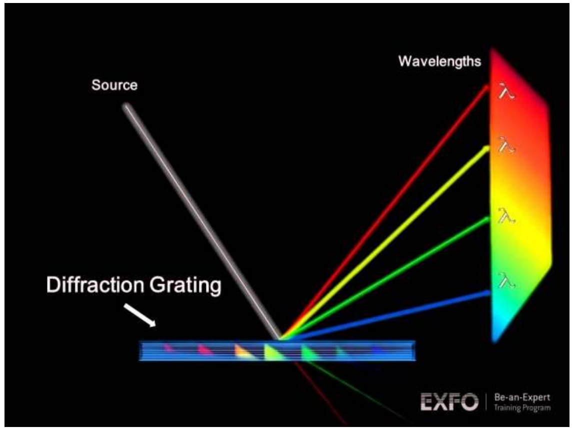
Opto-Electronics and Photonics

Lecture 14 : Diffraction Gratings

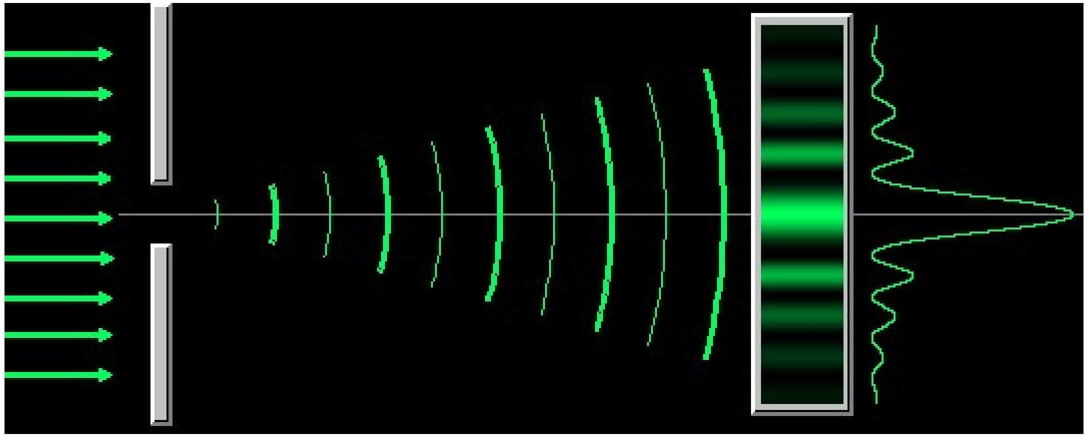
Woo-Young Choi

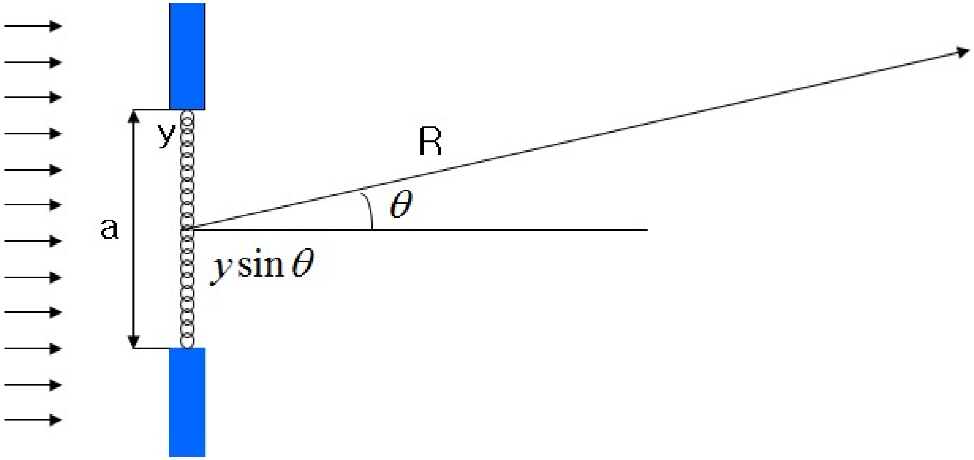
Dept. of Electrical and Electronic Engineering
Yonsei University

Lecture 14: Diffraction Grating



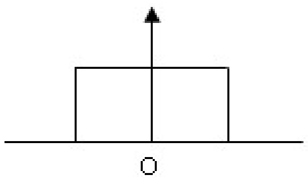
Lecture 14: Diffraction Grating





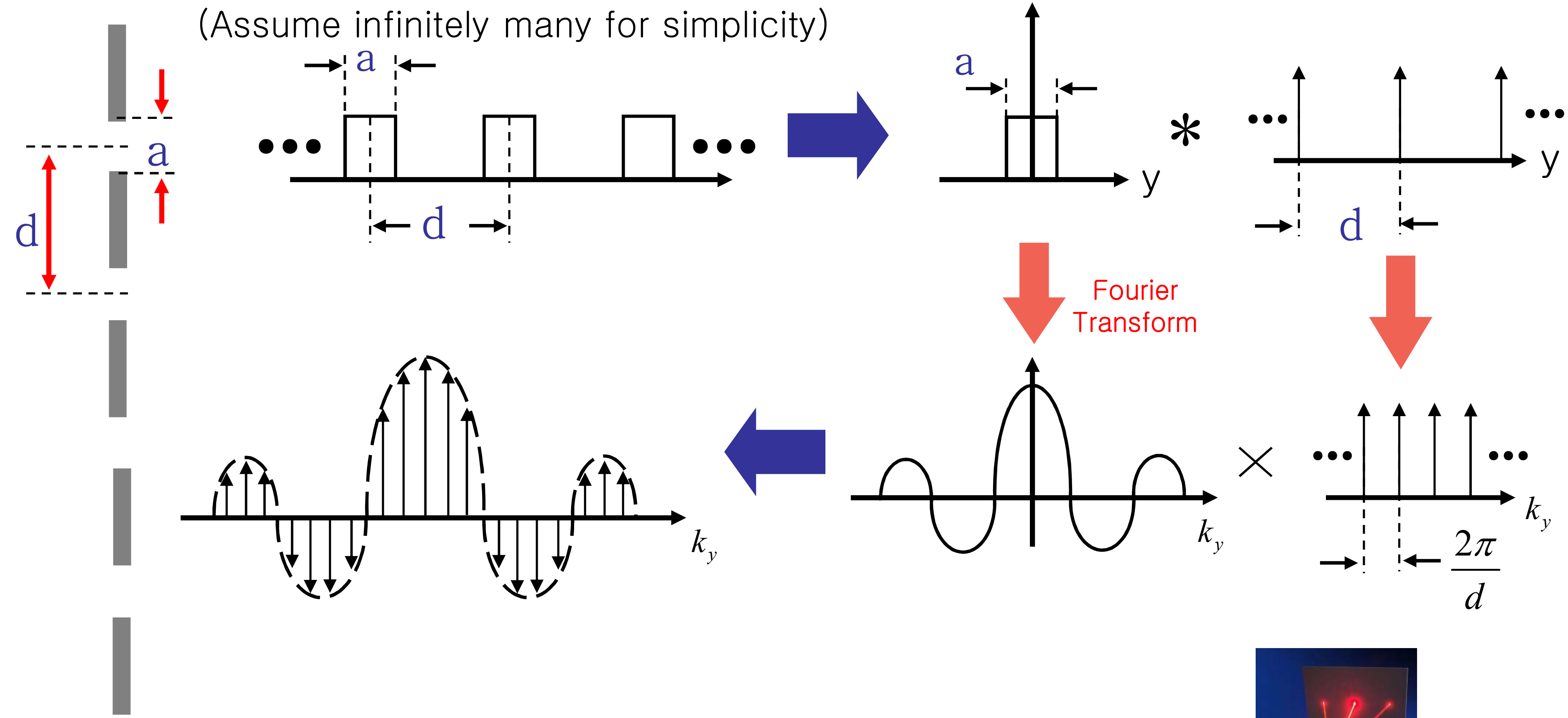
slit shape (y) ←→ far-field (k_y)

Fourier Transform



$$\frac{\sin\left(k_y \frac{a}{2}\right)}{k_y \frac{a}{2}}$$

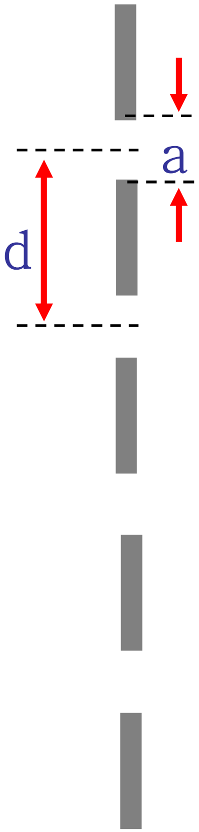
Lecture 14: Diffraction Grating

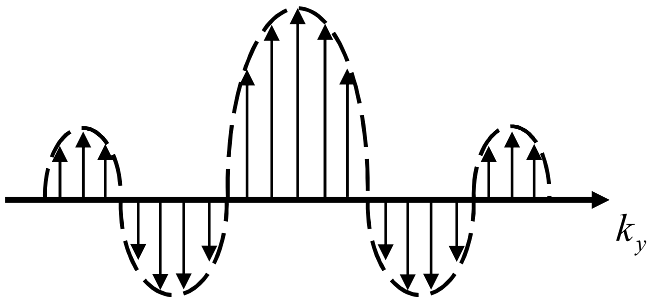


Diffracted light from periodic opening (Diffraction Grating)

==> Far-field only for discrete k_y's $\quad k_y = k\sin\theta = m\frac{2\pi}{d}$

Lecture 14: Diffraction Grating



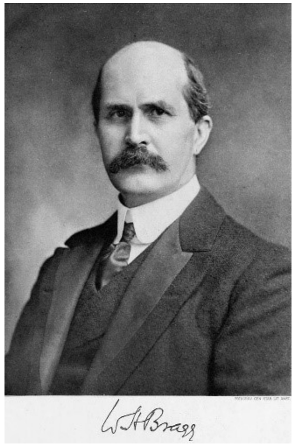


$$k_y = m\frac{2\pi}{d}$$

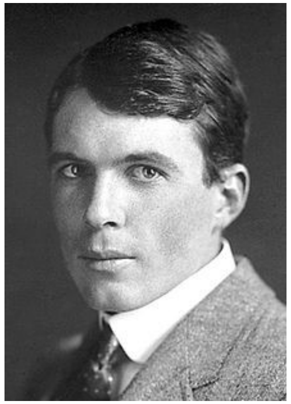
$$\frac{2\pi}{\lambda}\sin\theta = m\frac{2\pi}{d}$$

$$d\sin\theta = m\lambda$$

Grating equation (Bragg Condition)



William Henry Bragg
(1862-1942)



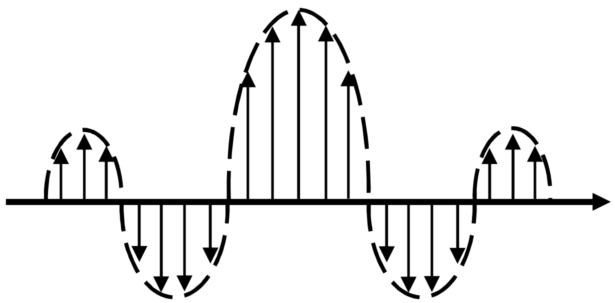
William Lawrence Bragg
(1890-1971)

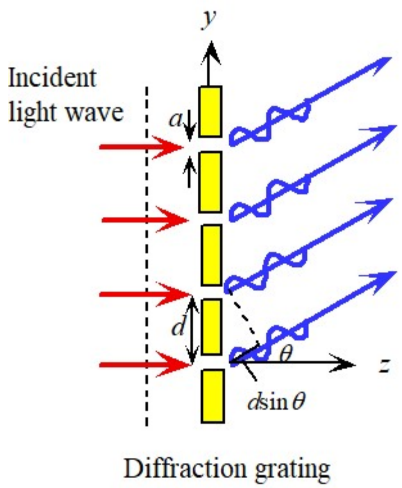
Nobel Prize in Physics (1915)

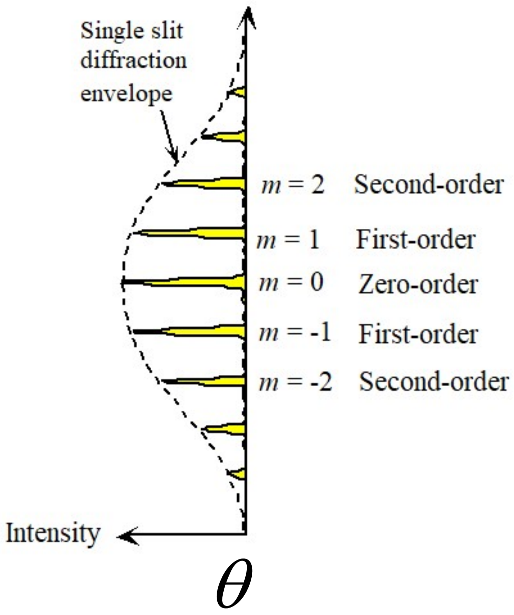
The only father-son joint Nobel winner

W. L. Bragg is the youngest Nobel Physics winner

Lecture 14: Diffraction Grating





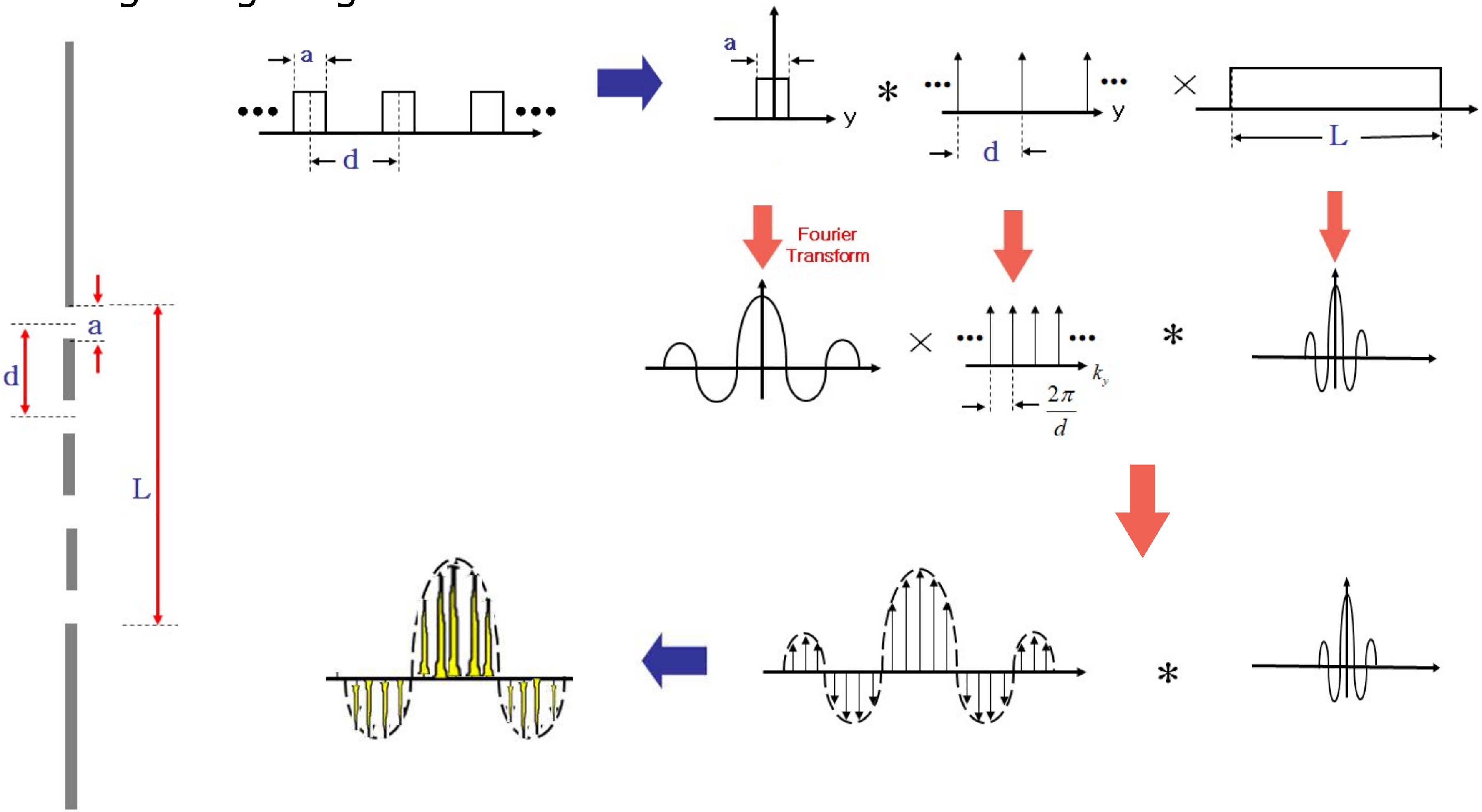


$$d \sin\theta = m\lambda$$

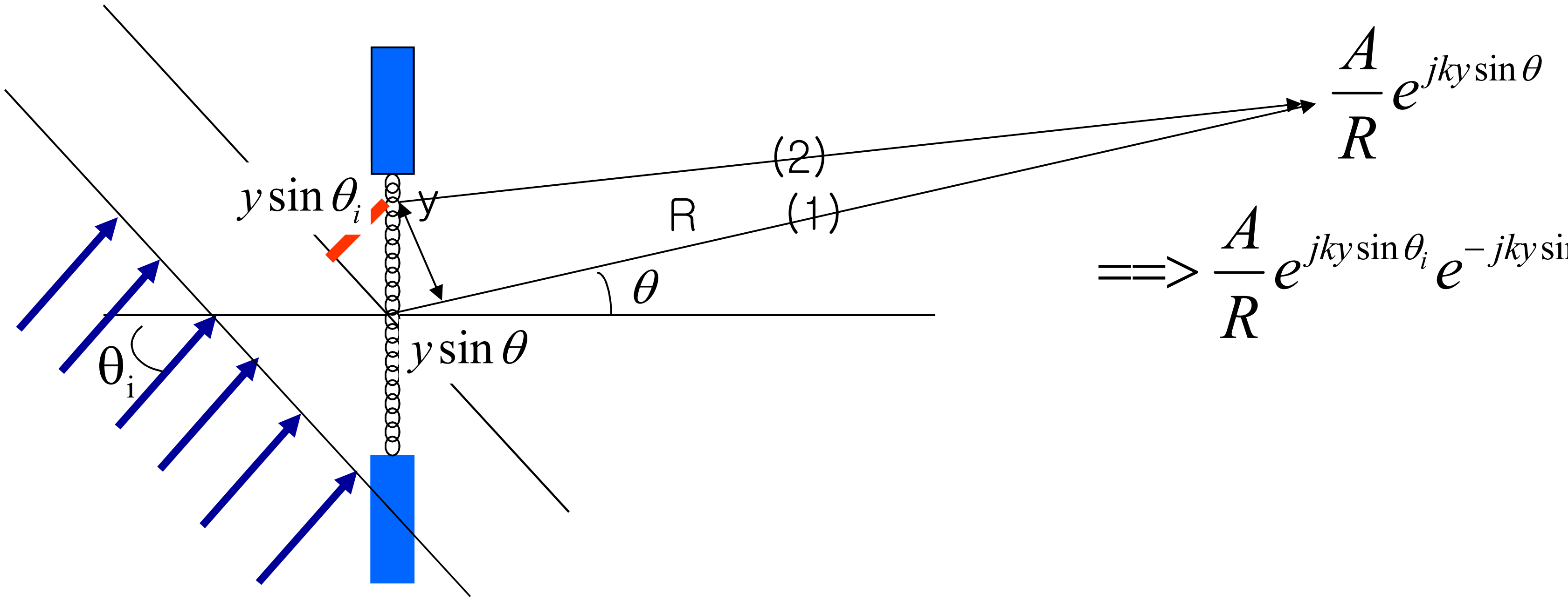
Width for each diffracted beam?

Lecture 14: Diffraction Grating

Finite grating length



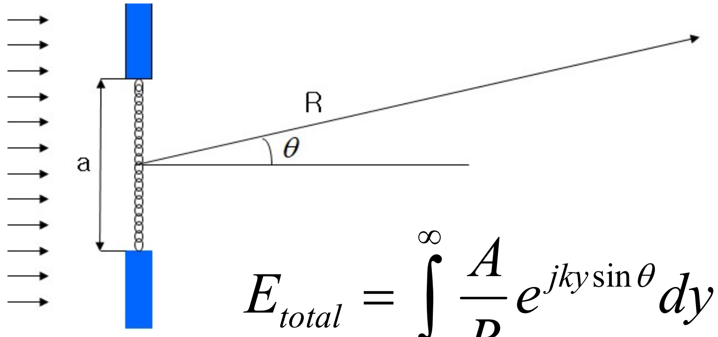
Input with tilted angle



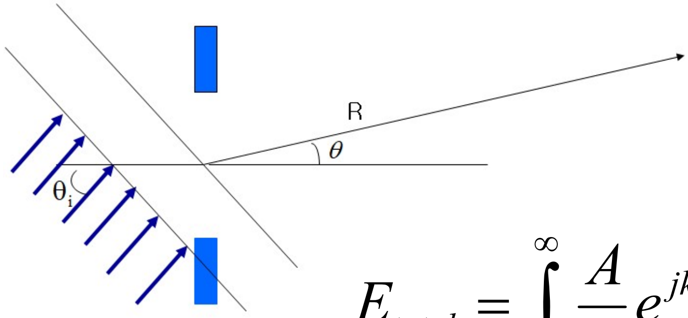
$$==> \frac{A}{R}e^{jky\sin\theta_i}e^{-jky\sin\theta_i}$$

$$E_{total} = \int_{-\infty}^{\infty}\frac{A}{R}e^{jky\sin\theta}dy \quad ==> \quad E_{total} = \int_{-\infty}^{\infty}\frac{A}{R}e^{jky(\sin\theta-\sin\theta_i)}dy$$

Lecture 14: Diffraction Grating



$$E_{total} = \int_{-\infty}^{\infty} \frac{A}{R} e^{jky\sin\theta} dy$$



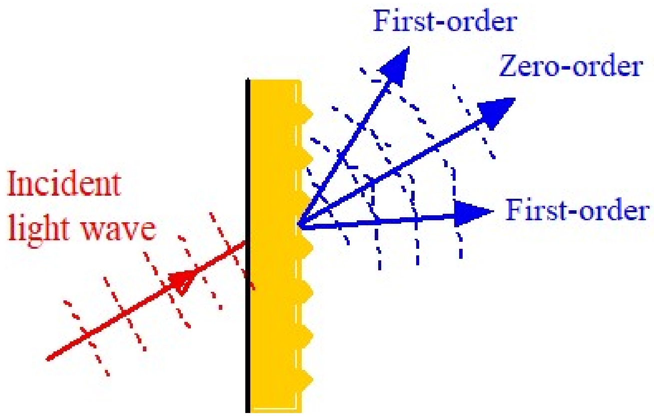
$$E_{total} = \int_{-\infty}^{\infty} \frac{A}{R} e^{jky(\sin\theta - \sin\theta_i)} dy$$

Normal Incidence on Grating

$$d\sin\theta = m\lambda$$

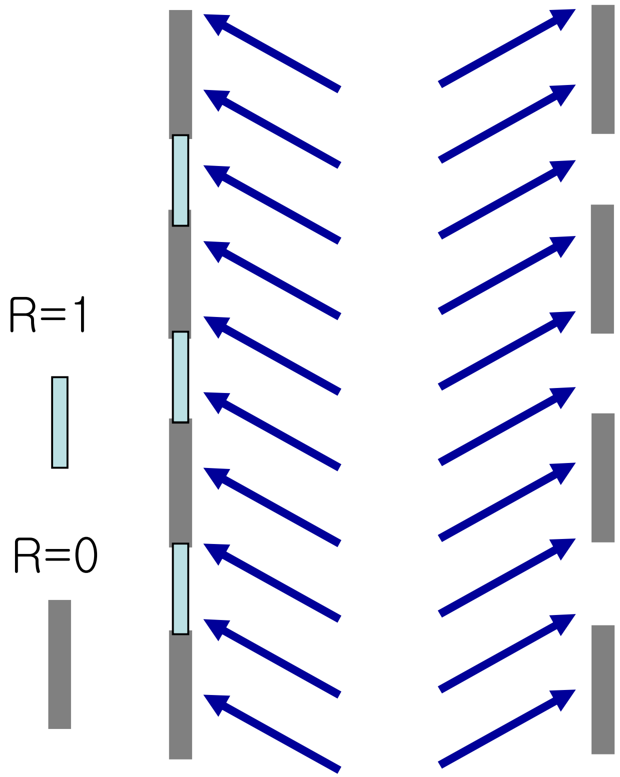
Tilted incidence on Grating

$$\mathrm{d}(\sin\theta - \sin\theta_i) = \mathrm{m}\cdot\lambda$$



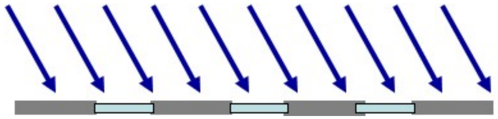
Lecture 14: Diffraction Grating

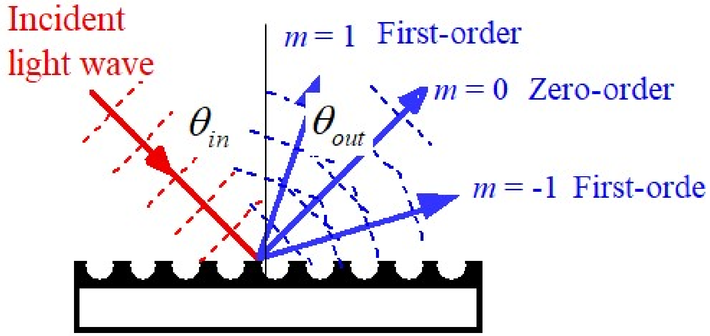
Reflection-type grating



Same diffraction equation

$$d\left(\sin\theta - \sin\theta_i\right) = m \cdot \lambda$$

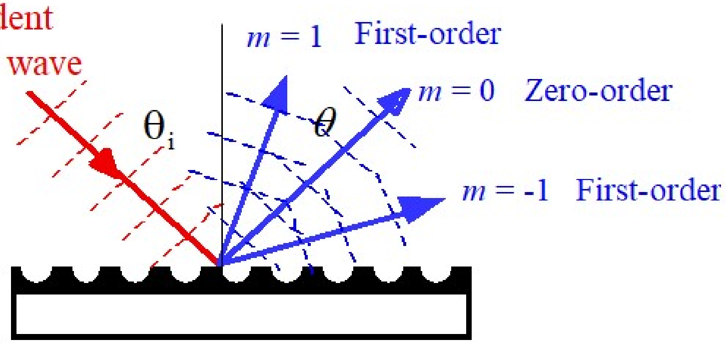




Reflection grating

Grating realized with periodic shaping of reflection surface

Lecture 14: Diffraction Grating



Reflection grating

$$d\left(\sin\theta_{out} - \sin\theta_{in}\right) = m \cdot \lambda$$

$$\frac{2\pi}{\lambda}\left(\sin\theta_{out} - \sin\theta_{in}\right) = m \cdot \frac{2\pi}{d}$$

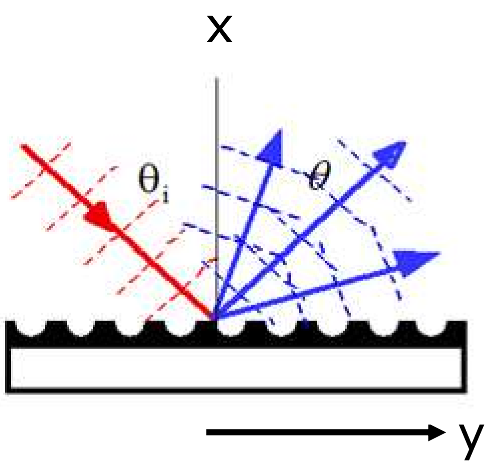
$$\frac{2\pi}{\lambda}\sin\theta_{out} = \frac{2\pi}{\lambda}\sin\theta_{in} + m \cdot \frac{2\pi}{d}$$

K-vector perspective?

$$k_{y,out} = k_{y,in} + m \cdot \frac{2\pi}{d}$$

Grating shifts $k_{y,in}$ by integer multiples of $2\pi/d$

Lecture 14: Diffraction Grating



$$k_{y,out} = k_{y,in} + m \cdot \frac{2\pi}{d}$$

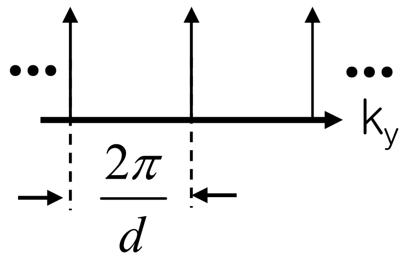
Grating imposes BC on the incident wave

$$E_r(x=0, y) = E_{in}(x=0, y) \text{x} f(y)$$

$$f(y) = \sum_m a_m \exp(jm\frac{2\pi}{d}y)$$

Far-Field Diffraction: F.T. of $E_{in}(x=0, y) \text{x} f(y)$

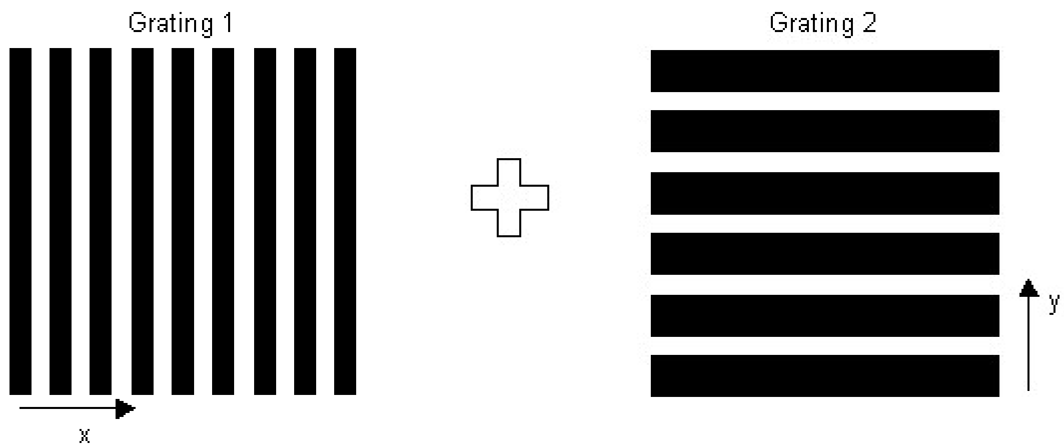
$$E(k_y)_{out} = E(k_y)_{in} * $$



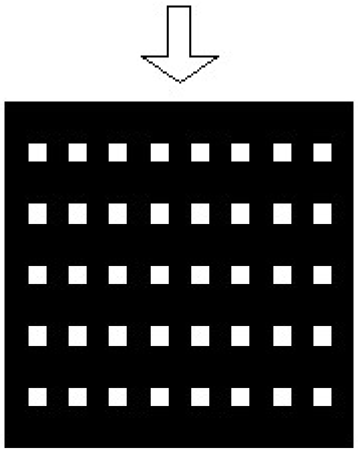
Spatial modulation ⟺ Sidebands formation

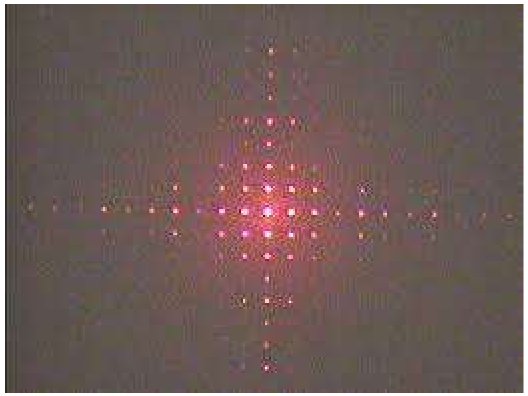
Lecture 14: Diffraction Grating

2-D Diffraction Grating



Take x and y separately

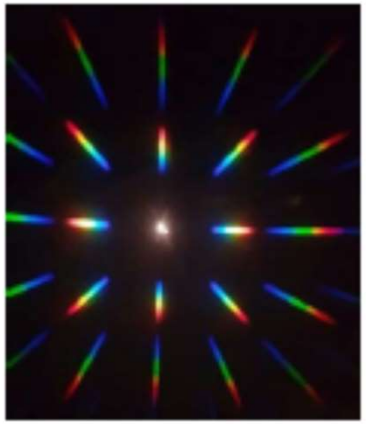




Lecture 14: Diffraction Grating

Homework (11/1)

The following figure shows the moon looked through a two-dimensional diffraction grating. The grating period in both x-direction and y-direction is d.



(a) Why are there discrete bands of diffracted light?

(b) Why is the red further away from the image of the moon in the center than the blue within the same band?

(c) Explain how you can estimate the distance between the grating and the observation plane from above figure. Use d, the grating period, and x, the distance between the center and a point of a particular color whose wavelength is λ.