Opto-Electronics and Photonics

Lecture 2: Electric Fields, Magnetic Fields

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Lect. 2: Electric Fields, Magnetic Fields

(Ref.: Cheng Chap. 3, 6)

Force between charges

q1 q2

Coulomb force $$\overline{F} = \overline{a}\frac{q_1 \cdot q_2}{4\pi\varepsilon_0 R^2}$$

ε_0 : permittivity of vacuum

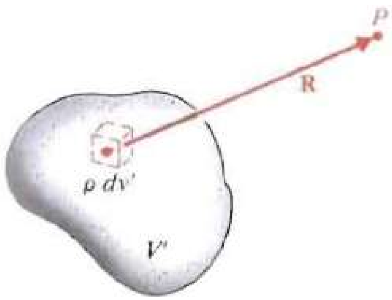
Electric field

q1

$$\overline{E} = \overline{a}\frac{q_1}{4\pi\varepsilon_0 R^2}$$ [Unit: V/m] $$\overline{F} = q_2\overline{E}$$

More than one point charge ➔ Superposition

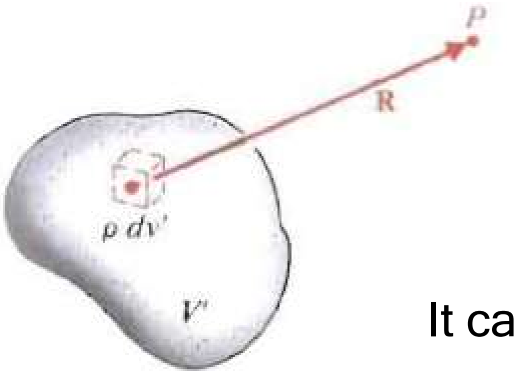
Continuous charge distribution



$$d\overline{E} = \overline{a}\frac{\rho dv'}{4\pi\varepsilon_0 R^2}$$

$$\overline{E} = \frac{1}{4\pi\varepsilon_0}\int_{v'} \overline{a}\frac{\rho(v')dv'}{R^2}$$

Lect. 2: Electric Fields, Magnetic Fields



$$\overline{E} = \frac{1}{4\pi\varepsilon_0} \int_{v'} \overline{a} \frac{\rho(v')dv'}{R^2}$$

It can be derived $\nabla \cdot \overline{E} = \frac{\rho}{\varepsilon_0}$ ➔ Charge produces E-field

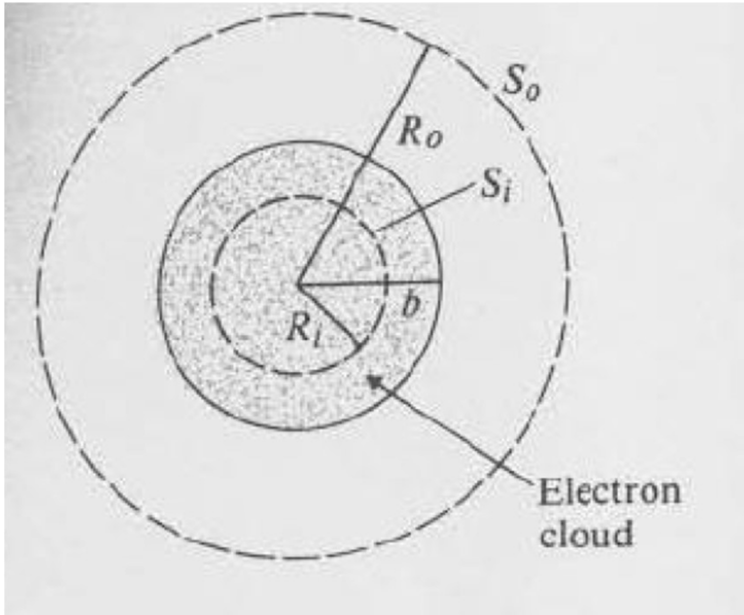
$$\int_v \nabla \cdot \overline{E}\, dv = \frac{1}{\varepsilon_0} \int_v \rho(v) dv \qquad \oint_S \overline{E} \bullet d\overline{s} = \frac{Q}{\varepsilon_0}$$

Unit for ε_0?

$$\frac{1}{m}\frac{V}{m} = \frac{C}{m^3}\frac{1}{[\varepsilon_0]} \qquad [\varepsilon_0] = \frac{C}{V}\frac{1}{m} = \frac{F}{m} \qquad \varepsilon_0 \sim 8.85 \times 10^{-12}\,\mathrm{F/m}$$

Determine electric filed for an charge cloud with uniform charge density ρ_0

(Example 3-7 in Cheng)



$$\oint \bar{E} \cdot d\bar{s} = \frac{Q}{\varepsilon_0}$$

$0 \le R \le b$

$$\bar{E} = \bar{R} E_R \qquad d\bar{s} = \bar{R}\, ds$$

$$\oint \bar{E} \cdot d\bar{s} = E_R \oint ds = E_R 4\pi R^2$$

$$Q = \int_V \rho \cdot dV = \rho_0 \cdot \frac{4\pi}{3} R^3$$

$$\bar{E} = \bar{R} \frac{\rho_0}{3\varepsilon_0} R$$

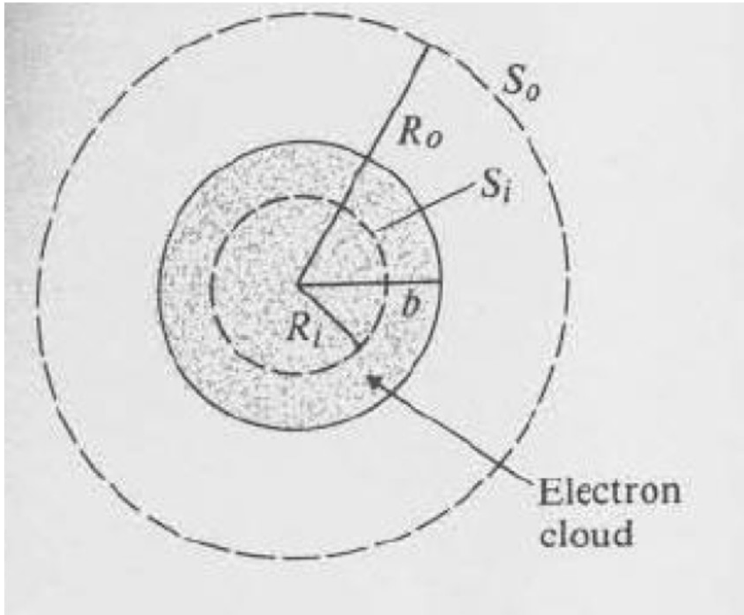
$R \ge b$

$$Q = \rho_0 \cdot \frac{4\pi}{3} b^3$$

$$\bar{E} = \bar{R} \rho_0 \cdot \frac{b^3}{3\varepsilon_0 R^2}$$

Determine electric filed for an charge cloud with uniform charge density ρ_0

(Example 3-7 in Cheng)



$$\oint \bar{E} \cdot d\bar{s} = \frac{Q}{\varepsilon_0}$$

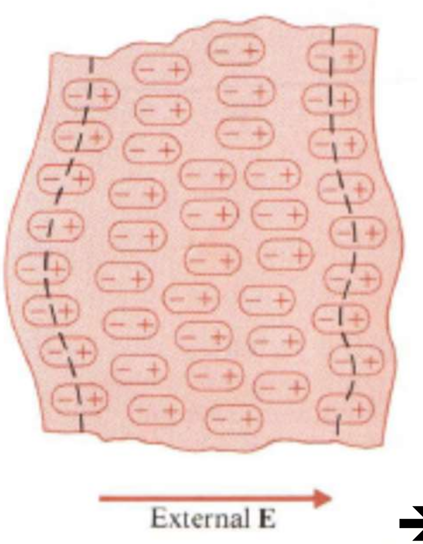
$0 \le R \le b$ $\quad \bar{E} = \bar{R} \cdot \frac{\rho_0}{3\varepsilon_0} R$

$R \ge b$ $\quad \bar{E} = \bar{R} \rho_0 \cdot \frac{b^3}{3\varepsilon_0 R^2}$

Lect. 2: Electric Fields, Magnetic Fields

- How do *dielectric* materials response to electric field?

(Insulator that is polarized with external E-field)



- Material is polarized

- Define Polarization Vector

$$\overline{P} = \chi_e \varepsilon_0 \overline{E} \qquad \chi_e$$: (electric susceptibility)

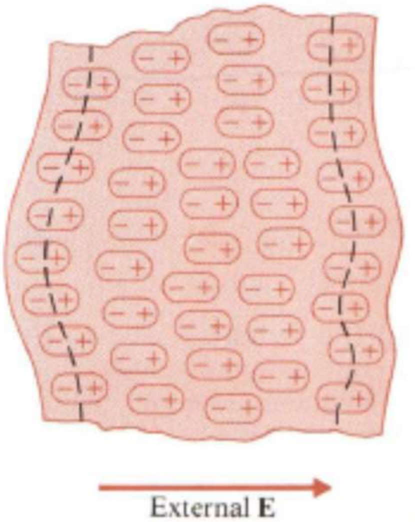
[P]: F/m x V/m = C/m^2

➔ Polarization reduces the electric field inside dielectric material

$$\nabla \cdot \overline{E} = \frac{\rho - \nabla \cdot \overline{P}}{\varepsilon_0}$$

Lect. 2: Electric Fields, Magnetic Fields

- How do *dielectric* materials response to electric field?



$$\overline{P} = \chi_e \varepsilon_0 \overline{E} \qquad \nabla \cdot \overline{E} = \frac{\rho - \nabla \cdot \overline{P}}{\varepsilon_0}$$

- Define Displacement Vector $\overline{D} = \varepsilon_0 \overline{E} + \overline{P}$

$$\overline{D} = (1+\chi_e)\varepsilon_0 \overline{E} = \varepsilon_r \varepsilon_0 \overline{E} = \varepsilon \overline{E} \qquad \overline{D} = \varepsilon \overline{E}$$

ε : permittivity ε_r : relative permittivity or dielectric constant

[D]: F/m x V/m = C/m^2

$$\nabla \cdot \overline{D} = \varepsilon_0 \nabla \cdot \overline{E} + \nabla \cdot \overline{P} = \rho$$

$$\oint_S \overline{D} \cdot d\overline{s} = Q$$

➔ Gauss's Law: Charge produces D-vector

Lect. 2: Electric Fields, Magnetic Fields

Magnetic fields produced by currents

Ampere's Law: $\nabla \times \overline{H} = \overline{J}$

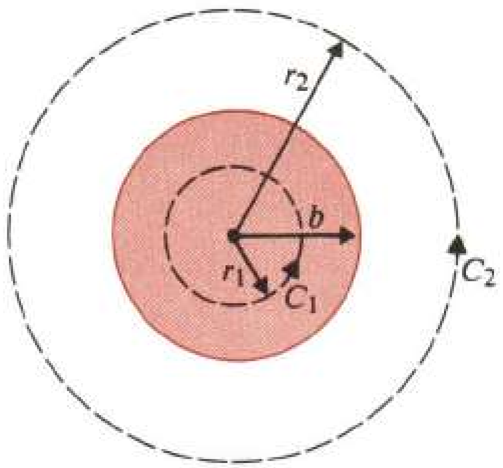
$\overline{J}$: Current density (A/m^2) $\overline{H}$: Magnetic field (A/m)

Integral form?

$$\int_S (\nabla \times \overline{H}) \cdot d\overline{s} = \int_S \overline{J} \cdot d\overline{s}$$

$$\int_C \overline{H} \cdot d\overline{l} = I$$

Determine H when uniform current I is flowing out.
(Example 6-1 in Cheng)



$$\oint_C \bar{H} \cdot d\bar{l} = I$$

Inside ($r \le b$)

$$\bar{H} = \bar{\phi} \cdot H_\phi \quad d\bar{l} = \bar{\phi}\, r\, d\phi$$

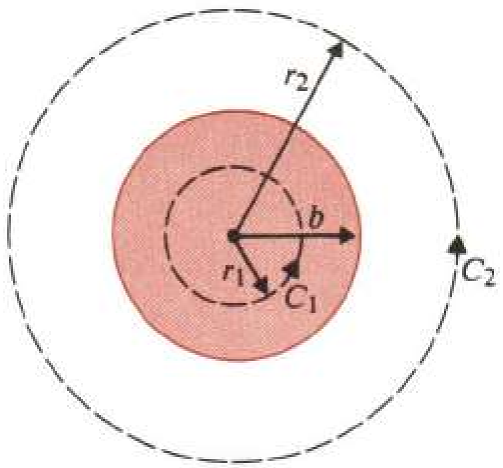
$$\oint_C H_\phi\, r\, d\phi = H_\phi \cdot r \cdot 2\pi = \frac{I}{\pi b^2} \cdot \pi r^2$$

$$H_\phi = \frac{rI}{2\pi b^2} \quad \bar{H} = \bar{\phi} \cdot \frac{rI}{2\pi b^2}$$

Outside ($r > b$)

$$H_\phi \cdot r \cdot 2\pi = I \quad H_\phi = \frac{I}{2\pi r} \quad \bar{H} = \bar{\phi} \cdot \frac{I}{2\pi r}$$

Determine H when uniform current I is flowing out.
(Example 6-1 in Cheng)



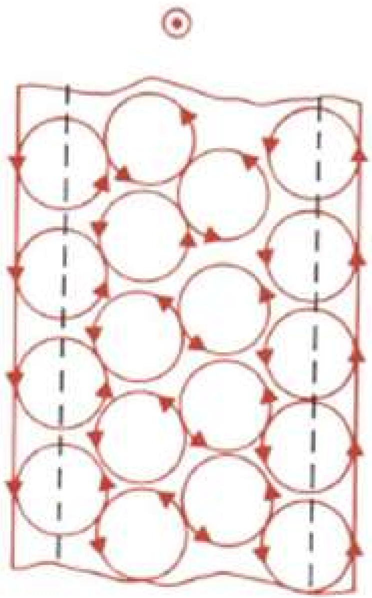
$$\int_C \bar{H} \cdot d\bar{l} = I$$

Inside $(r \leq b)$ $\bar{H} = \bar{\varphi} \cdot \frac{rI}{2\pi b^2}$

Outside $(r > b)$ $\bar{H} = \bar{\varphi} \cdot \frac{I}{2\pi r}$

Lect. 2: Electric Fields, Magnetic Fields

- How does magnetic field affect materials?



Magnetization: Microscopic current loops inside the material

$\bar{M} = \chi_m \bar{H}$ χ_m: Magnetic susceptibility

$$\bar{B} = \mu_0 \bar{H} + \mu_0 \bar{M} = \mu_0 \bar{H} + \mu_0 \chi_m \bar{H}$$
$$= (1+\chi_m)\mu_0 \bar{H} = \mu_r \mu_0 \bar{H} = \mu \bar{H}$$

$$\bar{B} = \mu \bar{H}$$

$\bar{B}$: Magnetic flux density

[B]: Weber/m^2 (also known as Tesla, 1Wb/cm^2 = 1T)

μ : Permeability [Henry/m]

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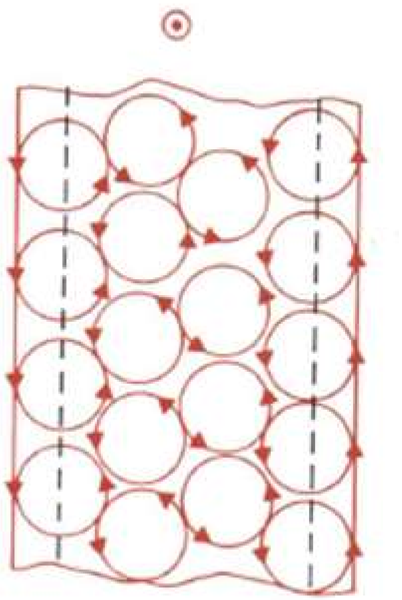
Magnetization: Microscopic current loops inside the material



$$\bar{B} = \mu \bar{H} \qquad \mu = \mu_r \mu_0$$

- Diamagnetic: $\mu_r < 1$
- Paramagnetic: $\mu_r > 1$
- Ferromagnetic: $\mu_r >> 1$

In this course, we assume $\mu = \mu_0$

➔ EM waves in non-magnetic dielectric materials

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$$\overline{E} \qquad\qquad \overline{H}$$

$$\overline{D} = \varepsilon \overline{E} \qquad\qquad \overline{B} = \mu \overline{H}$$

$$\nabla \cdot \overline{D} = \rho \qquad\qquad \nabla \times \overline{H} = \overline{J}$$

➔ E&M 1

No coupling between electric and magnetic fields

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Homework: Determine D, E, P for the following dielectric shell

Upload to YSCEC before 9/6 Sunday 24:00

Be prepared to explain your answer during Zoom session on 9/7

Post questions on YSCEC for class participation points