Opto-Electronics and Photonics

Lecture 6: EM Waves in Conductor

Woo-Young Choi

Dept. of Electrical and Electronic Engineering
Yonsei University

Lecture 6: EM Waves in Conductor

(Cheng 8-3, 8-6)

$$\nabla \cdot \overline{D} = \rho$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} ==> -j\omega\overline{B}$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} ==> \overline{J} + j\omega\overline{D}$$

$$\overline{D} = \varepsilon\overline{E}$$

$$\overline{B} = \mu\overline{H}$$

For EM wave equation derivation

$\overline{J} = 0$ assumed

What if E−field generates currents?

$\overline{J} = \sigma \cdot \overline{E}$, $\sigma \neq 0$

Lecture 6: EM Waves in Conductor

$$\nabla \cdot \overline{D} = \rho$$

$$\nabla \times \overline{E} = -j\omega \overline{B}$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{H} = \overline{J} + j\omega \overline{D}$$

$$\overline{D} = \varepsilon \overline{E}$$

$$\overline{B} = \mu \overline{H}$$

$$\overline{J} = \sigma \cdot \overline{E}, \ \sigma \neq 0$$

$$\nabla \times \overline{H} = \sigma \overline{E} + j\omega\varepsilon \overline{E} = j\omega(\varepsilon - j\frac{\sigma}{\omega})\overline{E}$$

$$= j\omega\varepsilon_c \overline{E}$$

➔ Complex permittivity $\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}$

Wave number k?

$$k = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu\varepsilon}\left(1 - j\frac{\sigma}{\varepsilon\omega}\right)^{\frac{1}{2}} = \beta - j\alpha,$$

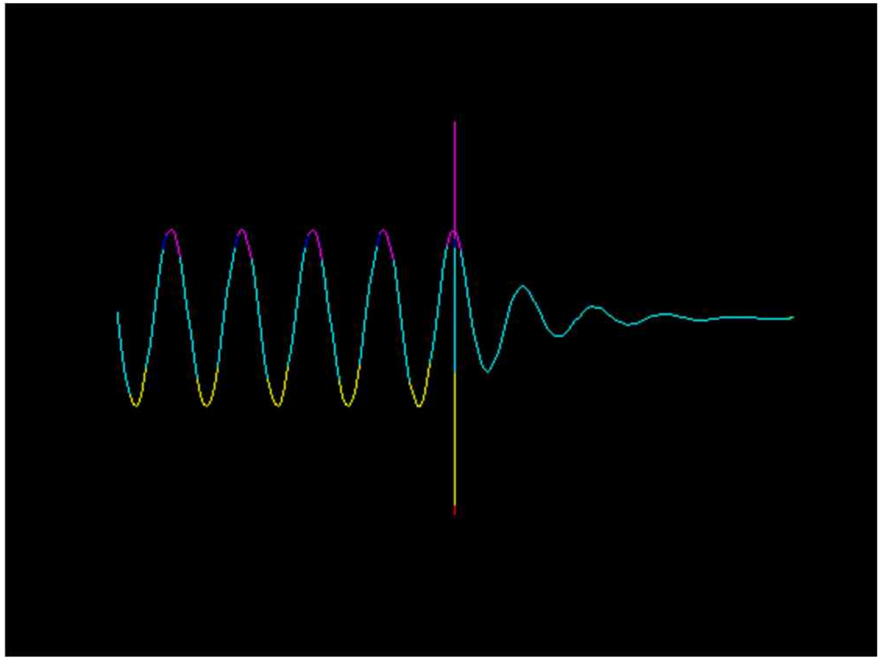
Consider plane wave solution:

$$\overline{E} = \overline{x}E_0 e^{-jkz} = \overline{x}E_0 e^{-j(\beta - j\alpha)z} = \overline{x}E_0 e^{-j\beta z} e^{-\alpha z}$$

Exponential Decay!

Lecture 6: EM Waves in Conductor

$$\overline{E} = \overline{x} E_0 e^{-j\beta z} e^{-\alpha z}$$



Why decay?

α : attenuation constant

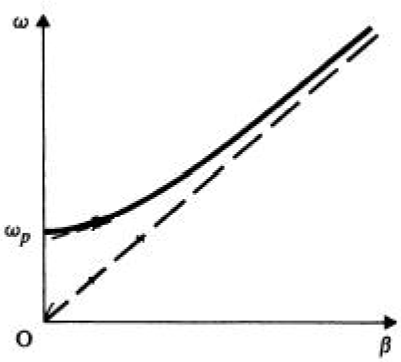
β : propagation constant

Lecture 6: EM Waves in Conductor

$$k = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu\varepsilon}\left(1 - j\frac{\sigma}{\varepsilon\omega}\right)^{\frac{1}{2}} = \beta - j\alpha, \qquad \varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}$$

Wavelength: $\dfrac{2\pi}{\beta}$

Phase velocity: $\dfrac{\omega}{\beta}$ (β is ω–dependent in certain materials) ==> $\dfrac{\omega}{\beta(\omega)}$

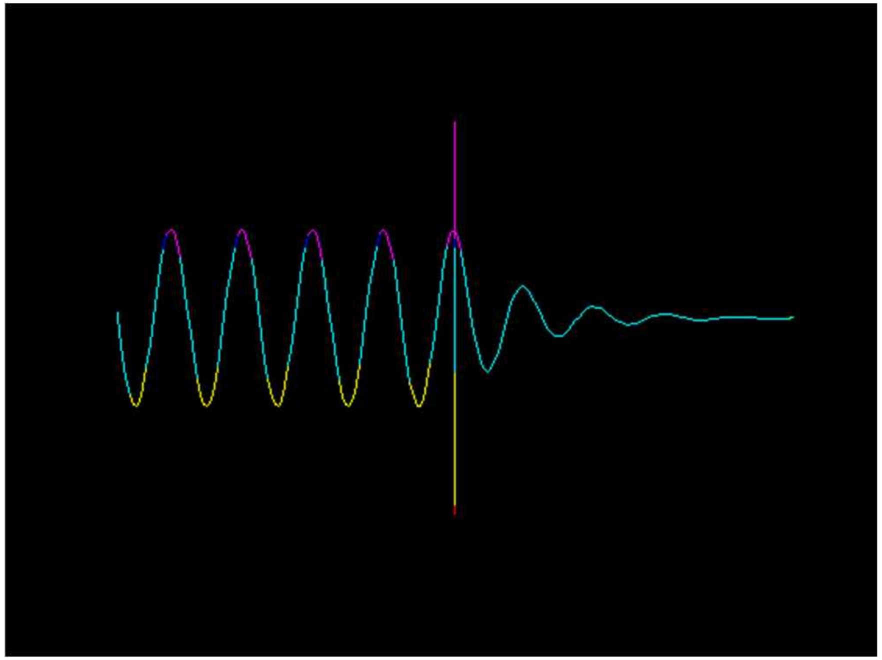


ω vs β for ionized gas
(Fig. 8-7 in Cheng)

For energy propagation: $\dfrac{\partial\omega}{\partial\beta}$ (Group velocity)

Lecture 6: EM Waves in Conductor

$$\overline{E} = \overline{x} E_0 e^{-j\beta z} e^{-\alpha z}$$



How much does the wave penetrate into the medium?

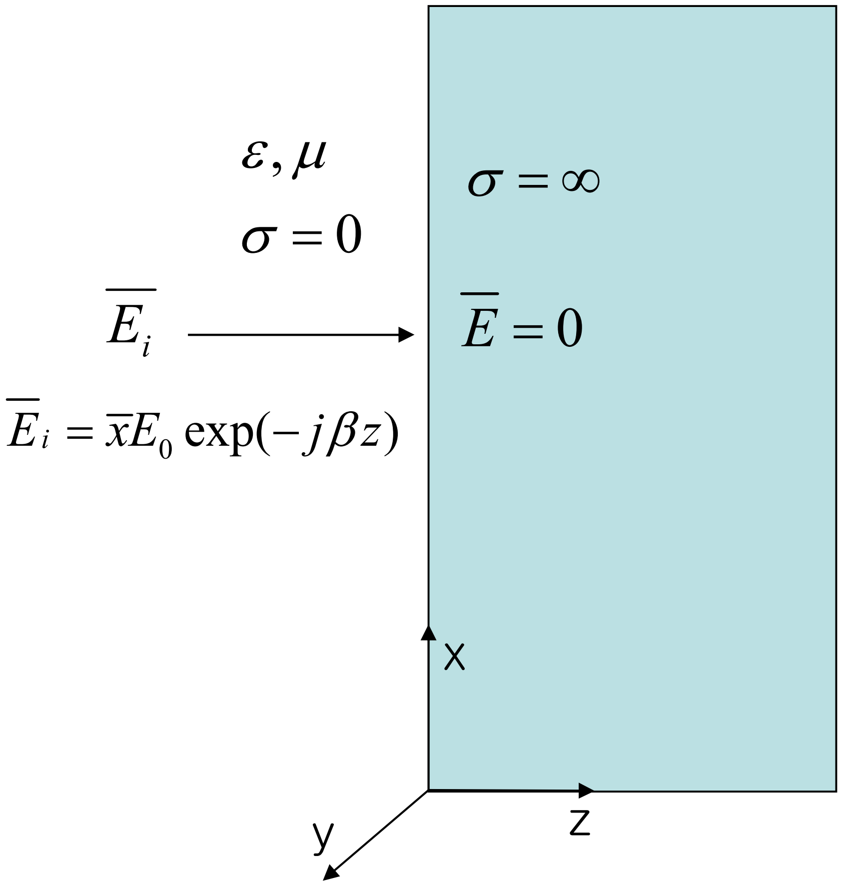
$$\delta = \frac{1}{\alpha}$$

Penetration depth, Or Skin depth

What if σ is infinitely large?

Lecture 6: EM Waves in Conductor

Consider EM waves normally incident at a perfect conductor at z=0



No penetration (skin depth = 0)

➔ Reflection

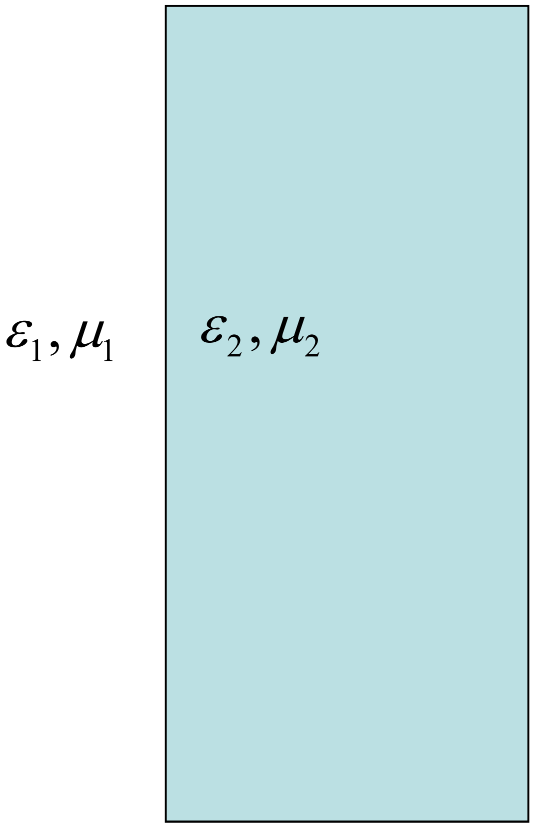
Reflected E-field?

$$\overline{E}_r = ?$$

Lecture 6: EM Waves in Conductor

✓ Boundary Conditions: Constraints on E,H fields at a boundary

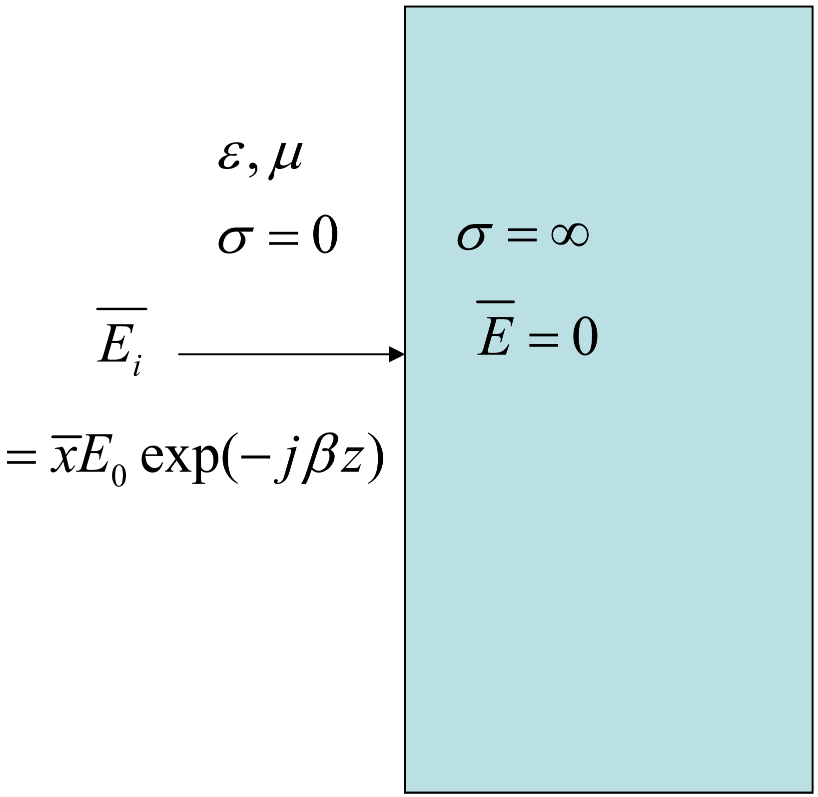
➔ Each of Maxwell's Equations provides one constraint on E or H



$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$E_{1,t} = E_{2,t}$$

➔ Same tangential E-fields across the boundary



$\overline{E}_r = ?$ $E_{1,t} = E_{2,t}$

At z = 0, $E_{2,t} = 0$

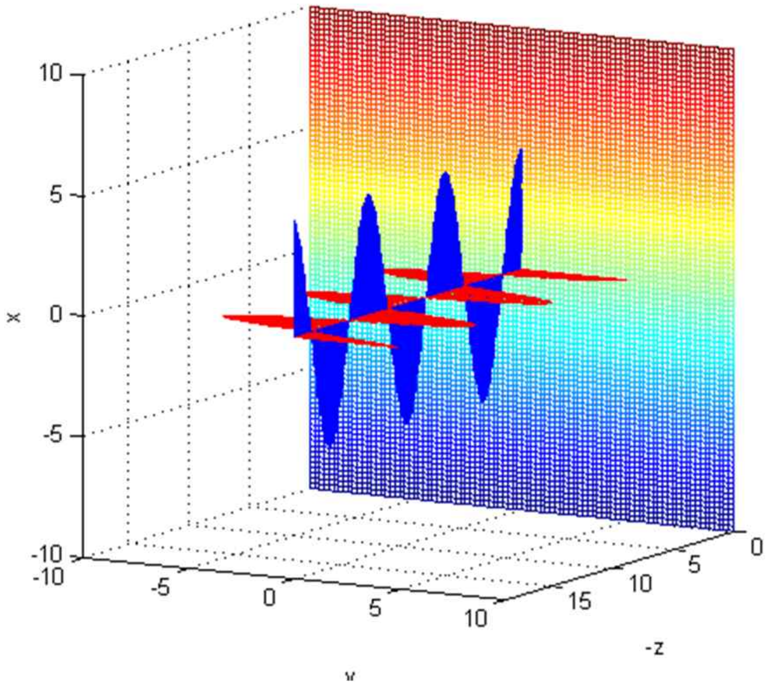
$$\overline{E}_i(z=0) + \overline{E}_r(z=0) = 0$$

$$\therefore \overline{E}_r = -\overline{x} E_0 \exp(j\beta z)$$

Lecture 6: EM Waves in Conductor

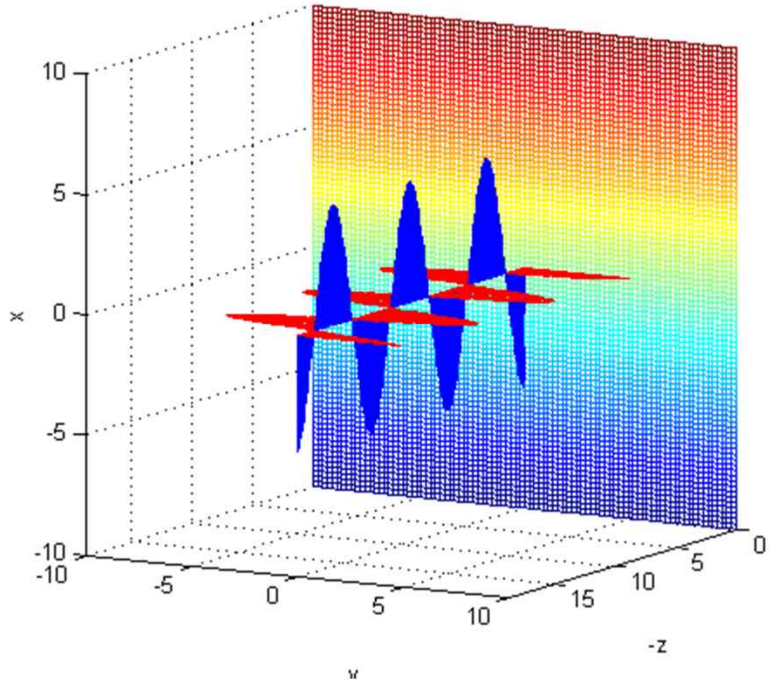
Incident Wave

$$\overline{E}_i = \overline{x} E_0 \exp(-j\beta z)$$



Reflected Wave

$$\overline{E}_r = -\overline{x} E_0 \exp(j\beta z)$$



Lecture 6: EM Waves in Conductor

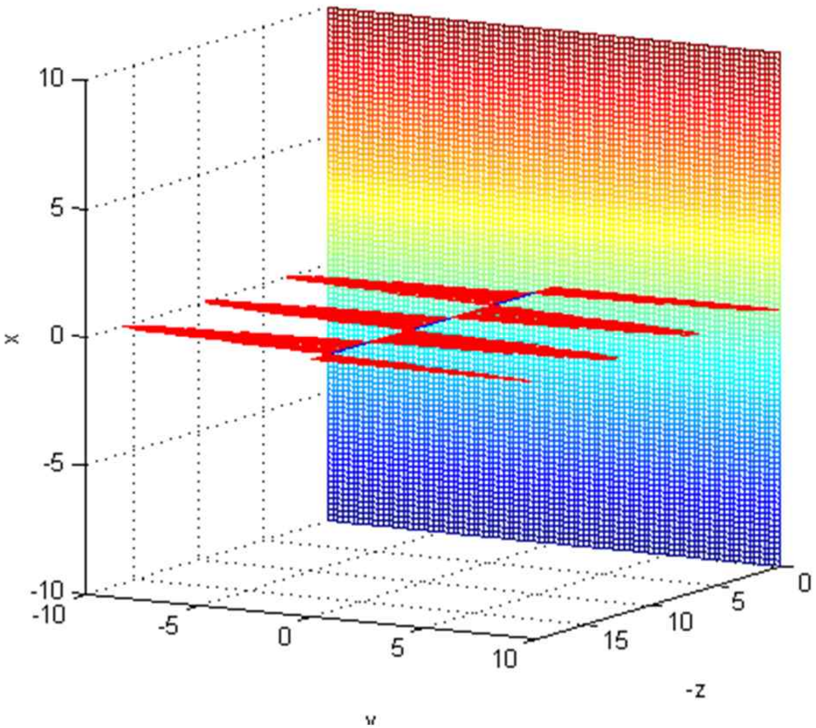
Total E−field for z < 0

$$\bar{E}_{total}(z) = \bar{x}E_0 \exp(-j\beta z) - \bar{x}E_0 \exp(j\beta z)$$

$$= \bar{x}E_0(-2j)\sin(\beta z)$$

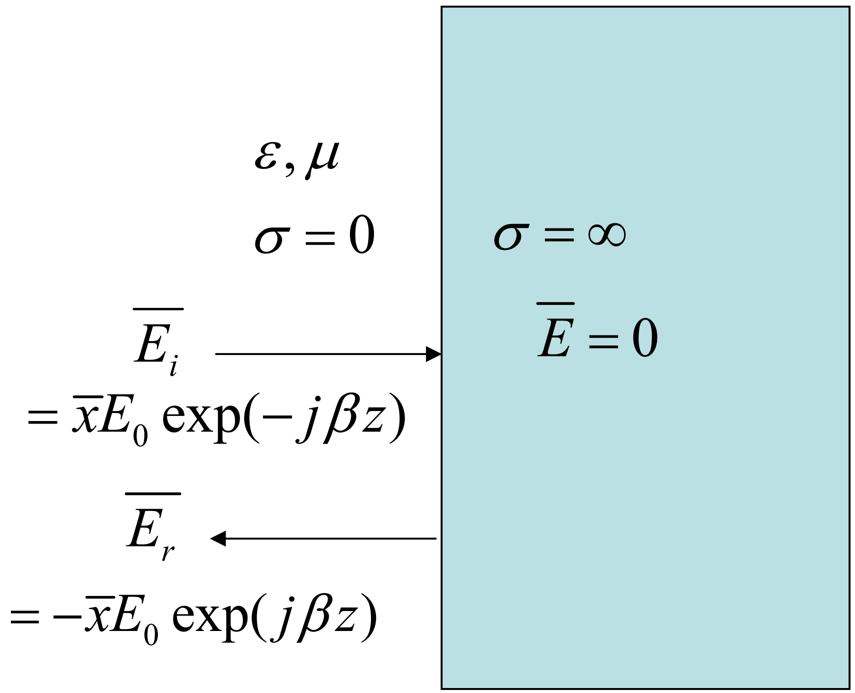
$$\bar{E}_{total}(z,t) = \bar{x}E_0(-2j)\sin(\beta z)e^{j\omega t}$$

$$\mathrm{Re}\left[\bar{E}_{total}(z,t)\right] = \bar{x}E_0 2\sin(\beta z)\sin(\omega t)$$



➔ Standing Wave!

Lecture 6: EM Waves in Conductor



H-field for z<0?

$$\overline{H}_i = \overline{y}\frac{E_0}{\eta}\exp(-j\beta z)$$

$$\overline{H}_r = \overline{y}\frac{E_0}{\eta}\exp(j\beta z)$$

$$\overline{H}_{total}(z) = \overline{y}\frac{E_0}{\eta}\exp(-j\beta z) + \overline{y}\frac{E_0}{\eta}\exp(j\beta z)$$

$$= \overline{y}\frac{2E_0}{\eta}\cos(\beta z)$$

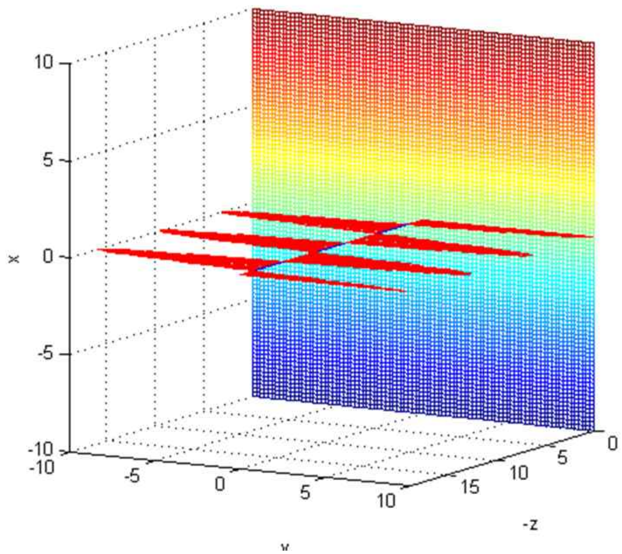
Lecture 6: EM Waves in Conductor

$$\bar{E}_{total}(z) = \bar{x}E_0 \exp(-j\beta z) - \bar{x}E_0 \exp(j\beta z)$$
$$= \bar{x}E_0(-2j)\sin(\beta z)$$

$$\bar{H}_{total}(z) = \bar{y}\frac{E_0}{\eta}\exp(-j\beta z) + \bar{y}\frac{E_0}{\eta}\exp(j\beta z)$$
$$= \bar{y}\frac{2E_0}{\eta}\cos(\beta z)$$

$$\mathrm{Re}\left[\bar{E}_{total}(z,t)\right] = \bar{x}E_0 2\sin(\beta z)\sin(\omega t)$$

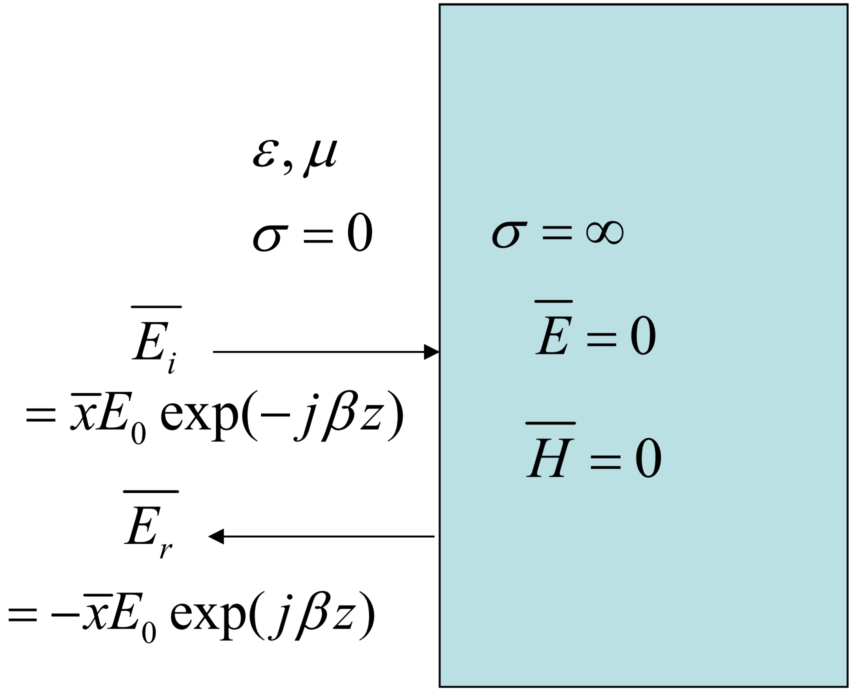
$$\mathrm{Re}\left[\bar{H}_{total}(z,t)\right] = \bar{y}\frac{2E_0}{\eta}\cos(\beta z)\cos(\omega t)$$



Power flow? $\frac{1}{2}\mathrm{Re}\left[\bar{E}\times\bar{H}^*\right]$

(Flow of power density time average)

Lecture 6: EM Waves in Conductor



H-field for z<0?

$$\overline{H}_i = \overline{y}\frac{E_0}{\eta}\exp(-j\beta z)$$

$$\overline{H}_r = \overline{y}\frac{E_0}{\eta}\exp(j\beta z)$$

$$\overline{H}_{total}(z) = \overline{y}\frac{E_0}{\eta}\exp(-j\beta z) + \overline{y}\frac{E_0}{\eta}\exp(j\beta z)$$

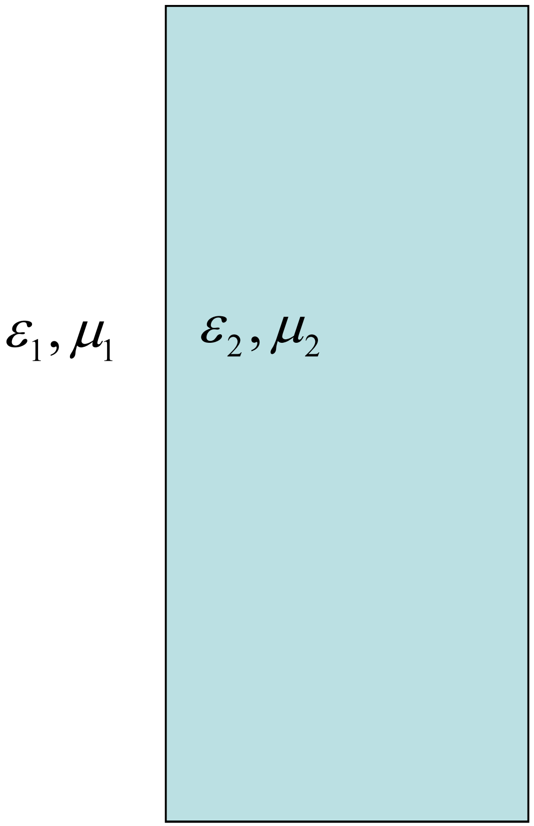
H-field for z>0?

Boundary condition for H?

Lecture 6: EM Waves in Conductor

✓ Boundary Conditions: Constraints on E,H fields at a boundary

➔ Each of Maxwell's Equations provides one constraint on E or H



$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \qquad E_{1,t} = E_{2,t}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} \qquad H_{2,t} - H_{1,t} = J_s$$

$$\nabla \cdot \overline{D} = \rho \qquad \varepsilon_2 E_{2,n} - \varepsilon_1 E_{1,n} = \rho_s$$

$$\nabla \cdot \overline{B} = 0 \qquad \mu_2 H_{2,n} - \mu_1 H_{1,n} = 0$$

Lecture 6: EM Waves in Conductor

Homework: (Due 9/20)

Determine ρ and $\overline{J}_s$ at the interface (z=0)