Si Photonics

Lecture 10: Passive Waveguide Devices

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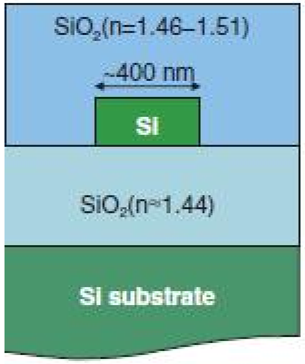
Lecture 10: Passive Waveguide Devices

SOI waveguides: Si (core)/ SiO_2 (cladding)



– Si is transparent to 1.3, 1.55μm light

– Si fabrication technology very mature

➔ Waveguide devices with high integration level

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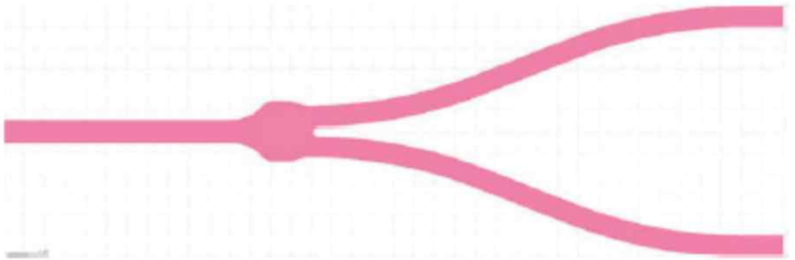
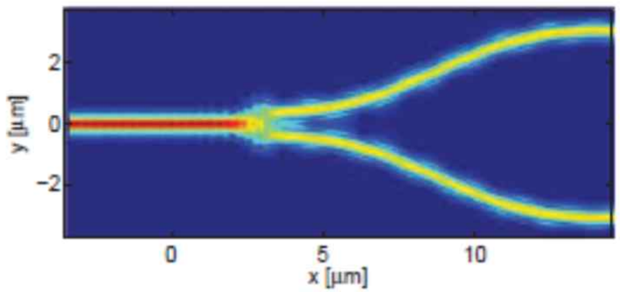
Power division



➔ Y-branch, Y-junction,
Beam-splitter



$$E_1 = E_2 = \frac{E_i}{\sqrt{2}}$$

3-dB beam splitter

- Transition region shape/length optimized for uniformity, bandwidth, insertion loss

- Numerical simulation required

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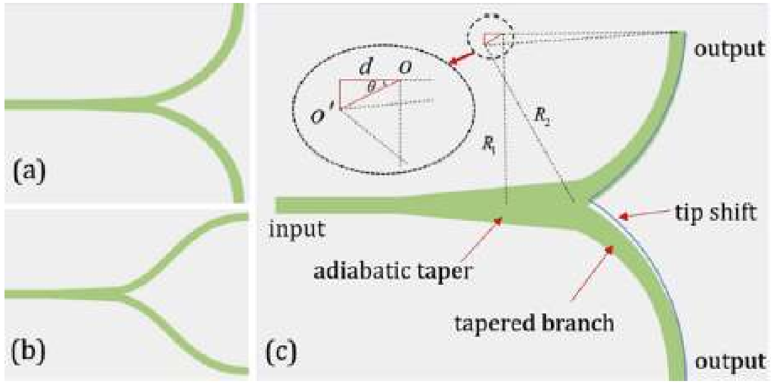
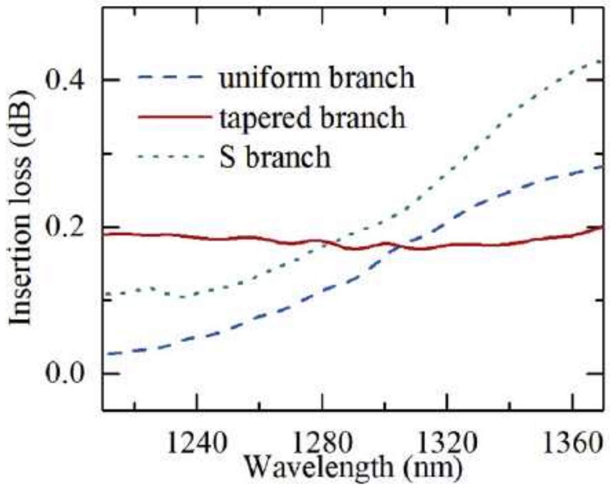


(a) Uniform

(b) S branch

(c) Tapered



"Broadband and high uniformity Y junction optical beam splitter …" *Optik*, 180 (2019) p. 866

Optical beam combiner?

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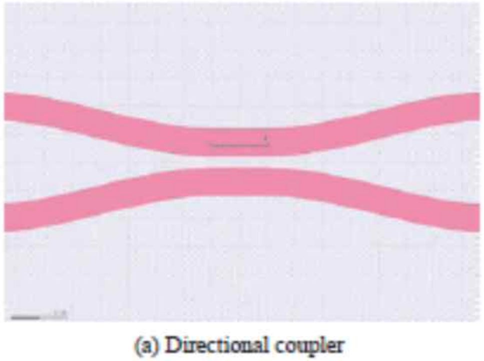
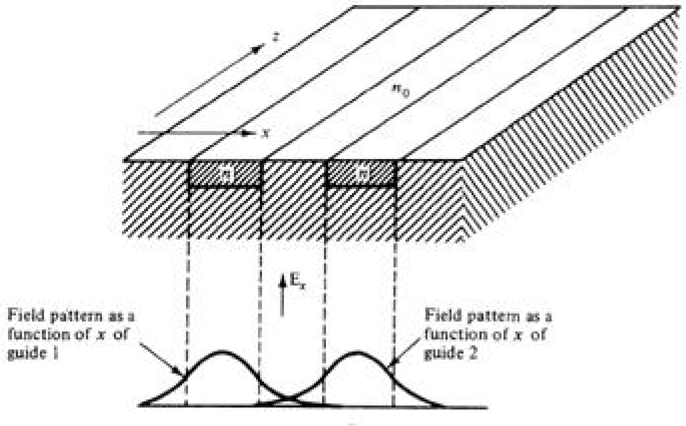
- Directional Coupler

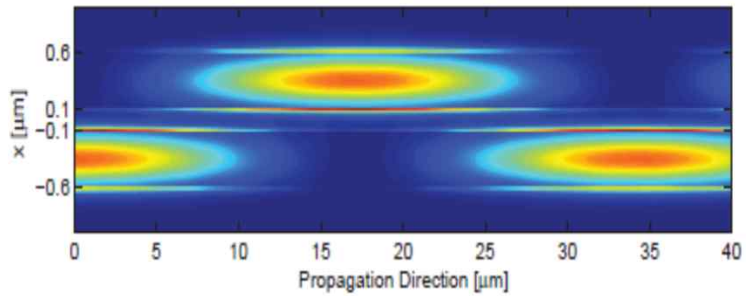




(Top View)

Each waveguide has single mode

Two waveguides placed very close to each other ➔ Coupling



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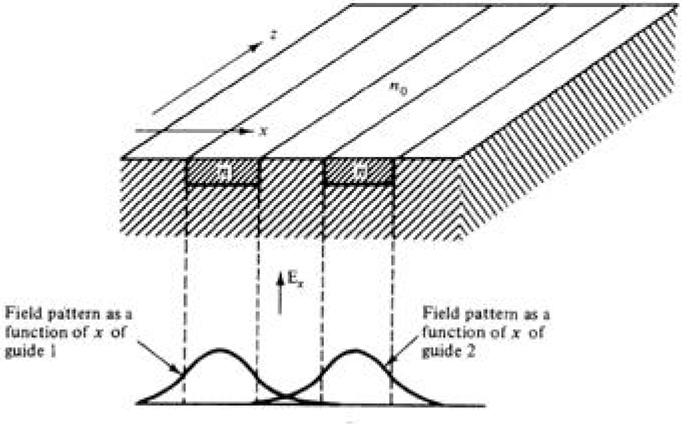

Analyze with the coupled-mode theory for symmetric directional coupler

Guided mode without coupling $\sim e^{j(\omega t-\beta z)}$

$$\frac{da_1}{dz} = -j\beta_1 a_1 \qquad \frac{da_2}{dz} = -j\beta_2 a_2 \qquad \beta = N_{eff} k_0$$

With a small amount of coupling

$$\frac{da_1}{dz} = -j\beta_1 a_1 + \kappa_{12} a_2$$

$$\frac{da_2}{dz} = -j\beta_2 a_2 + \kappa_{21} a_1$$

If two waveguides are identical $\beta_1 = \beta_2 = \beta_0$ $\qquad \kappa_{12} = \kappa_{21} = \kappa'$

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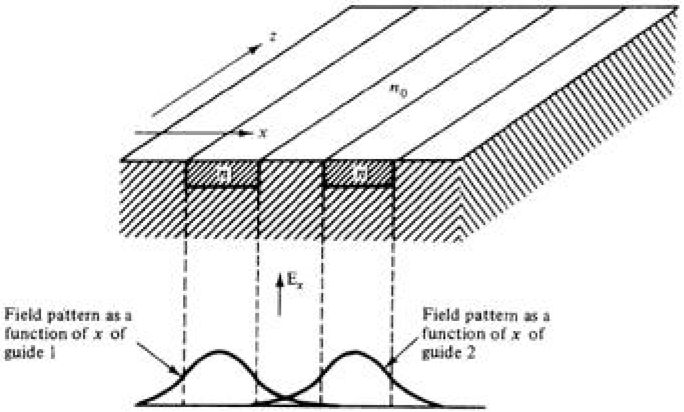

$$\frac{da_1}{dz} = -j\beta_0 a_1 + \kappa' a_2 \qquad \frac{da_1}{dz} = -j\beta_0 a_1 - j\kappa a_2$$

$$\frac{da_2}{dz} = -j\beta_0 a_2 + \kappa' a_1 \quad \rightarrow \quad \frac{da_2}{dz} = -j\beta_0 a_2 - j\kappa a_1$$

For energy conservation,

$$\frac{d\left(\left|a_1\right|^2 + \left|a_2\right|^2\right)}{dz} = 0 \quad = \frac{d\left(a_1 a_1{}^* + a_2 a_2{}^*\right)}{dz} = a_1 \frac{da_1^*}{dz} + a_1^* \frac{da_1}{dz} + a_2 \frac{da_2^*}{dz} + a_2^* \frac{da_2}{dz}$$

$$= a_1(j\beta_0 a_1{}^* + \kappa'^* a_2{}^*) + a_1^*(-j\beta_0 a_1 + \kappa' a_2) + a_2(j\beta_0 a_2{}^* + \kappa'^* a_1{}^*) + a_2^*(-j\beta_0 a_2 + \kappa' a_1)$$

$$= (a_1 a_2{}^* \kappa'^*) + (a_1{}^* a_2 \kappa') + (a_1{}^* a_2 \kappa'^*) + (a_1 a_2{}^* \kappa') = a_1 a_2{}^*(\kappa' + \kappa'^*) + a_1{}^* a_2(\kappa' + \kappa'^*)$$

$$= (a_1 a_2{}^* + a_1{}^* a_2)(\kappa' + \kappa'^*) = 0$$

$\kappa'^* = -\kappa'$ $\quad \kappa'$: purely imaginary

Let $\kappa' = -j\kappa$ (κ real)

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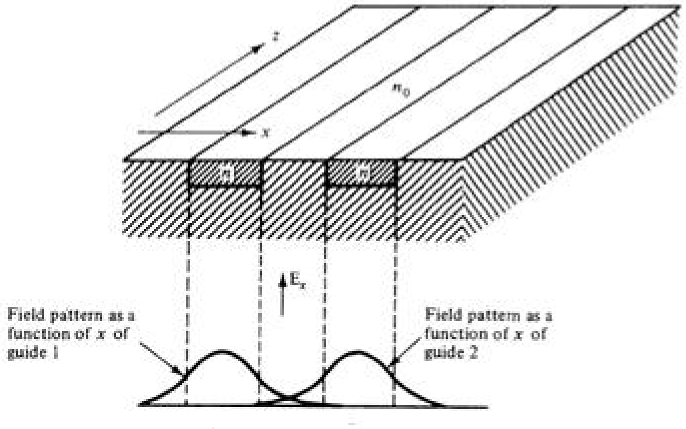

$$\frac{da_1}{dz} = -j\beta_0 a_1 - j\kappa a_2$$

$$\frac{da_2}{dz} = -j\beta_0 a_2 - j\kappa a_1$$

Solve the coupled differential equations

Or how much β_0 changes with coupling?

Assume a_1,a_2 are exponential : $a_1, a_2 \sim e^{-j\beta z}$ ➔ Determine β

$$-j\beta a_1 = -j\beta_0 a_1 - j\kappa a_2 \qquad (\beta_0 - \beta)a_1 + \kappa a_2 = 0$$

$$-j\beta a_2 = -j\beta_0 a_2 - j\kappa a_1 \qquad \kappa a_1 + (\beta_0 - \beta)a_2 = 0$$

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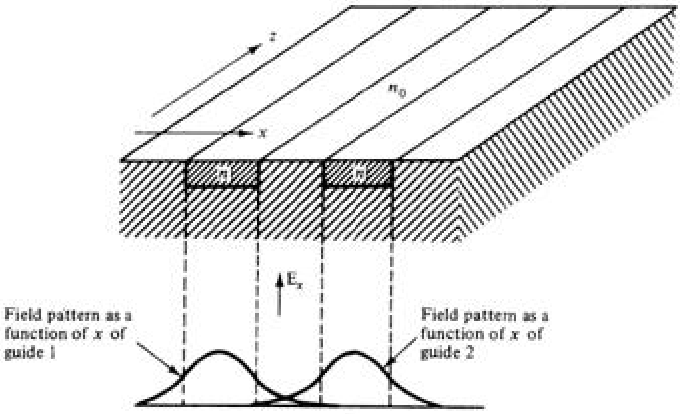

$$(\beta_0 - \beta)a_1 + \kappa a_2 = 0$$

$$\kappa a_1 + (\beta_0 - \beta)a_2 = 0$$

$$\begin{bmatrix} \beta_0 - \beta & \kappa \\ \kappa & \beta_0 - \beta \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0$$

For non-trivial solution, $\det\left(\begin{bmatrix} \beta_0 - \beta & \kappa \\ \kappa & \beta_0 - \beta \end{bmatrix}\right) = 0 \Rightarrow (\beta_0 - \beta)^2 - \kappa^2 = 0$

$\beta = \beta_0 \pm \kappa$ ➔ Two Eigen values

For $\beta = \beta_0 + \kappa$

$$-\kappa a_1 + \kappa a_2 = 0 \qquad a_1 = a_2$$

For $\beta = \beta_0 - \kappa$

$$\kappa a_1 + \kappa a_2 = 0 \qquad a_1 = -a_2$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \sim \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

➔ Two Eigen vectors

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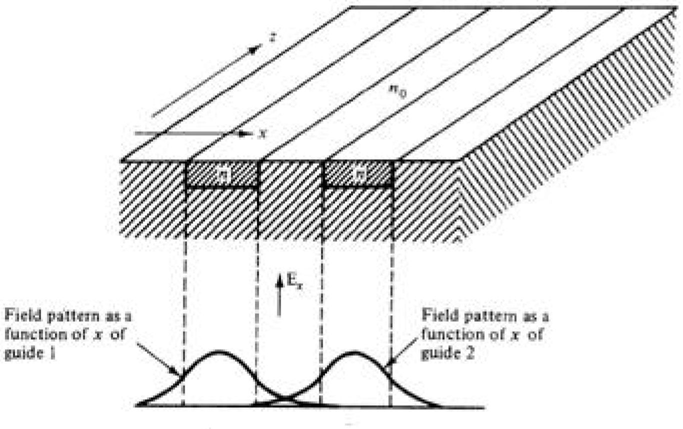

For $\beta = \beta_0 + \kappa \qquad a_1 = a_2$

For $\beta = \beta_0 - \kappa \qquad a_1 = -a_2$

$$\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} = A\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-j(\beta_0+\kappa)z} + B\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-j(\beta_0-\kappa)z}$$

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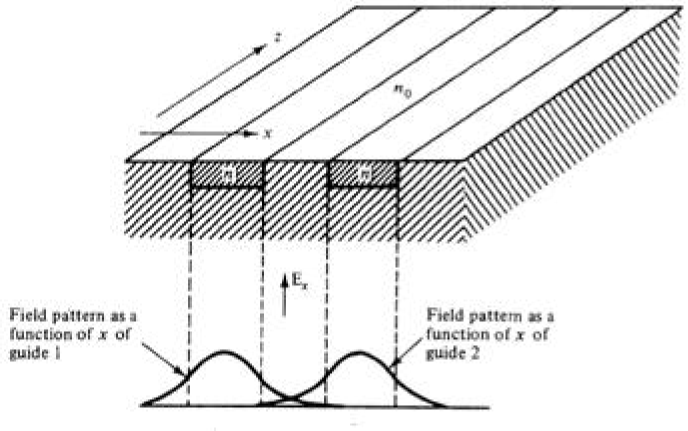
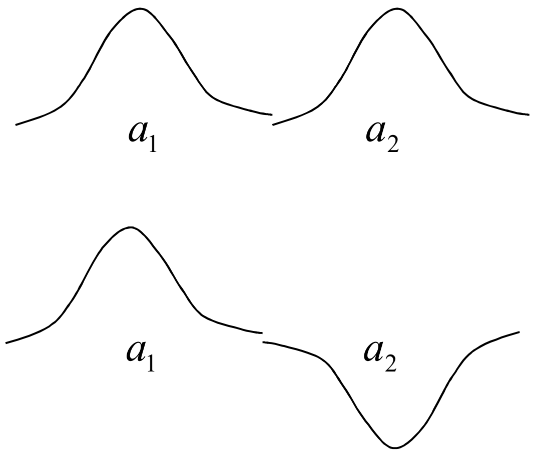


$$\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} = A \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-j(\beta_0+\kappa)z} + B \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-j(\beta_0-\kappa)z}$$

$$\beta = \beta_0 + \kappa$$

$$\beta = \beta_0 - \kappa$$

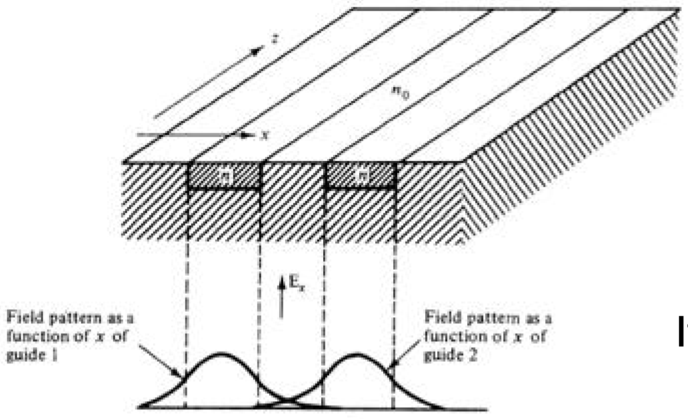


(Even, Symmetric mode)

➔ These two solutions propagate without changing their profiles

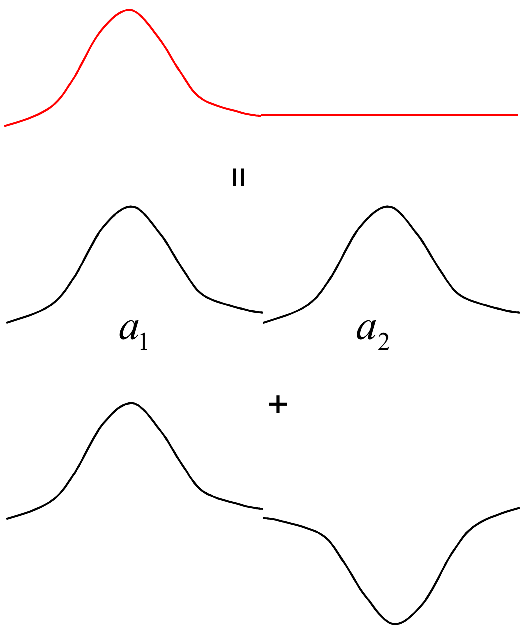
(Odd, Anti-Symmetric mode)

How to determine *A*, *B* ?



$$\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} = A\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-j(\beta_0+\kappa)z} + B\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-j(\beta_0-\kappa)z}$$

If input light is introduced only to the left waveguide



even :

odd :

A=B

Two Eigen solutions propagate
with different phase velocity

➔ Field profile changes as it propagates

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$$\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} = A\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-j(\beta_0+\kappa)z} + B\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-j(\beta_0-\kappa)z}$$

$$= A\begin{bmatrix} e^{-j(\beta_0+\kappa)z} + e^{-j(\beta_0-\kappa)z} \\ e^{-j(\beta_0+\kappa)z} - e^{-j(\beta_0-\kappa)z} \end{bmatrix} = Ae^{-j\beta_0 z}\begin{bmatrix} e^{-j\kappa z} + e^{j\kappa z} \\ e^{-j\kappa z} - e^{j\kappa z} \end{bmatrix}$$

$$= 2Ae^{-j\beta_0 z}\begin{bmatrix} \cos(\kappa z) \\ -j\sin(\kappa z) \end{bmatrix}$$

$$\begin{bmatrix} |a_1(z)|^2 \\ |a_2(z)|^2 \end{bmatrix} \sim \begin{bmatrix} \cos^2(\kappa z) \\ \sin^2(\kappa z) \end{bmatrix}$$

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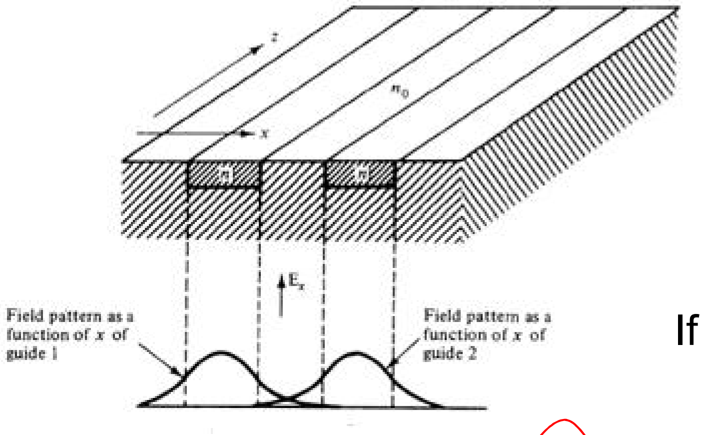
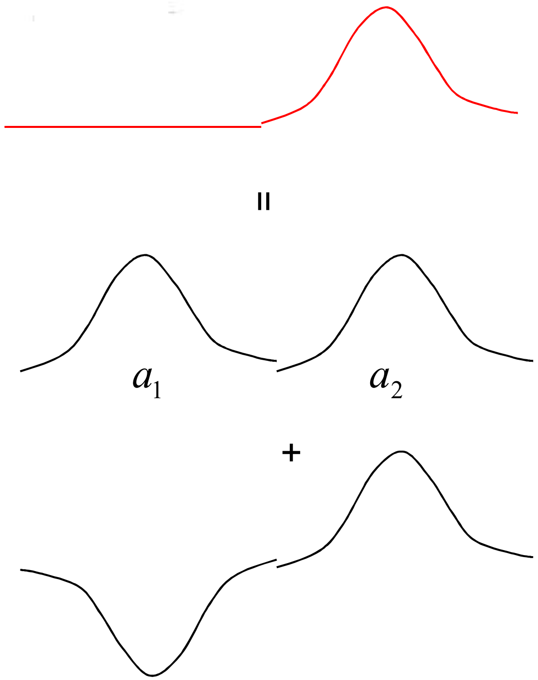


$$\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} = A\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-j(\beta_0+\kappa)z} + B\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-j(\beta_0-\kappa)z}$$

If input light is introduced only to the right waveguide



A = - B

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$$\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} = A\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-j(\beta_0+\kappa)z} + B\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-j(\beta_0-\kappa)z}$$

$$= A\begin{bmatrix} e^{-j(\beta_0+\kappa)z} - e^{-j(\beta_0-\kappa)z} \\ e^{-j(\beta_0+\kappa)z} + e^{-j(\beta_0-\kappa)z} \end{bmatrix} = Ae^{-j\beta_0 z}\begin{bmatrix} e^{-j\kappa z} - e^{j\kappa z} \\ e^{-j\kappa z} + e^{j\kappa z} \end{bmatrix}$$

$$= 2Ae^{-j\beta_0 z}\begin{bmatrix} -j\sin(\kappa z) \\ \cos(\kappa z) \end{bmatrix}$$

$$\begin{bmatrix} |a_1(z)|^2 \\ |a_2(z)|^2 \end{bmatrix} \sim \begin{bmatrix} \sin^2(\kappa z) \\ \cos^2(\kappa z) \end{bmatrix}$$

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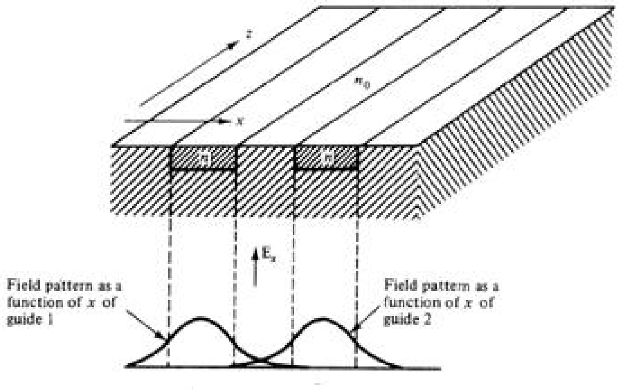
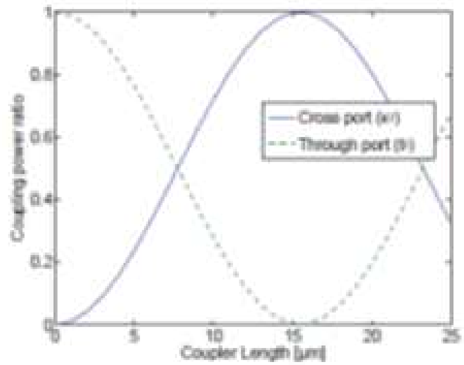


$$\begin{bmatrix} |a_1(z)|^2 \\ |a_2(z)|^2 \end{bmatrix} \sim \begin{bmatrix} \sin^2(\kappa z) \\ \cos^2(\kappa z) \end{bmatrix}$$



Cross-over length : Length required for 100% power transfer

$$\kappa z = \frac{\pi}{2}$$ With $\beta_0 + \kappa$ and $\beta_0 - \kappa$ $$[(\beta_0+\kappa)-(\beta_0-\kappa)]z = \pi$$

$$\beta_0 + \kappa = N_{eff,even} k_0 \qquad \beta_0 - \kappa = N_{eff,odd} k_0$$

$$z = \frac{\pi}{(N_{eff,even} - N_{eff,odd})k_0} = \frac{\lambda}{2(N_{eff,even} - N_{eff,odd})}$$

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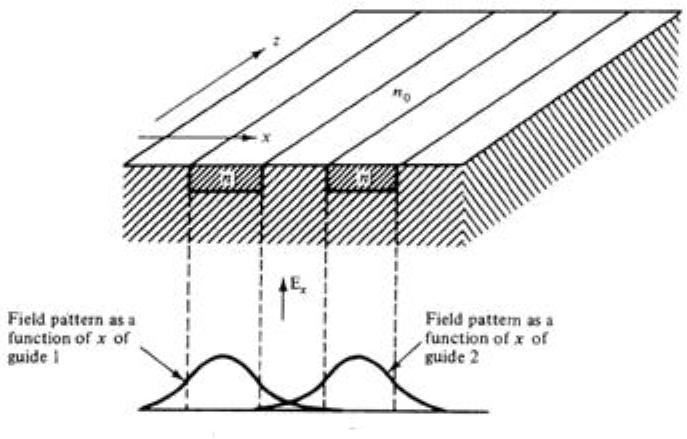
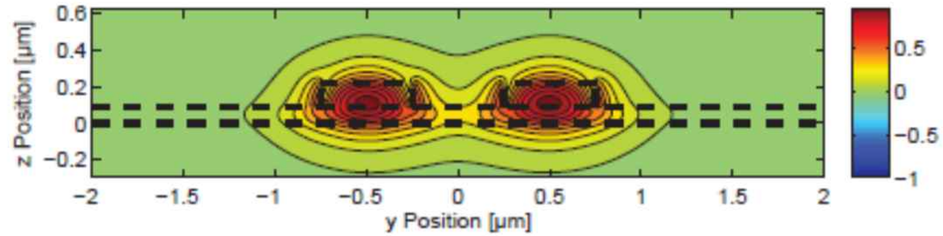
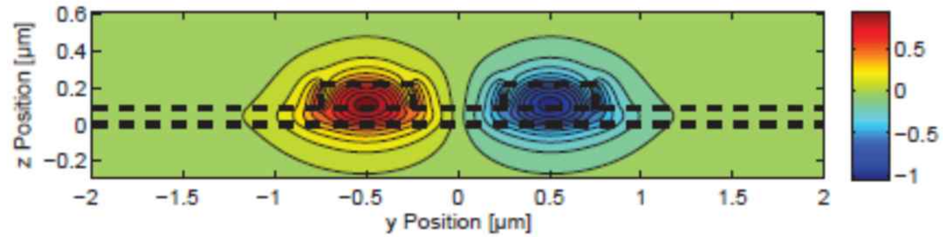
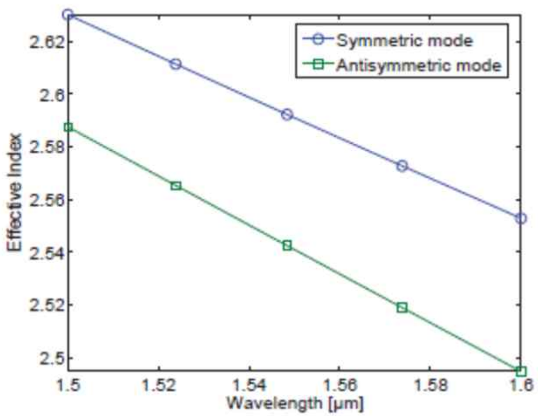




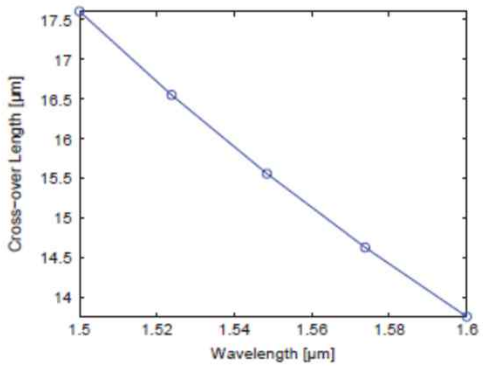
(a) Symmetric mode, Real(E_y); cp. Figure 4.13a



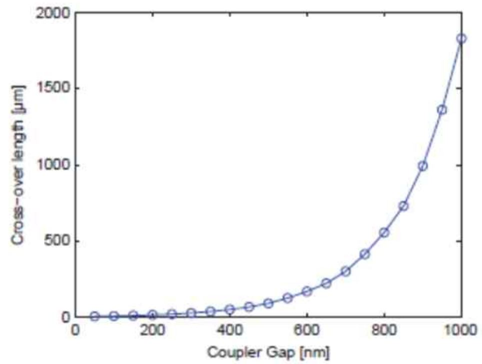
(b) Antisymmetric mode, Real(E_y)



(a) n_{eff} of the two modes

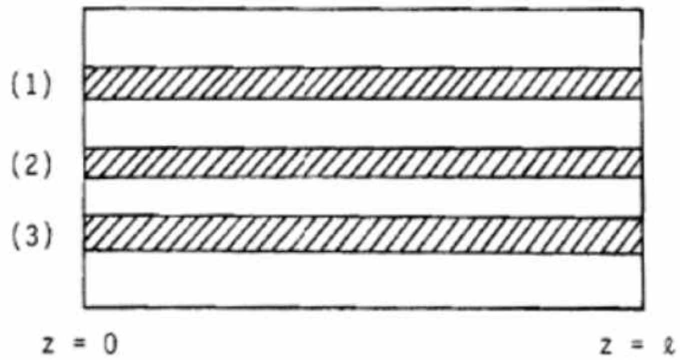


(b) Cross-over length



Homework #1

Consider a directional coupler made up of three identical waveguides as below. Each uncoupled waveguide has only one guided mode which propagates into z-direction with $\exp(-j\beta_0 z)$ dependence. Assume that coupling occurs only between neighboring waveguides or $\kappa_{12} = \kappa_{23} = -j\kappa$ and $\kappa_{13} = 0$.



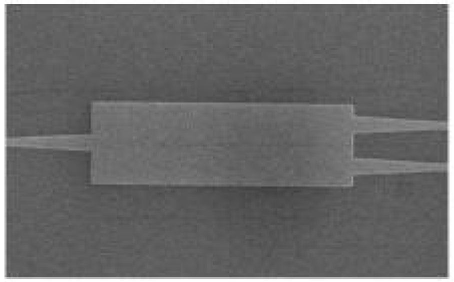
(a) Write the coupled-mode equations involving a1(z), a2(z) and a3(z).

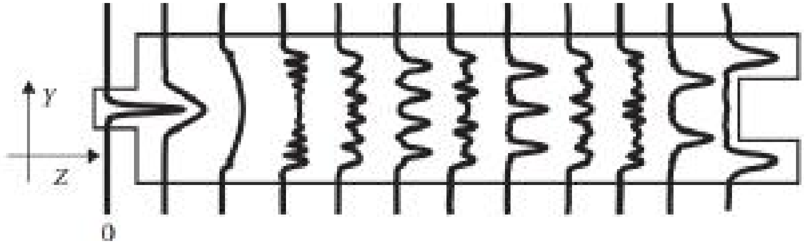
(b) Determine three propagation constants, β, that govern the wave propagation in the coupled waveguides.

(c) Assume a light signal is launched into the top waveguide, or a1(z=0)=1, a2(z=0) =a3 (z=0)=0. Determine the expressions for a1(z), a2(z), and a3(z).

NxM Multi-Mode Interferometer (MMI) Coupler

Example: 1x2 MMI





Input: One single-mode waveguide

Output: Two single-mode waveguides

Multi-mode waveguide in the middle

Beam profile changes due to interference among multiple guided modes

By carefully selecting multi-mode waveguide length, various functions possible

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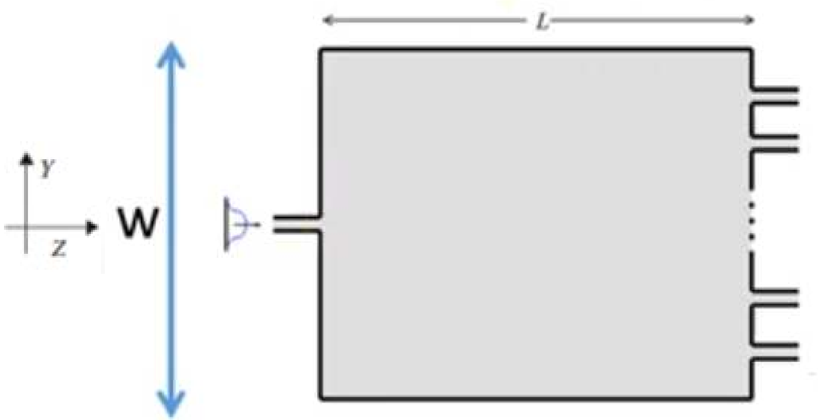
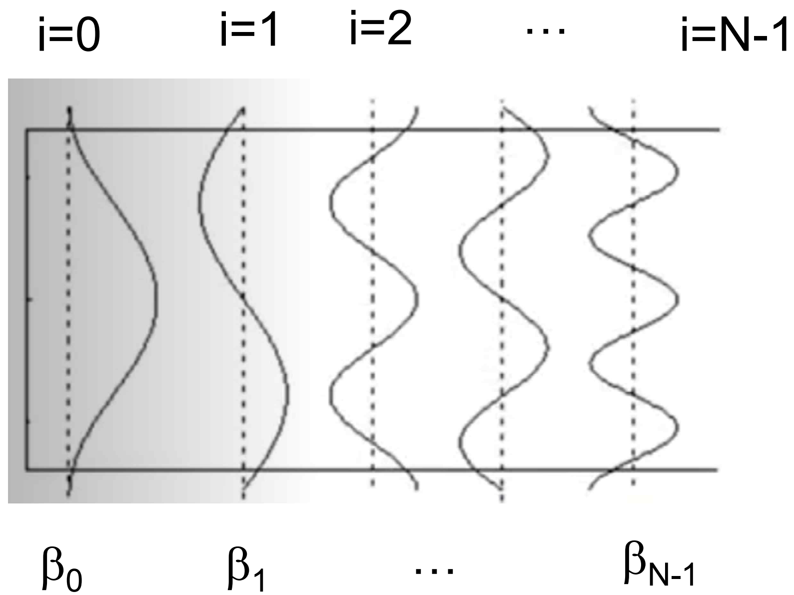


Mode Profile



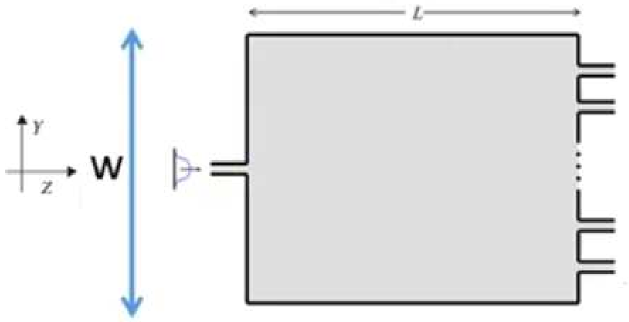
Assume W is much larger than 220nm so that we can treat the problem as 2-D

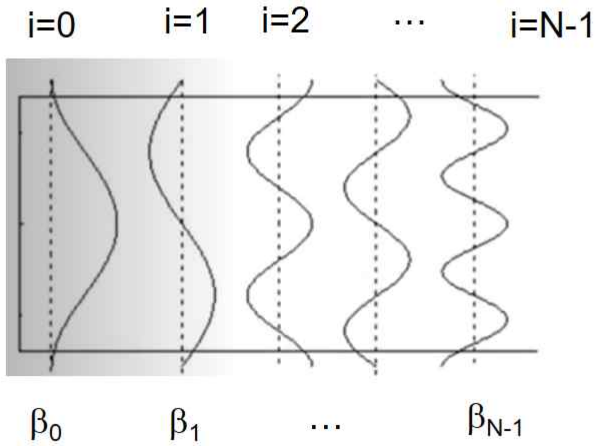
i even $E(y) \sim \cos(k_{y,i} y)$

i odd $E(y) \sim \sin(k_{y,i} y)$

$$k_{y,i} = \frac{(i+1)\pi}{W}$$

$$k_{y,i}{}^2 + \beta_i{}^2 = (n_M k_0)^2$$





i even $E(y) \sim \cos(k_{y,i} y)$

i odd $E(y) \sim \sin(k_{y,i} y)$

$$k_{y,i} = \frac{(i+1)\pi}{W}$$

$$E(y,z) = \sum_{i=0}^{N-1} C_i E_i(y) \exp(-j\beta_i z) \;= \exp(-j\beta_0 z) \sum_{i=0}^{N-1} C_i E_i(y) \exp[j(\beta_0 - \beta_i) z]$$

$$k_{y,i}{}^2 + \beta_i^2 = (n_M k_0)^2$$

$$\beta_i = \sqrt{(n_M k_0)^2 - k_{y,i}{}^2} = n_M k_0 \sqrt{1 - \frac{k_{y,i}{}^2}{(n_M k_0)^2}} \sim n_M k_0 \left(1 - \frac{k_{y,i}{}^2}{2(n_M k_0)^2}\right) = n_M k_0 - \frac{k_{y,i}{}^2}{2 n_M k_0}$$

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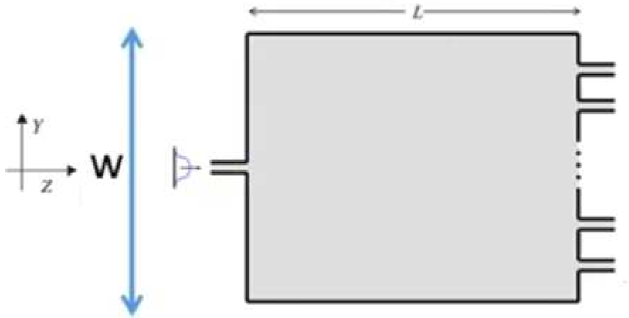

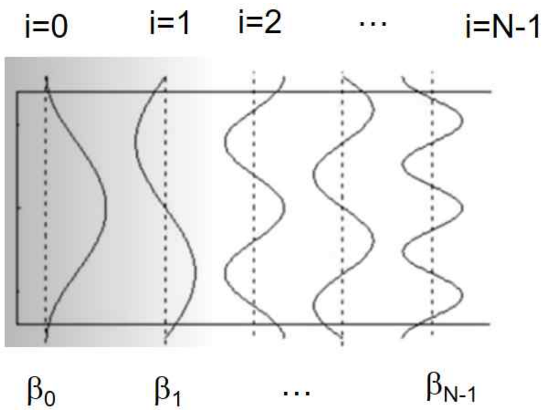

i even $E(y) \sim \cos(k_{y,i} y)$

i odd $E(y) \sim \sin(k_{y,i} y)$

$$k_{y,i} = \frac{(i+1)\pi}{W}$$

$$\beta_i = n_M k_0 - \frac{k_{y,i}^{\ 2}}{2 n_M k_0} \qquad E(y,z) = \exp(-j\beta_0 z)\sum_{i=0}^{N-1} C_i E_i(y)\exp[j(\beta_0 - \beta_i)z]$$

$$\beta_0 - \beta_i = \frac{i(i+2)\pi^2}{2 n_M k_0 W^2} = \frac{i(i+2)\pi\lambda}{4 n_M W^2} = \frac{i(i+2)\pi}{3 L_{eff}}$$

$$\beta_0 - \beta_1 = \frac{3\pi\lambda}{4 n_M W^2} = \frac{\pi}{L_{eff}} \quad \text{With } L_{eff} = \frac{4 n_M W^2}{3\lambda} \text{ or } (\beta_0\text{-}\beta_1)L_{eff} = \pi$$

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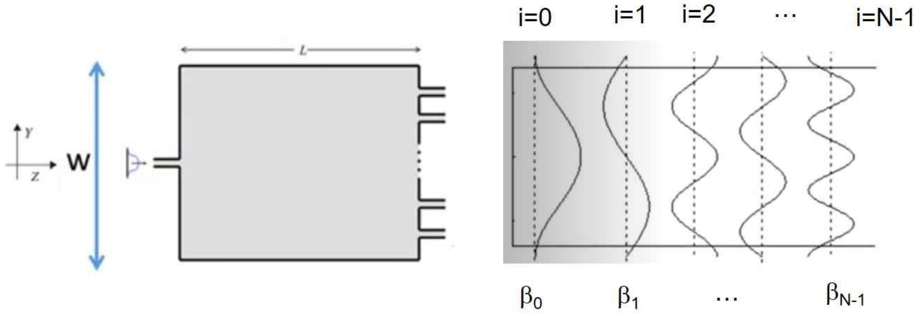


$$E(y,z) = \exp(-j\beta_0 z)\sum_{i=0}^{N-1} C_i E_i(y)\exp[j\frac{i(i+2)\pi}{3L_{eff}}z]$$

$$L_{eff} = \frac{4n_M W^2}{3\lambda} \text{ or } (\beta_0\text{-}\beta_1)L_{eff} = \pi$$

At $z = 6L_{eff}$ $\quad E(y,z) = \exp(-j\beta_0 6L_{eff})\sum_{i=0}^{N-1} C_i E_i(y)\exp[j2i(i+2)\pi]$

➔ Same intensity profile as z=0

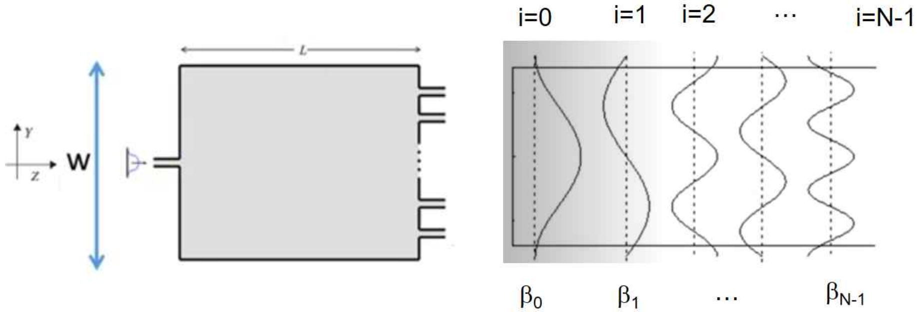
At $z = 3L_{eff}$ $\quad E(y,z) = \exp(-j\beta_0 3L_{eff})\sum_{i=0}^{N-1} C_i E_i(y)\exp[ji(i+2)\pi]$

$$= \exp(-j\beta_0 3L_{eff})\left\{\sum_{i=0,\text{even}}^{N-1} C_i E_i(y) - \sum_{i=0,\text{odd}}^{N-1} C_i E_i(y)\right\}$$

$$E_i(y) = E_i(-y) \qquad E_i(y) = -E_i(-y)$$

$$= \exp(-j\beta_0 3L_{eff})\left\{\sum_{i=0,\text{even}}^{N-1} C_i E_i(-y) + \sum_{i=0,\text{odd}}^{N-1} C_i E_i(-y)\right\}$$

Lecture 10: Passive Waveguide Devices

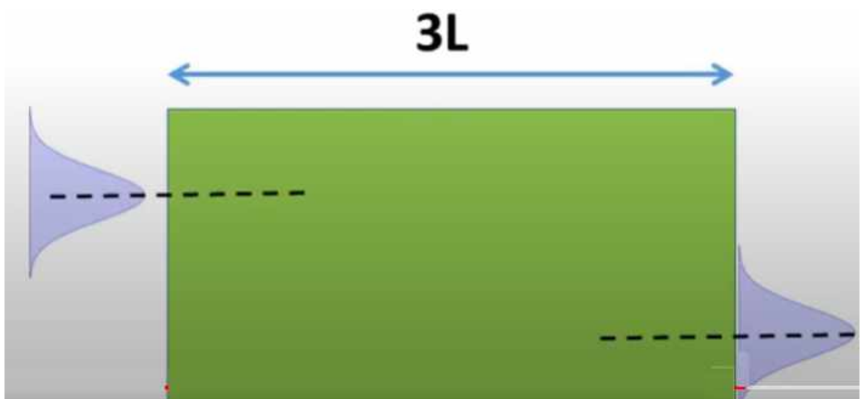


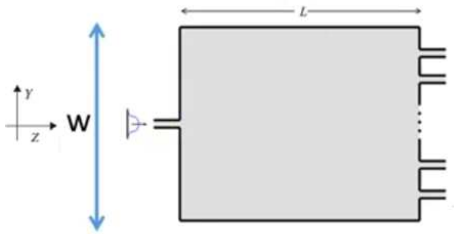
$$E(y,z)=\exp(-j\beta_0 z)\sum_{i=0}^{N-1} C_i E_i(y)\exp[j\frac{i(i+2)\pi}{3L_{eff}}z]$$

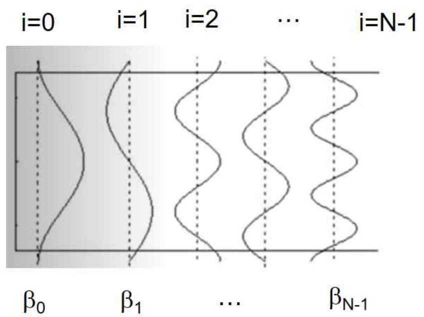
$$L_{eff}=\frac{4n_M W^2}{3\lambda} \text{ or } (\beta_0\text{-}\beta_1)L_{eff}=\pi$$

At $z=3L_{eff}$ $\quad E(y,z)=\exp(-j\beta_0 3L_{eff})\left\{\sum_{i=0,\text{even}}^{N-1} C_i E_i(-y)+\sum_{i=0,\text{odd}}^{N-1} C_i E_i(-y)\right\}$

Anti-symmetric intensity profile to z=0







$$E(y,z) = \exp(-j\beta_0 z)\sum_{i=0}^{N-1} C_i E_i(y)\exp[j\frac{i(i+2)\pi}{3L_{eff}}z]$$

$$L_{eff} = \frac{4n_M W^2}{3\lambda} \text{ or } (\beta_0\text{-}\beta_1)L_{eff} = \pi$$

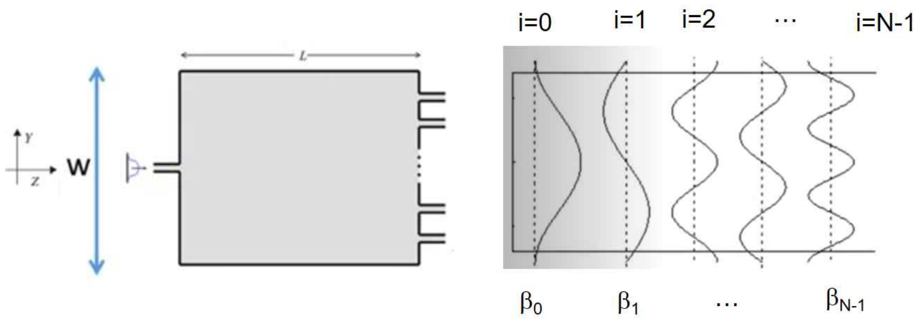
At $z = \frac{3}{2}L_{eff}$ $\quad E(y,z) = \exp(-j\beta_0\frac{3L_{eff}}{2})\sum_{i=0}^{N-1} C_i E_i(y)\exp[j\frac{i(i+2)\pi}{2}]$

$$= \exp(-j\beta_0\frac{3L_{eff}}{2})\left\{\sum_{i=0,\text{even}}^{N-1} C_i E_i(y) + \sum_{i=0,\text{odd}}^{N-1} -jC_i E_i(y)\right\}$$

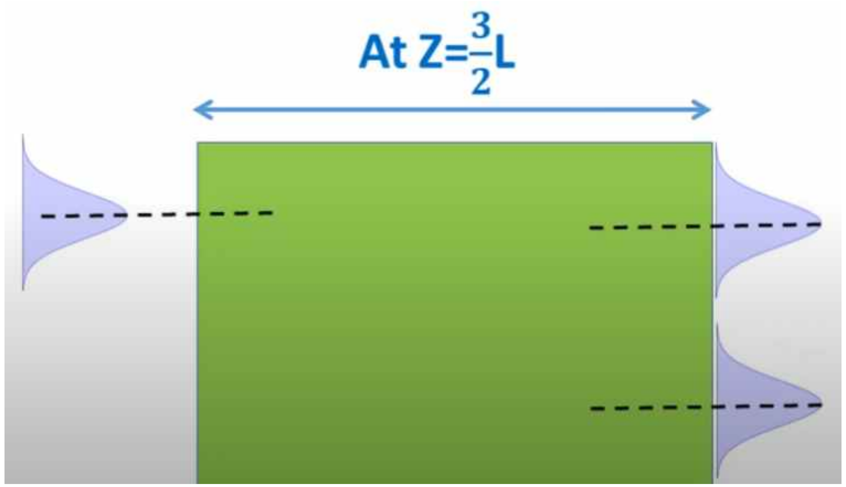
$$= \exp(-j\beta_0\frac{3L_{eff}}{2})\left[\left\{\left(\frac{1-j}{2}\right)\sum_{i=0,\text{even}}^{N-1} C_i E_i(y) + \left(\frac{1+j}{2}\right)\sum_{i=0,\text{even}}^{N-1} C_i E_i(y)\right\} + \left\{\left(\frac{1-j}{2}\right)\sum_{i=0,odd}^{N-1} C_i E_i(y) + \left(\frac{-1-j}{2}\right)\sum_{i=0,odd}^{N-1} C_i E_i(y)\right\}\right]$$

$$= \exp(-j\beta_0\frac{3L_{eff}}{2})\left[\left(\frac{1-j}{2}\right)\sum_{i=0,\text{all}}^{N-1} C_i E_i(y) + \left(\frac{1+j}{2}\right)\sum_{i=0,\text{all}}^{N-1} C_i E_i(-y)\right]$$

Lecture 10: Passive Waveguide Devices



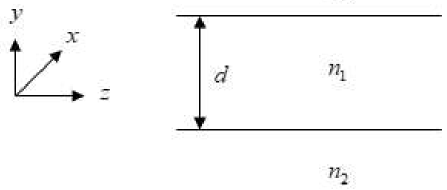
$$\text{At } z = \frac{3}{2}L_{eff} \quad E(y,z) = \exp(-j\beta_0 \frac{3L_{eff}}{2}) \left[\left(\frac{1-j}{2}\right) \sum_{i=0,\text{all}}^{N-1} C_i E_i(y) + \left(\frac{1+j}{2}\right) \sum_{i=0,\text{all}}^{N-1} C_i E_i(-y) \right]$$

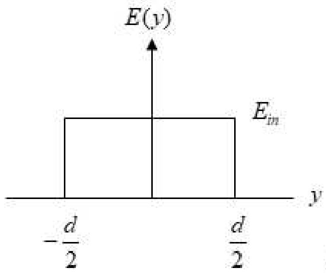


➔ 3-dB coupler

Homework #2

Assume that the symmetric three-layer dielectric waveguide shown below left supports 3 TE guided modes.





(a) Estimate the propagation constants β for each guided mode. For the estimation, assume that the confinement factor for each mode is very close to one so that the evanescent fields in the cladding can be assumed zero. Or assume the E-field becomes zero at both top and bottom boundaries.

(b) An input field with TE polarization having the rectangular shape as shown above right is introduced to the waveguide at z=0. Among three guided modes traveling in z-direction, only two modes are excited. Determine which mode is not excited and briefly explain why.

(c) Two excited modes can experience interference as they travel along z-direction. Determine the condition for z, where the E-field intensity profile is identical to that of z=0.

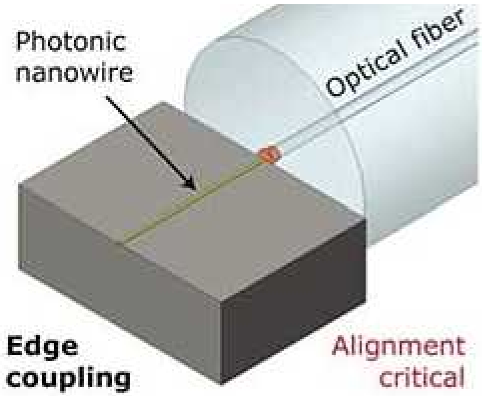
(d) Determine how many much of the input power is coupled to the fundamental mode.

Lecture 10: Passive Waveguide Devices

How do we get light
in and out of Si PIC?

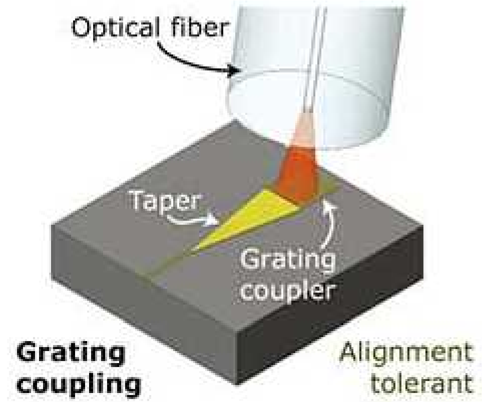
Si light source
is not available

➔ Externally supplied



Good coupling possible
with spot-size conversion

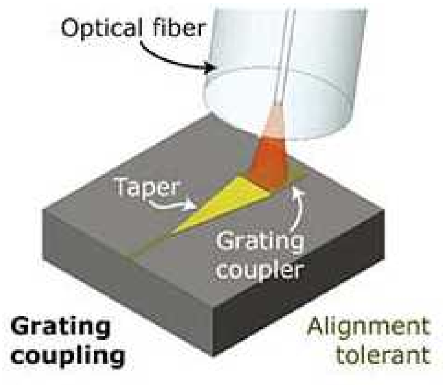
Requires
- Precise alignment
- Polishing fiber facet
- AR coating
- Dicing of a chip



No dicing required

Wafer-level optical
testing possible

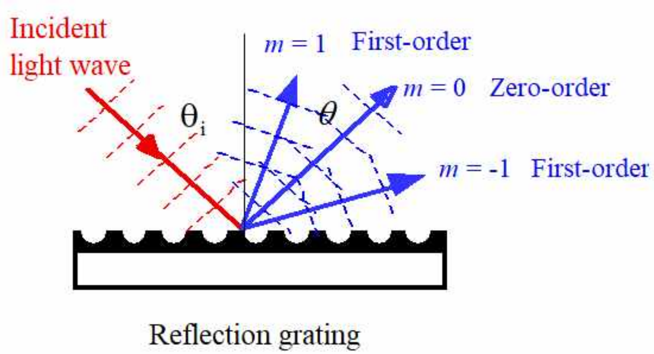
Lecture 10: Passive Waveguide Devices



Out

In

Reflection type grating with period d



Reflection grating

Condition for vertical coupling out?

$$\theta_i = 90^\circ \qquad \theta = 0^\circ$$

$d = \lambda$ With m=−1

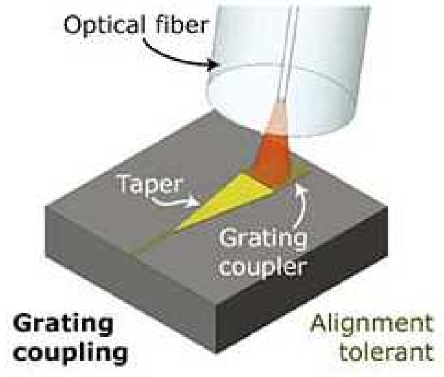
$$\mathrm{d}(\sin\theta - \sin\theta_i) = \mathrm{m} \cdot \lambda$$

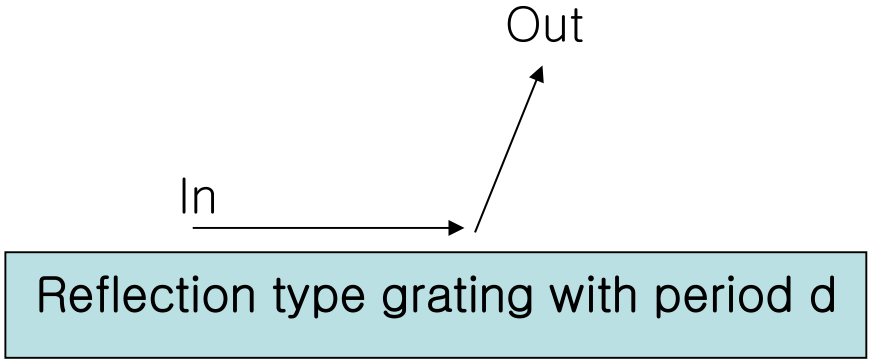
But with d=λ m=0, $\theta = 90^\circ$

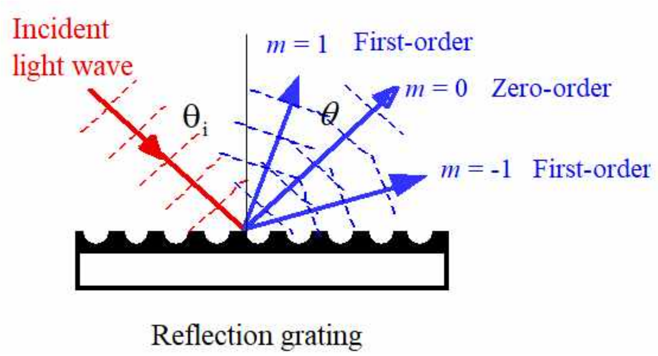
m=−2, $\theta = -90^\circ$

Also satisfied ➔ Not desired

Lecture 10: Passive Waveguide Devices







$$\mathrm{d}(\sin\theta - \sin\theta_i) = \mathrm{m}\cdot\lambda$$

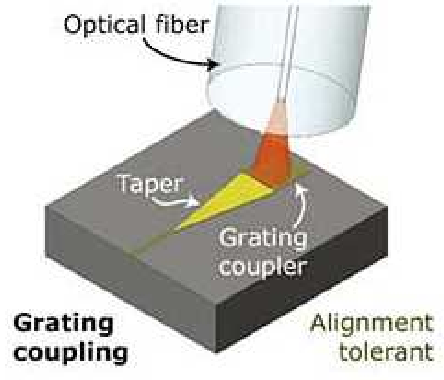
$$\theta_i = 90^\circ$$

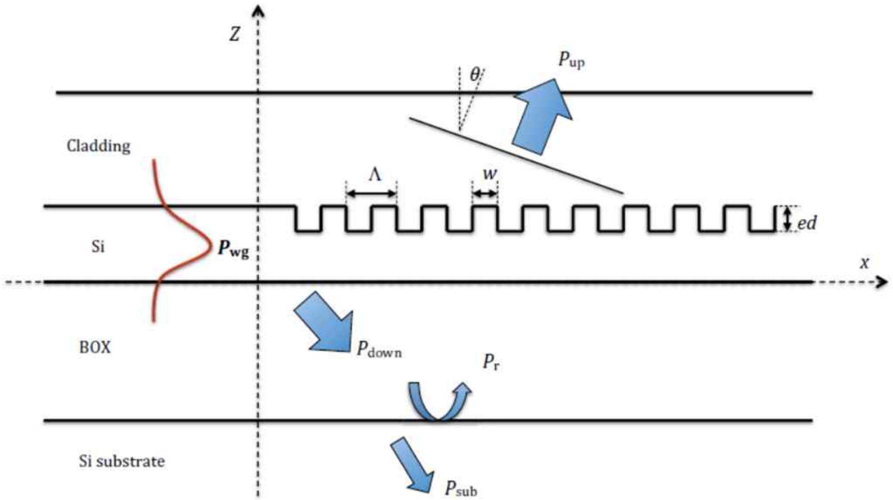
$$d(\sin\theta - 1) = m\cdot\lambda$$

If $$d = \frac{\lambda}{1-\sin\theta} \quad (m=-1)$$

Output only at θ and 180− θ

Lecture 10: Passive Waveguide Devices





$$k_{x,out} = k_{x,in} + m \cdot \frac{2\pi}{d}$$

$$n_c \frac{2\pi}{\lambda} \sin\theta_c = n_{eff} \frac{2\pi}{\lambda} + m \cdot \frac{2\pi}{d}$$

For m=-1

Snell's Law

$$d = \frac{\lambda}{n_{eff} - n_c \sin\theta_c} = \frac{\lambda}{n_{eff} - \sin\theta_{air}}$$

Diffraction at 180°- θ_c is also allowed