13: Diffraction



(1) $\frac{A}{R}\exp(-jkR)$

(2) $\frac{A}{R}\exp\left[-jk(R-y\sin\theta)\right]$

$$E_{total}(R,\theta) = \int_{y=-a/2}^{y=a/2} \frac{A}{R} e^{-jk(R-y\sin\theta)} dy = \frac{A}{R} e^{-jkR} \int_{y=-a/2}^{y=a/2} e^{jky\sin\theta} dy$$

Since interference is due to ***phase difference***, ignore the constant phase term

$$E_{total}(R,\theta) = \frac{A}{R} \int_{y=-a/2}^{y=a/2} e^{jky\sin\theta} dy$$

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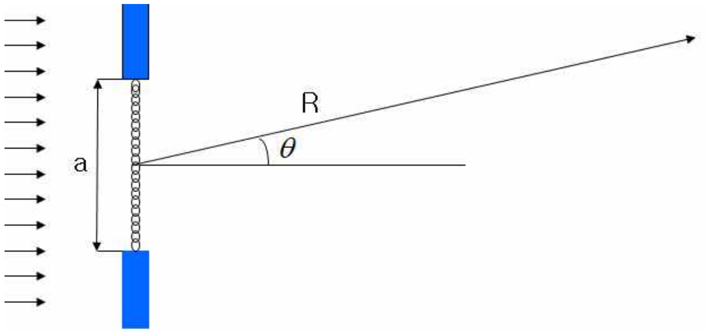

Evaluate $E_{total}(R,\theta) = \frac{A}{R}\int_{-a/2}^{a/2} e^{jky\sin\theta} dy$

Let $y' = jky\sin\theta$ => $dy' = jk\sin\theta dy$

$$e^{jky\sin\theta}dy = e^{y'}\frac{dy'}{jk\sin\theta}$$

$$E_{total}(R,\theta) = \frac{A}{R}\int_{y'=-jk\frac{a}{2}\sin\theta}^{y'=jk\frac{a}{2}\sin\theta} e^{y'}\frac{dy'}{jk\sin\theta} = \frac{A}{R}\frac{1}{jk\sin\theta}\left(e^{jk\frac{a}{2}\sin\theta} - e^{-jk\frac{a}{2}\sin\theta}\right)$$

$$= \frac{A}{R}\frac{2j}{jk\sin\theta}\sin(k\frac{a}{2}\sin\theta) = \frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}$$

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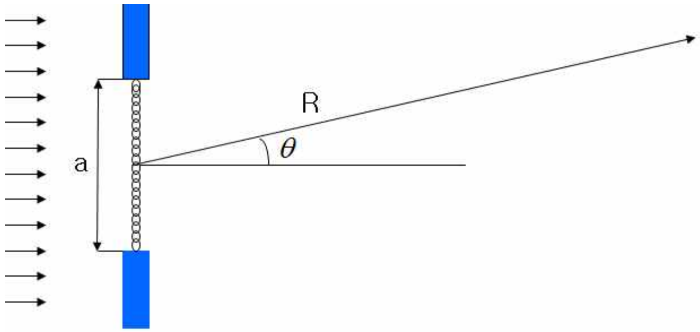

$$E_{total}(R,\theta) = \frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}$$

$$\frac{E_{total}(R,\theta)}{E_{total}(R,0)} = ?$$

$$E_{total}(R,0) = \frac{2A}{R}\frac{\cos(k\frac{a}{2}\sin\theta)k\frac{a}{2}\cos\theta\Big|_{\theta=0}}{k\cos\theta\big|_{\theta=0}} = \frac{2A}{R}\frac{a}{2}$$

$$\therefore \frac{E_{total}(\theta)}{E_{total}(0)} = \frac{\frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}}{\frac{2A}{R}\frac{a}{2}} = \frac{\sin(k\frac{a}{2}\sin\theta)}{k\frac{a}{2}\sin\theta} = \frac{\sin(k_y\frac{a}{2})}{k_y\frac{a}{2}} \qquad k_y = k\sin\theta$$

➔ sinc function

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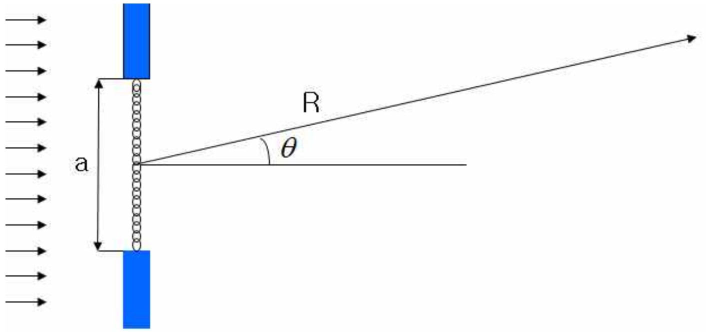
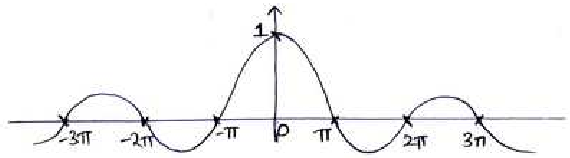
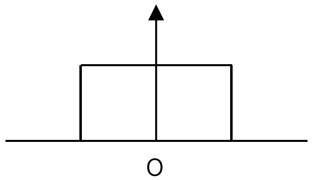


$$\frac{E_{total}(\theta)}{E_{total}(0)} = \frac{\sin(k_y \frac{a}{2})}{k_y \frac{a}{2}}$$

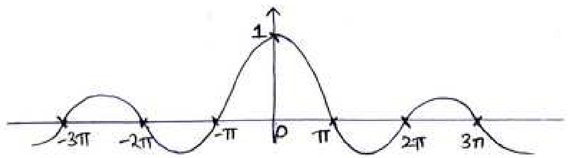


Source



FT relationship

Diffraction



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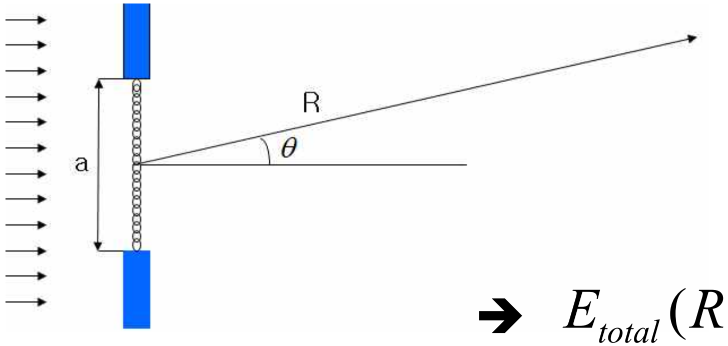

$$E_{total}(R,\theta) = \frac{A}{R}\int_{y=-a/2}^{y=a/2} e^{jky\sin\theta}dy$$

$$\Rightarrow \; E_{total}(R,k_y) = \frac{1}{R}\int_{y=-\infty}^{y=\infty} A(y)e^{jk_y y}dy \qquad \left(k_y = k\sin\theta\right)$$

From Signals and Systems $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

$$\mathrm{f}(t) \quad <=> \quad F(\omega)$$

$$E_{total}(k_y) <=> A(y)$$

Diffraction of A(y) is F.T. of *A(y)*

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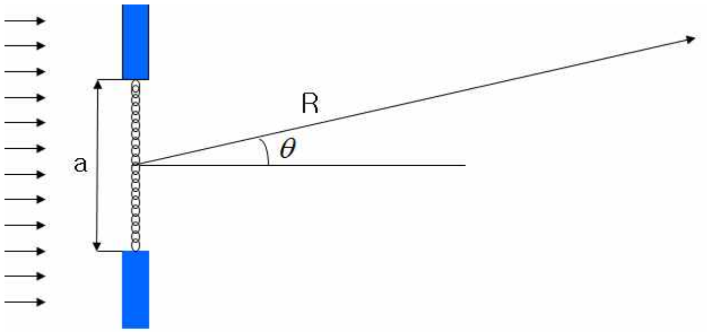

$$E_{total}(R, k_y) = \frac{1}{R} \int_{y=-\infty}^{y=\infty} A(y) e^{jk_y y} dy$$

Far-field diffraction

Fraunhoffer Diffraction

Jeseph Ritter von Fraunhoffer

(1787-1826)

German physicist

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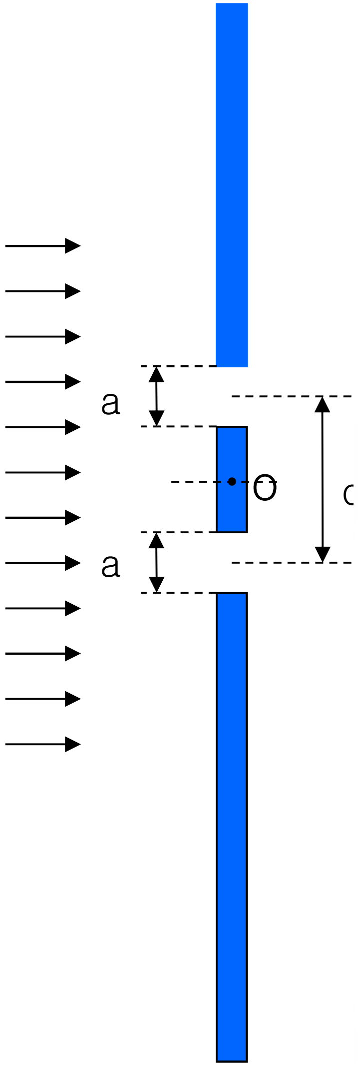

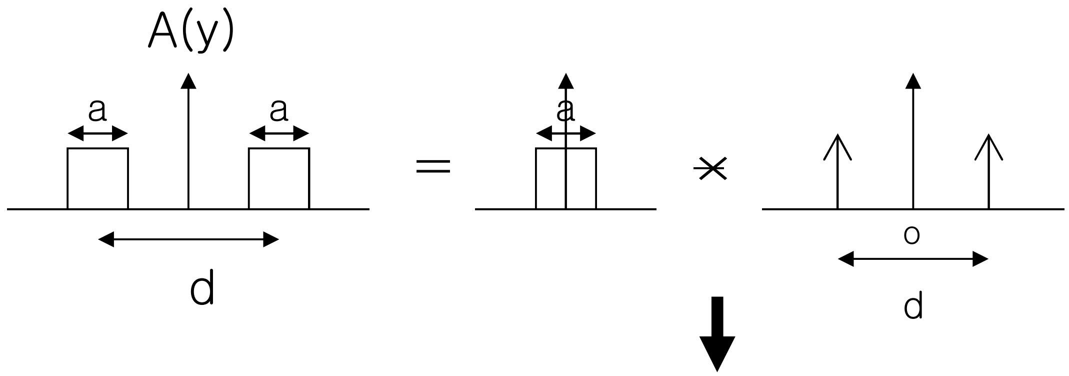

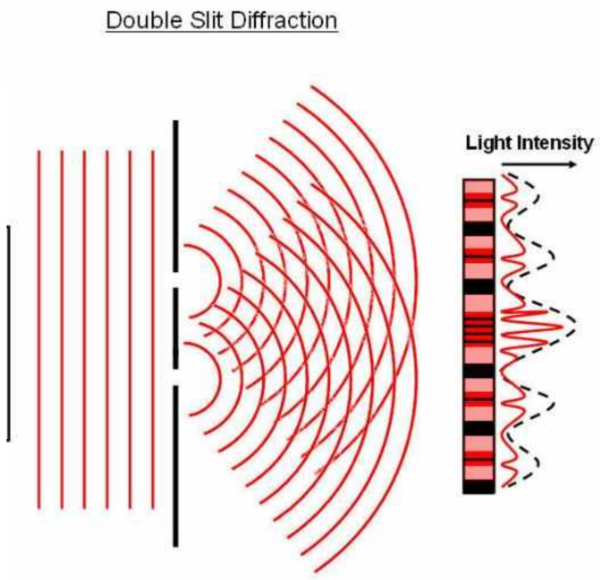

$$\frac{\sin(k_y \frac{a}{2})}{k_y \frac{a}{2}} \quad \text{X} \quad e^{jk_y \frac{d}{2}} + e^{-jk_y \frac{d}{2}} = 2\cos(k_y \frac{d}{2})$$

$$\frac{E(k_y)}{E(0)} = 2\cos(k_y \frac{d}{2}) \frac{\sin(\frac{k_y a}{2})}{\frac{k_y a}{2}}$$

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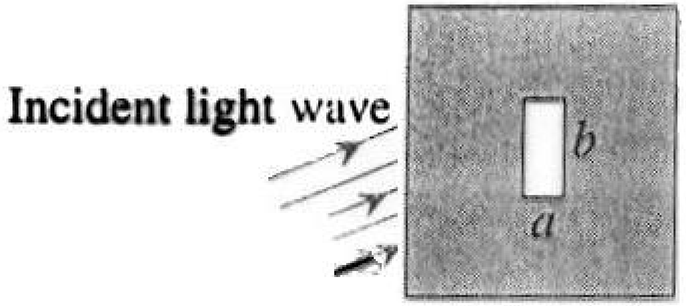

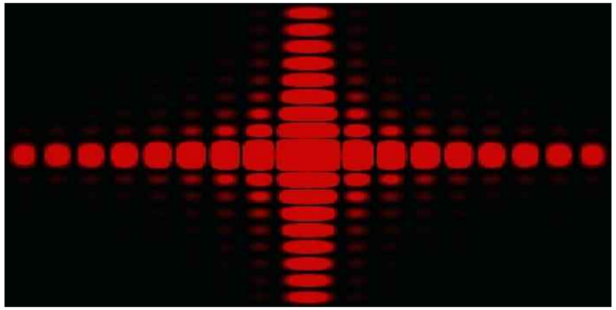

$$E_{total}(k_x, k_y) \sim \int_{y=-\infty}^{y=\infty} \int_{x=-\infty}^{x=\infty} A(x,y) e^{jk_x x} e^{jk_y y} dx dy = \int_{y=-\frac{b}{2}}^{y=\frac{b}{2}} \int_{x=-\frac{a}{2}}^{x=\frac{a}{2}} e^{jk_x x} e^{jk_y y} dx dy$$

$$= \int_{y=--\frac{b}{2}}^{y=\frac{b}{2}} e^{jk_x y} dy \int_{x=-\frac{a}{2}}^{x=\frac{a}{2}} e^{jk_x x} dx$$

$$\frac{E_{total}(k_x, k_y)}{E_{total}(0,0)} = \frac{\sin(k_y \frac{b}{2})}{k_y \frac{b}{2}} \frac{\sin(k_x \frac{a}{2})}{k_x \frac{a}{2}}$$

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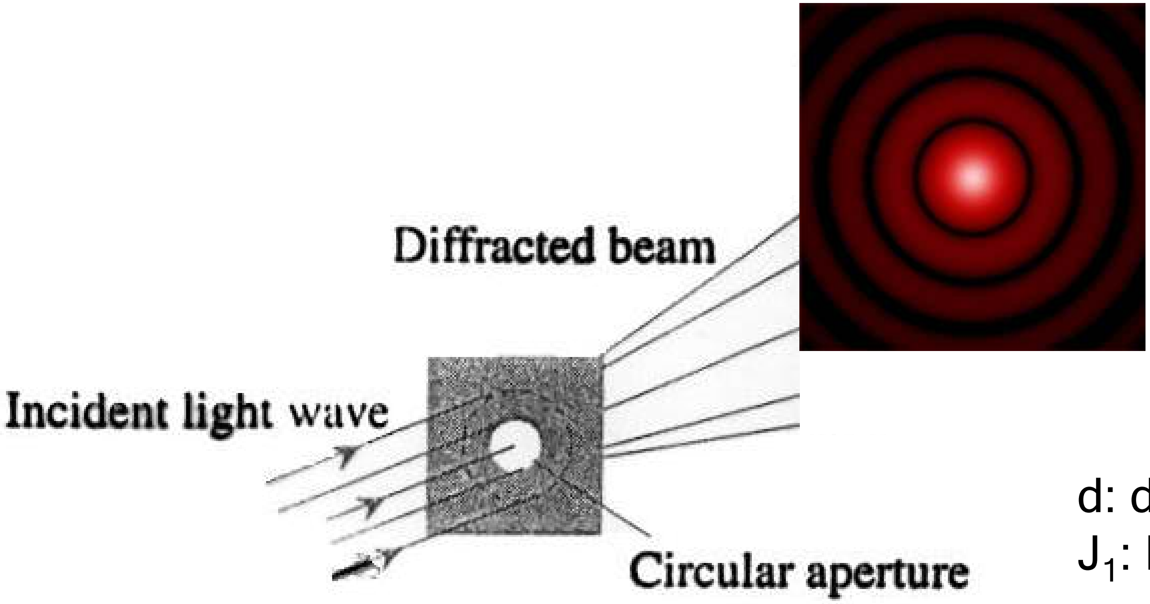

Airy pattern

$$\frac{I(k_y)}{I_0} = \left(\frac{2J_1(k_y d)}{k_y d}\right)^2$$

d: diameter
J_1: Bessel function of first order

For the first dark ring,

$$k_y d \sim 3.83 \quad \frac{2\pi}{\lambda}\sin(\theta)d \sim 3.83 \quad \sin\theta \sim \frac{3.83}{d}\frac{\lambda}{2\pi} \sim 1.22\frac{\lambda}{d}$$

θ determines imaging resolution

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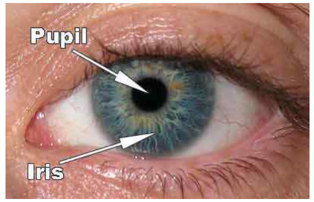
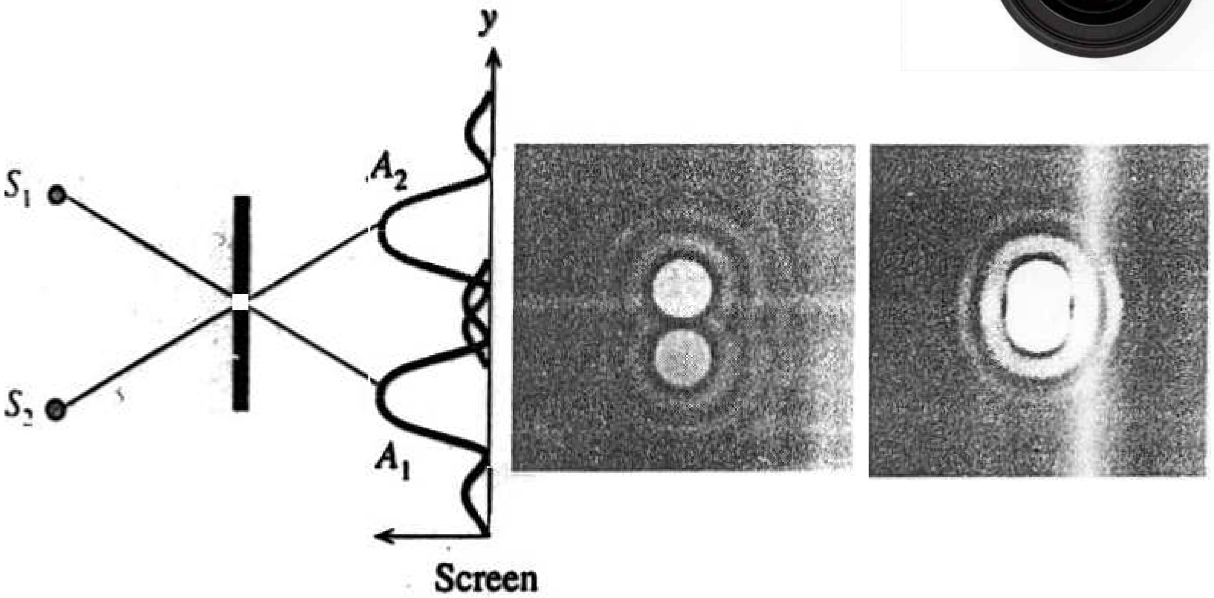
$$\sin\theta \sim 1.22\frac{\lambda}{d}$$

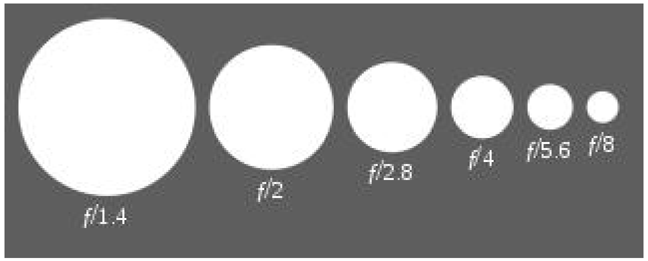
Many imaging systems have circular aperture





Larger d ➔ smaller θ ➔ Better resolution

(But smaller depth of field)



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Gran Telescopio Canarias (GTC)

Aperture diameter: 10.4 m
(World's largest optical telescope)

In Spain

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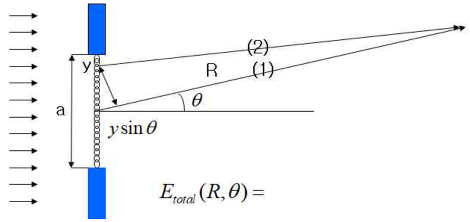
In our analysis,

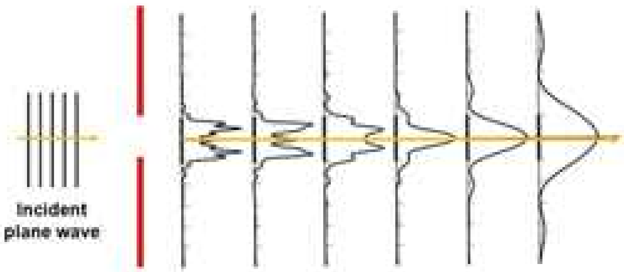
- Diffraction is observed at locations having same R

➔ In reality, observation is made at a flat surface. Consequently, there is additional phase shifts, requiring more complicated analysis



- It is assumed R>>λ (far field)

If not, FT relation cannot be used ➔ Near-field or Fresnel diffraction.



Augustin-Jean Fresnel
(1788~1827)
French Physicist

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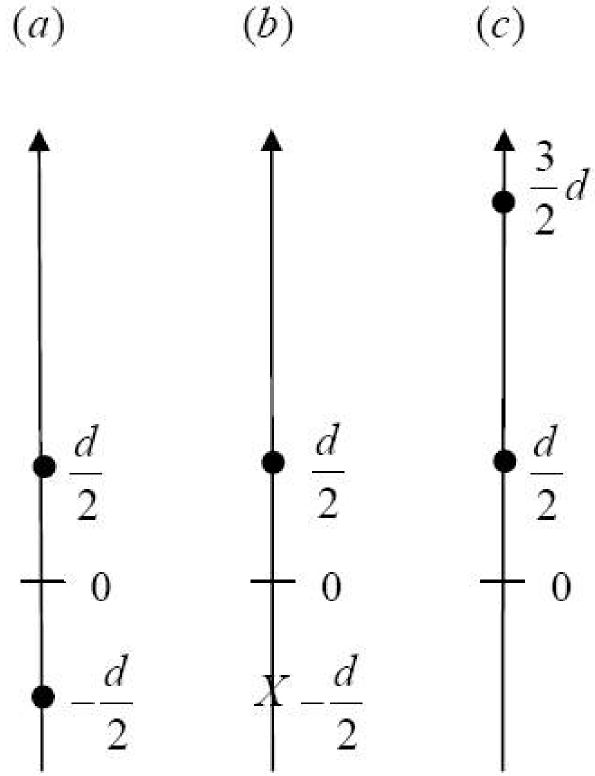
Homework (Due 10/19)

Two point light sources are located as shown below. We are interested in the far-field pattern produced by the interference of these point sources. For each of cases given below, sketch $|E(\Theta)|$ normalized to $|E_{max}|$. For the sketch, use $2\pi\sin(\Theta)/\lambda$ as the x-axis. On the sketch, clearly indicate the locations of the max. and min. magnitudes.

(a) Two source are located near origin and E-fields from two sources are in-phase when they are produced at the source.

(b) Two source are located near origin and E-fields from two sources are out-of-phase when they are produced at the source.

(c) Same as in (a) but the location of sources are shifted by d.



Mid-term on 10~12 am on 10/21 Wed. Open book