

Si Photonics

Lecture 11 : Si Modulators

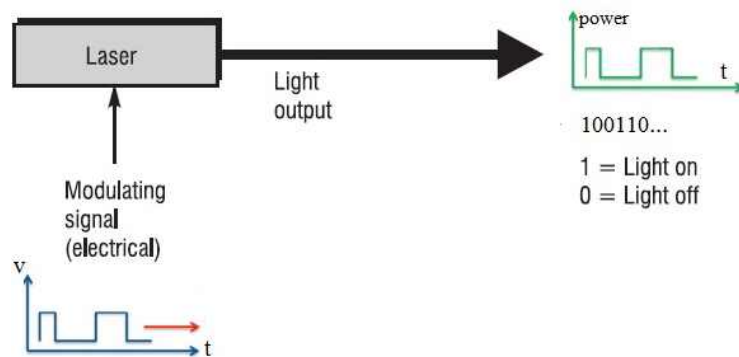
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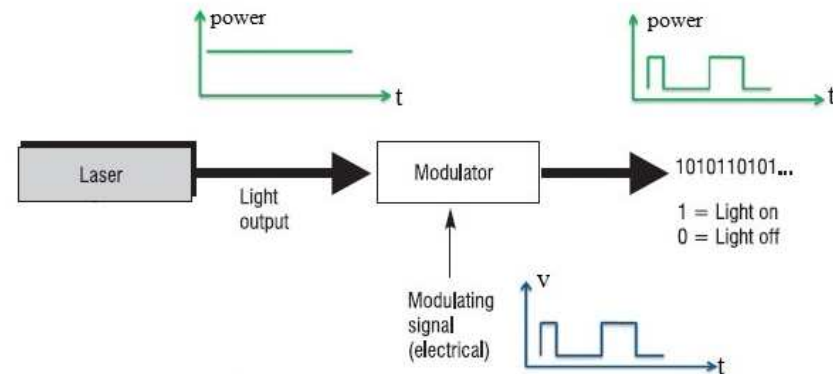
Lecture 11: Si Waveguide Devices

How to generate light signals with (digital) information?

Direct Modulation



External Modulation



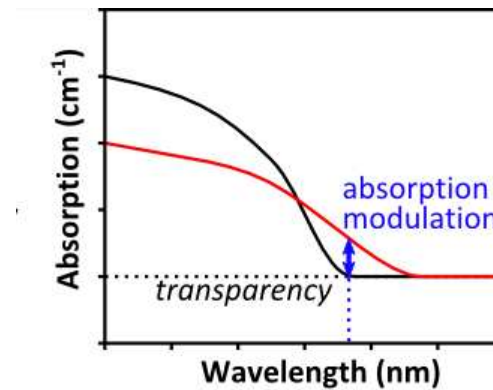
- Simple
- For high-speed modulation, semiconductor laser suffers from frequency chirp
(Change in lasing frequency → Suffers from fiber dispersion in long-distance optical comm.)
- No Si laser

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External modulation

I. Refractive index modulation

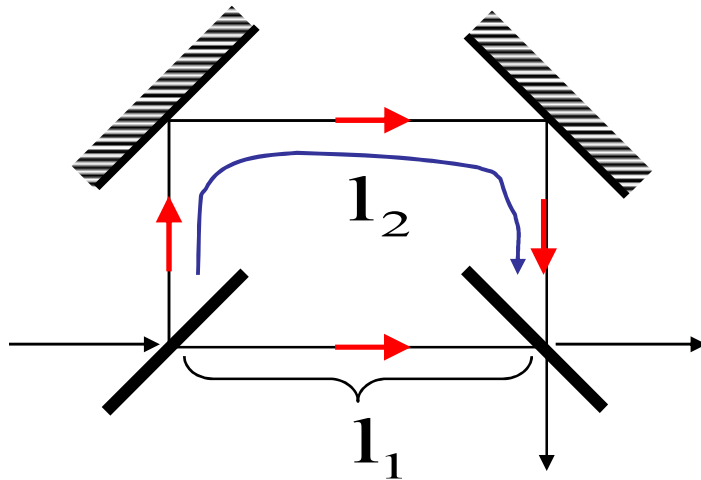
II. Bandgap (absorption) modulation



In Si photonics, Type I is often used

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Mach-Zehnder Interferometer



$$I_{out, bottom} = \frac{I_{in}}{2} [1 + \cos(kl_1 - kl_2)]$$

What happens when $(kl_1 - kl_2)$ changes?

Output intensity changes

Intensity modulation

➔ MZ Modulator (MZM)

MZM with waveguides?

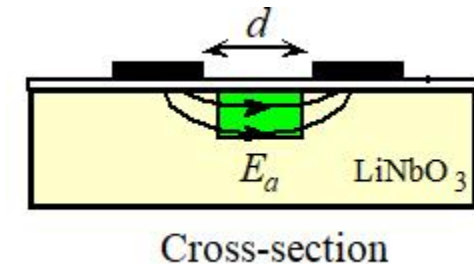
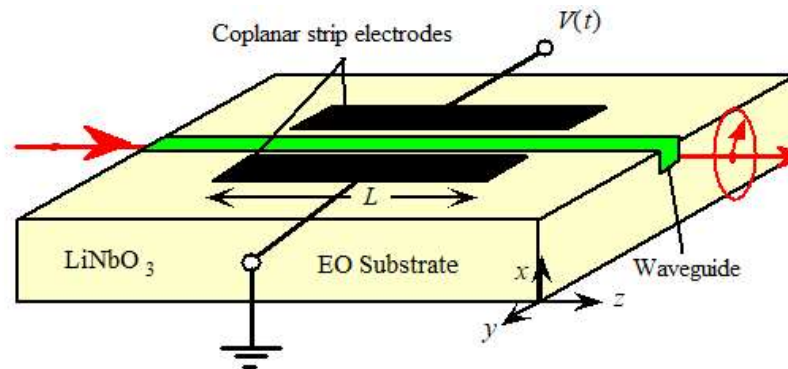
Modulation with voltage signals?

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LiNbO₃ (Lithium Niobate) waveguide



Waveguides are formed by metal (Ti) doping



Very small loss for the wavelength of interest

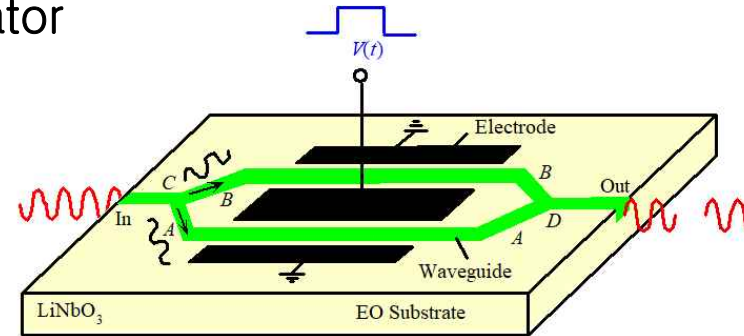
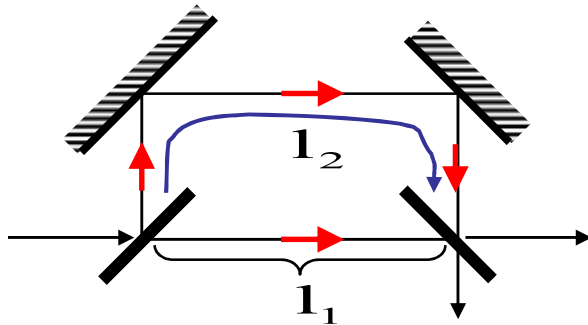
Refractive index of LiNbO₃ can be changed by E-field
(Large linear electro-optic effect or Pockels effect)

➔ Voltage controlled phase shift or delay line

EO effect is very fast (Speed is limited by electrodes)

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LiNbO₃ (Lithium Niobate) MZ Modulator



$$I_{out} = \frac{I_{in}}{2} [1 + \cos(kl_1 - kl_2)] \Rightarrow \frac{I_{in}}{2} [1 + \cos(\beta_1 - \beta_2)L] \quad \beta_1 = n_{eff,1}k_o \quad \beta_2 = n_{eff,2}k_o$$

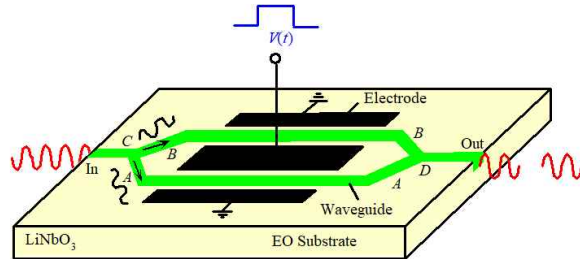
With $V(t)$, $n_{eff,1} = n_{eff,0} + \Delta n_{eff}(V(t))$ $n_{eff,2} = n_{eff,0} - \Delta n_{eff}(V(t))$

$$\beta_1 - \beta_2 = 2\Delta n_{eff}(V(t))k_o$$

$$I_{out} = \frac{I_{in}}{2} [1 + \cos(2\Delta n_{eff}(V(t))k_o L)]$$

➔ Widely used in high-performance optical communication systems

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$$I_{out} = \frac{I_{in}}{2} \left[1 + \cos \left(2\Delta n_{eff}(V(t))k_0L \right) \right]$$

How to realize Si MZM?

Change Si waveguide effective index with $V(t)$

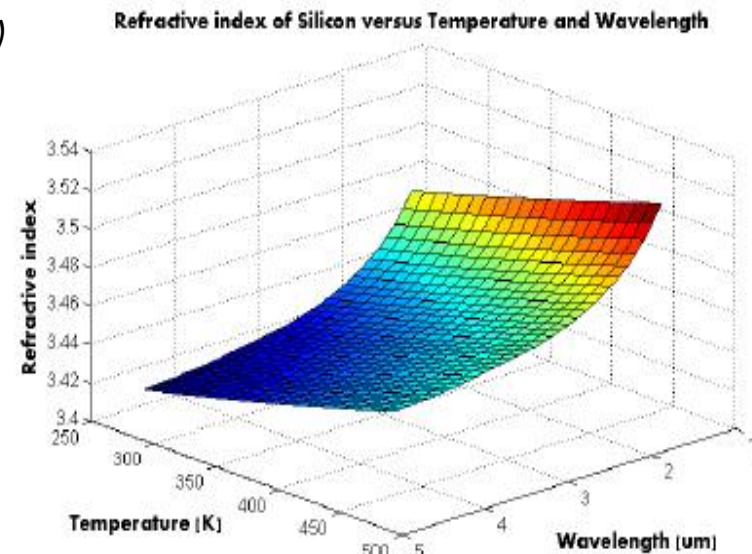
But Si does not have any linear E-O effect

Si refractive index changes with temperature

For 1500nm at RT, $dn/dT = 1.87 \times 10^{-4} \text{ K}^{-1}$

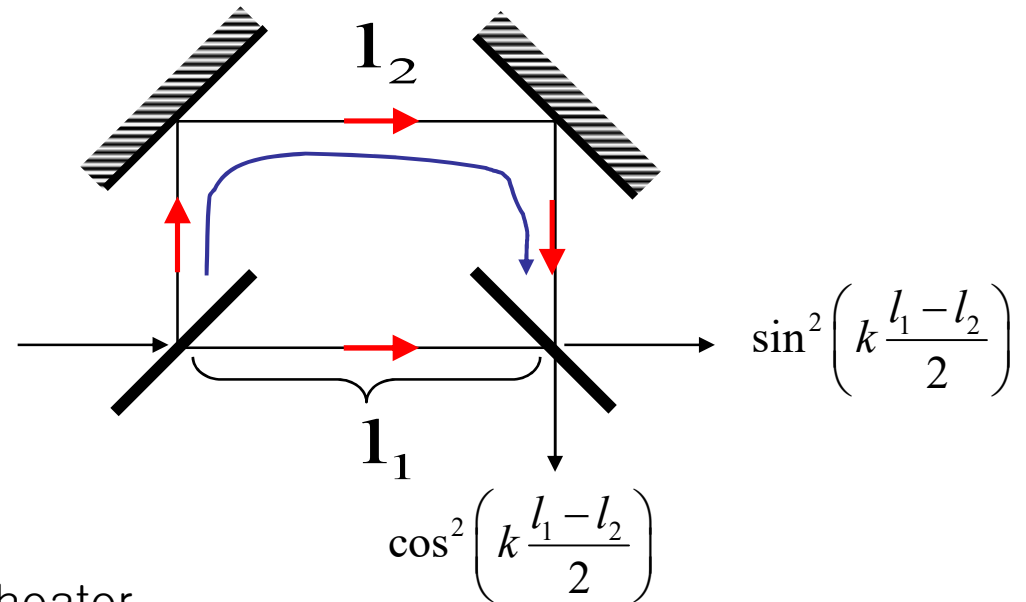
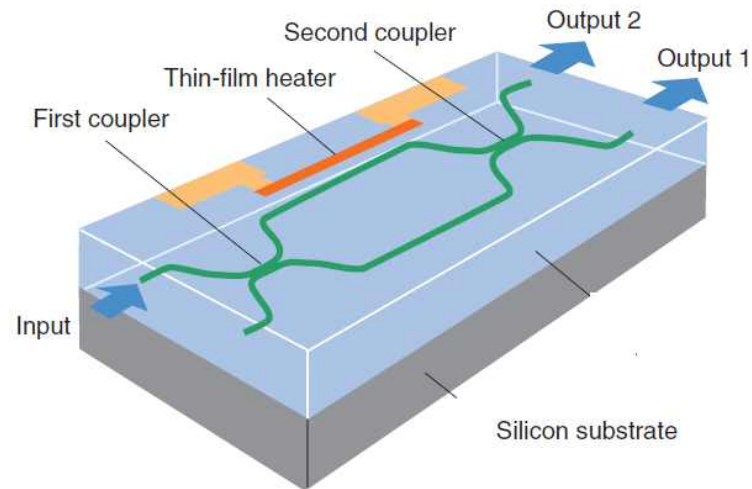
Efficient but slow

➔ Used for slow switching devices

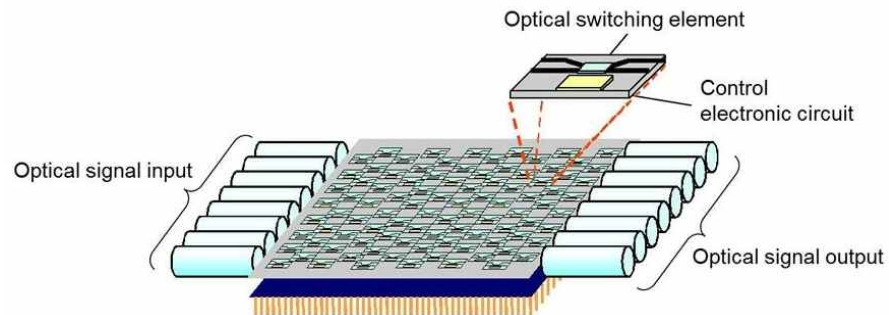


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Si MZI Switch



Temperature change with a built-in heater



Optical switching

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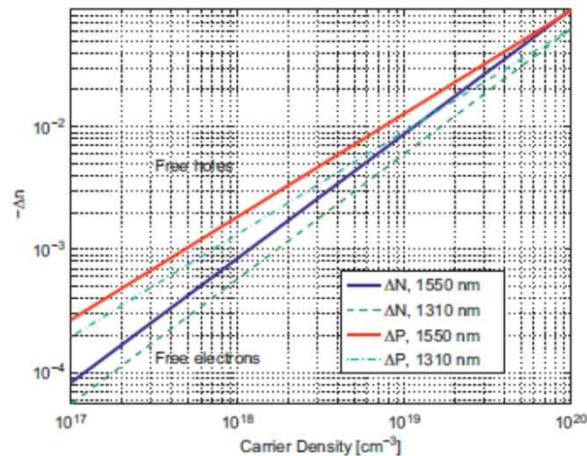
Si MZM



$$\frac{I_{in}}{2} \left[1 + \cos(n_{eff,1} - n_{eff,2})k_0L \right]$$

How to change Si refractive index with voltage fast?

Change amount of carriers (N and P) inside Si waveguides



Plasma dispersion effect

$$\Delta n \text{ (at 1550 nm)} = -8.8 \times 10^{-22} \Delta N - 8.5 \times 10^{-18} \Delta P^{0.8}$$

$$\Delta n \text{ (at 1310 nm)} = -6.2 \times 10^{-22} \Delta N - 6 \times 10^{-18} \Delta P^{0.8}$$

(Soref and Bennett)

More carriers → Smaller refractive index

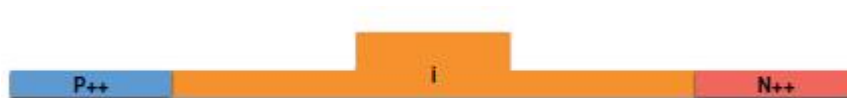
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How to change N, P?

Forward-biased PIN Junction



$$\frac{I_{in}}{2} \left[1 + \cos(n_{eff,1} - n_{eff,2})k_0L \right]$$



More carriers

Smaller refractive index

➔ Smaller effective index

Very efficient

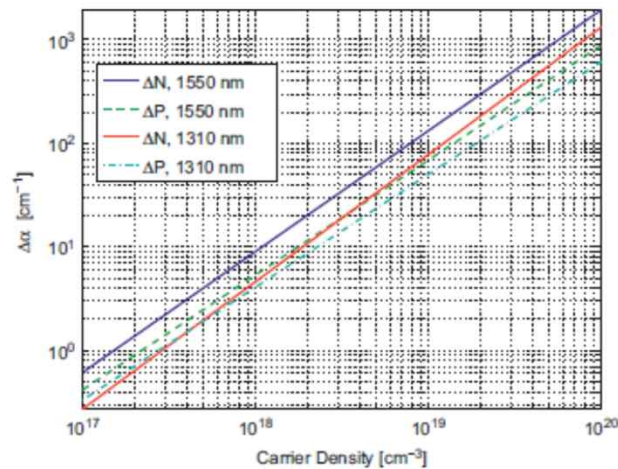
But not fast enough due to carrier life time (~ 1 nsec)

Injected carriers induce loss

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How to change N, P?

Forward-biased PIN Junction



➔ Variable Optical Attenuator



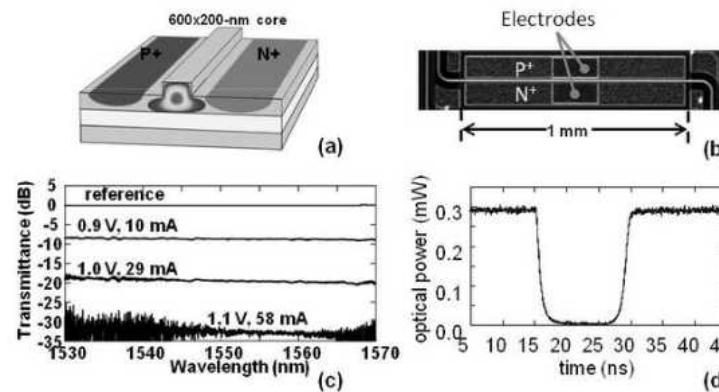
$$\frac{I_{in}}{2} \left[1 + \cos(n_{eff,1} - n_{eff,2})k_0L \right]$$

$$\Delta\alpha \text{ (at 1550 nm)} = 8.5 \times 10^{-18} \Delta N + 6 \times 10^{-18} \Delta P \text{ [cm}^{-1}\text{]}$$

$$\Delta\alpha \text{ (at 1310 nm)} = 6 \times 10^{-18} \Delta N + 4 \times 10^{-18} \Delta P \text{ [cm}^{-1}\text{]},$$

Refractive index change \longleftrightarrow Loss change

(Kramers–Kronig relationship)



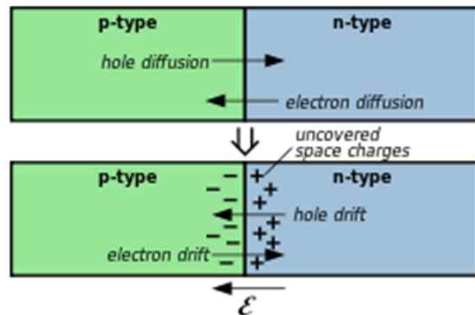
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How to change N, P very fast?

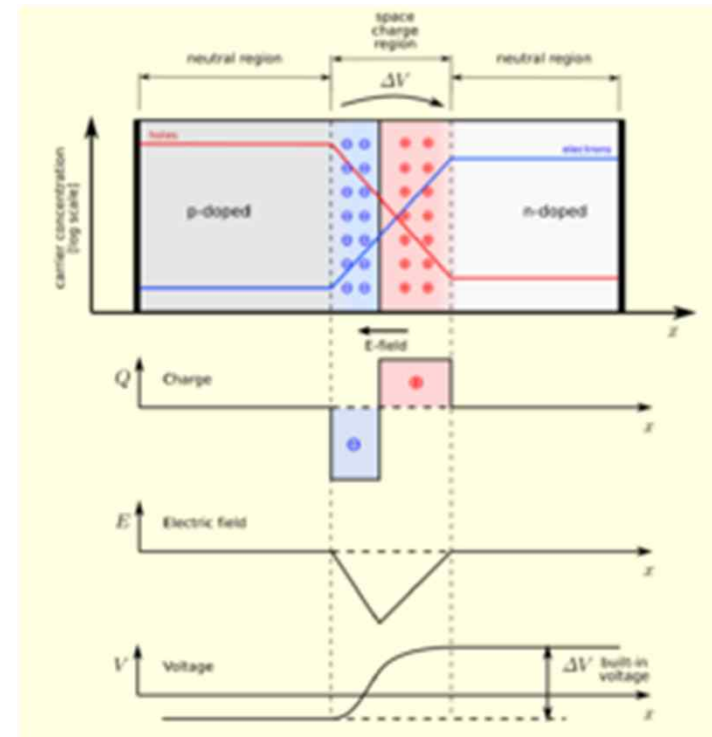


$$\frac{I_{in}}{2} \left[1 + \cos(n_{eff,1} - n_{eff,2})k_0L \right]$$

Reversed-Biased PN Junction



Formation of depletion region
Between P and N regions



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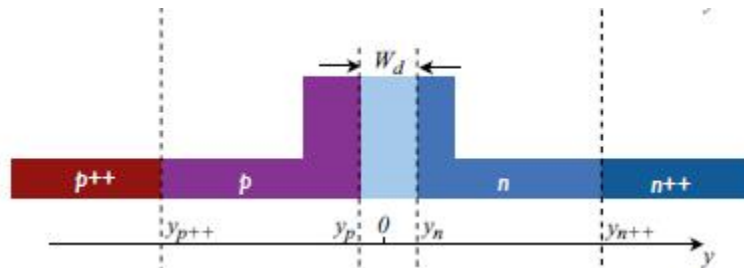
How to change N, P very fast?



$$\frac{I_{in}}{2} \left[1 + \cos(n_{eff,1} - n_{eff,2})k_0L \right]$$

Reverse-biased PN Junction

Assuming uniform abrupt junction



$$W_d = \sqrt{\frac{2\epsilon_0\epsilon_s(N_A + N_D)(V_{bi} - V)}{qN_A N_D}}$$

With larger reverse bias, larger W_d

Less plasma dispersion effect because less carriers interact with guided light

Larger refractive index

→ Larger effective index

Not as efficient as thermal, PIN phase shifter

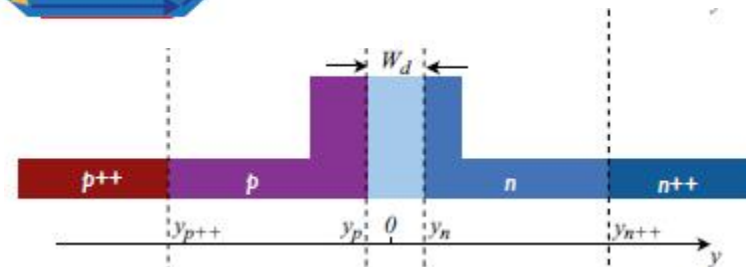
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How to change N, P very fast?



$$\frac{I_{in}}{2} \left[1 + \cos(n_{eff,1} - n_{eff,2})k_0L \right]$$

Reverse-biased PN Junction



$$W_d = \sqrt{\frac{2\epsilon_0\epsilon_s(N_A + N_D)(V_{bi} - V)}{qN_A N_D}}$$

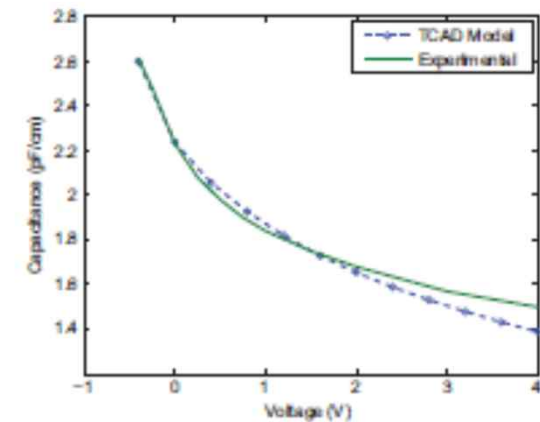
Frequency response:

Dominated by RC time constant

R: Series resistance


C: Junction capacitance = Area $\times \epsilon_{si} / W_d$

$$\sim \sqrt{\frac{q\epsilon_0\epsilon_s}{2(1/N_D + 1/N_A)(V_{bi} - V)}}$$

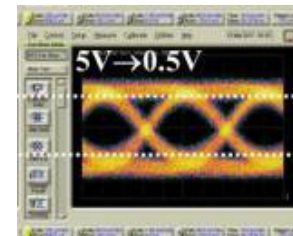
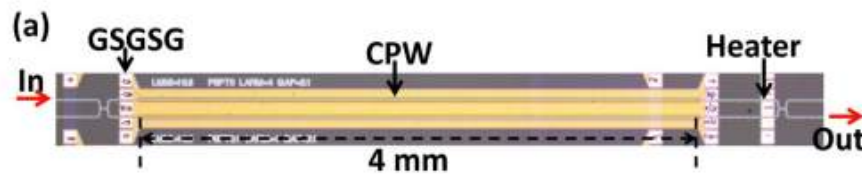
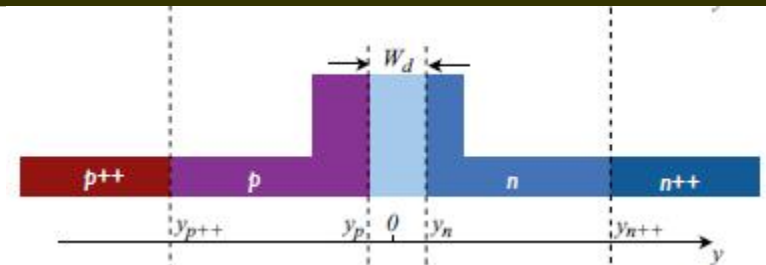


➔ Most popular method for high-speed Si phase shifter

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$$\frac{I_{in}}{2} \left[1 + \cos(n_{eff,1} - n_{eff,2})k_0L \right]$$



25 Gbps
operation

(Optics Letters, Vol. 24, No. 3, p. 403, 2018)

Built-in heater required → Compensating phase shift due to process variation

Very long → Requires transmission line electrodes with impedance termination

Smaller Si modulator?

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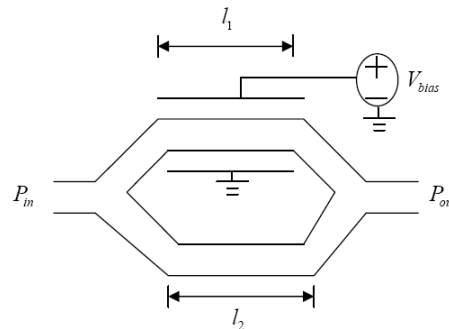
Homework #1

Consider a Mach-Zehnder interferometer shown below. The effective index that $1.5\ \mu\text{m}$ light experiences while traveling inside the interferometer is 3.5 when no bias voltage is applied. Due to manufacturing problems, $l_1 = 100\ \mu\text{m}$ and $l_2 = 100.1\ \mu\text{m}$ are not the same.

(a) What is the output power when the input power is 1mW at $1.5\ \mu\text{m}$ and no bias is applied? Assume P_{out} has \cos^2 dependence on P_{in} .

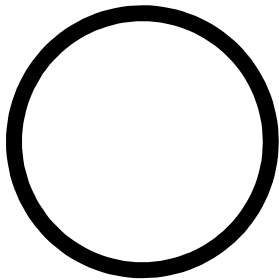
We want to use the interferometer as an optical on/off switch by applying voltage to the upper arm as shown. The effective index of the upper arm increases 0.001 per 1 volt applied.

(b) What is the voltage with the smallest absolute value that needs to be applied to make the switch on?



Lecture 11: Si Waveguide Devices

Consider a ring waveguide



Assuming there is light circulating inside the ring

Resonance condition:

$$e^{-j\beta(2\pi r)} = 1$$

$$n_{eff} \frac{2\pi}{\lambda} 2\pi r = 2m\pi \quad L = m \frac{\lambda}{n_{eff}}$$

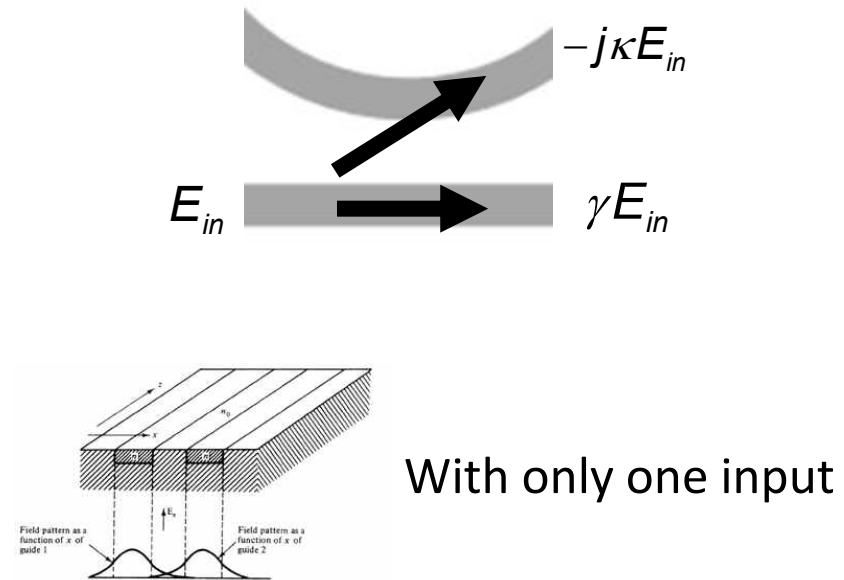
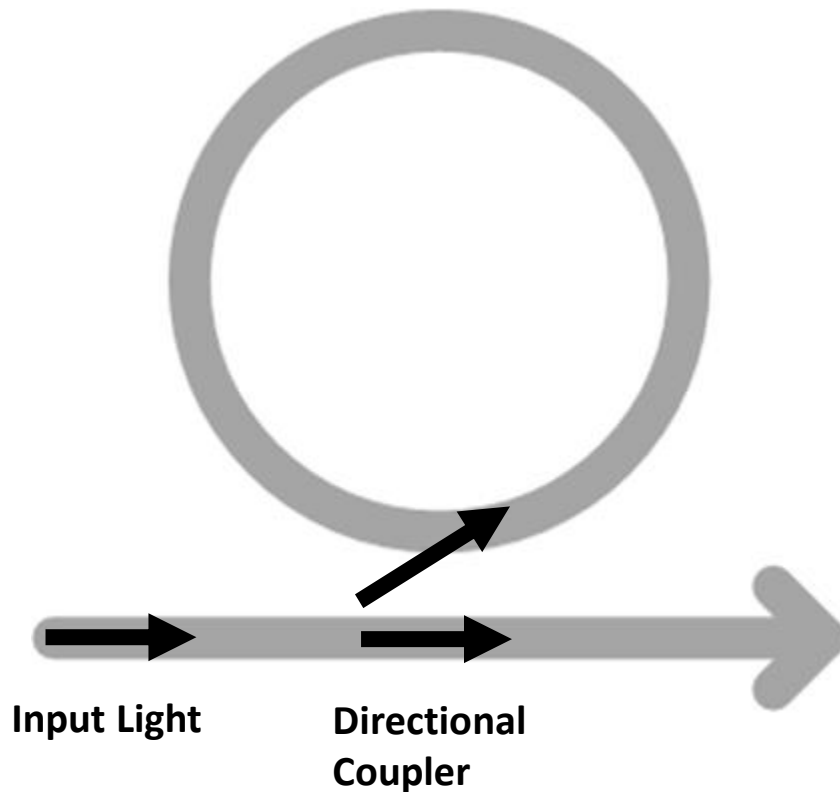
What can we do with this?

Effectively F-P resonator

How to couple light in and out?

Lecture 11: Si Waveguide Devices

- Ring Interferometer (Top View)



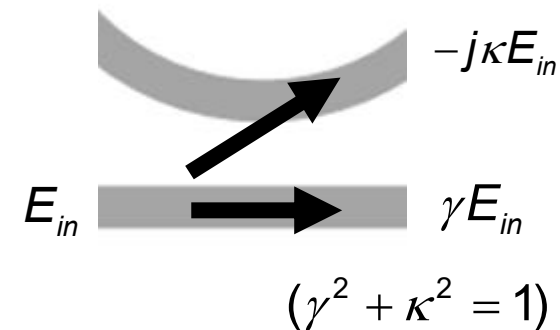
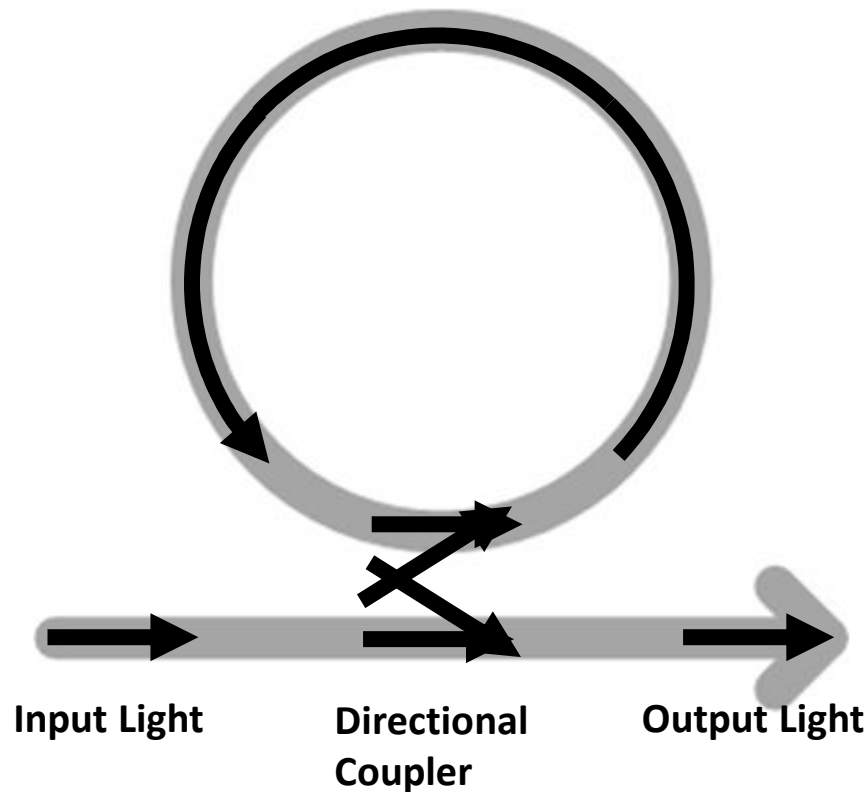
$$\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} \sim \begin{bmatrix} -j \sin(\kappa' z) \\ \cos(\kappa' z) \end{bmatrix}$$

$$\gamma = \cos(\kappa' d) \quad \kappa = \sin(\kappa' d)$$

$$(\gamma^2 + \kappa^2 = 1)$$

Lecture 11: Si Waveguide Devices

- Ring Interferometer (Top View)



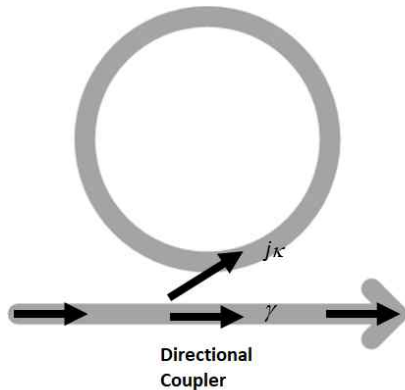
α : amount after one circulation

L : ring circumference

$$E_{out} = E_{in}\gamma + E_{in}(-j\kappa)(\alpha e^{-jn_{eff}k_0L})(-j\kappa) \\ + E_{in}(-j\kappa)(\alpha e^{-jn_{eff}k_0L})\gamma(\alpha e^{-jn_{eff}k_0L})(-j\kappa) \\ \dots$$

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Ring Interferometer



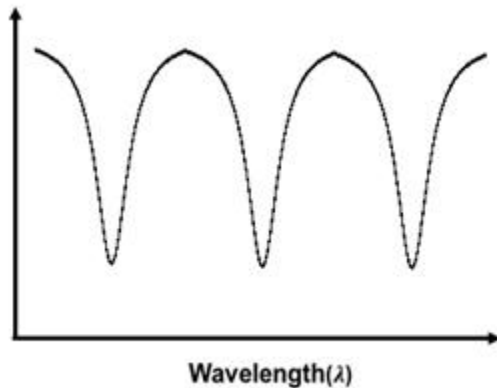
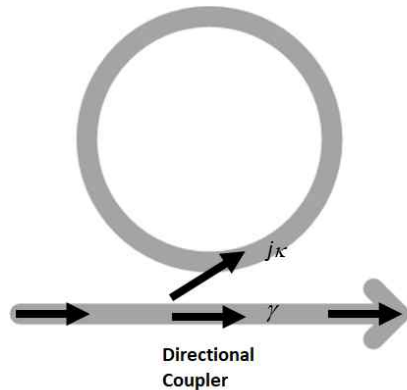
$$E_{out} = E_{in}\gamma + E_{in}(-j\kappa)(\alpha e^{-jn_{eff}k_0L})(-j\kappa) \\ + E_{in}(-j\kappa)(\alpha e^{-jn_{eff}k_0L})\gamma(\alpha e^{-jn_{eff}k_0L})(-j\kappa) \\ \dots$$

$$(\beta = n_{eff}k_0)$$

$$\frac{E_{out}}{E_{in}} = \gamma - \kappa^2 \alpha e^{-j\beta L} - \kappa^2 \alpha^2 \gamma e^{-j2\beta L} - \kappa^2 \alpha^3 \gamma^2 e^{-j3\beta L} - \dots \\ = \gamma - \kappa^2 \sum_{n=1}^{\infty} \alpha^n \gamma^{n-1} e^{-jn\beta L} = \gamma - \frac{\kappa^2 \alpha e^{-j\beta L}}{1 - \alpha \gamma e^{-j\beta L}} = \frac{\gamma - \alpha e^{-j\beta L}}{1 - \alpha \gamma e^{-j\beta L}}$$

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Ring Interferometer



$$E_{out} = E_{in}\gamma + E_{in}(-j\kappa)(\alpha e^{-j\beta L})(-j\kappa) \\ + E_{in}(-j\kappa)(\alpha e^{-j\beta L})\gamma(\alpha e^{-j\beta L})(-j\kappa) + \dots$$

$$\frac{E_{out}}{E_{in}} = \frac{\gamma - \alpha e^{-j\beta L}}{1 - \alpha\gamma e^{-j\beta L}} \quad T = \left| \frac{E_{out}}{E_{in}} \right|^2 = \frac{\alpha^2 + \gamma^2 - 2\alpha\gamma \cos(\beta L)}{1 + (\alpha\gamma)^2 - 2\alpha\gamma \cos(\beta L)}$$

It can be shown min. transmission occurs when $\beta L = 2m\pi$ (Resonance condition)

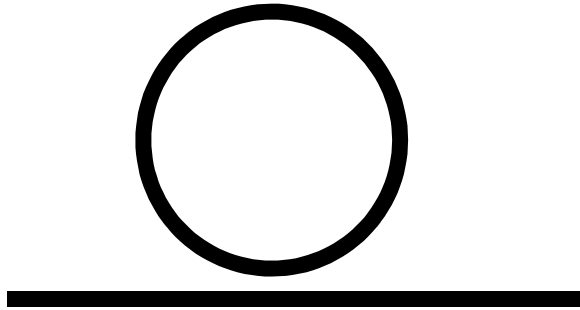
$$n_{eff} \frac{2\pi}{\lambda} L = 2m\pi \quad L = m \frac{\lambda}{n_{eff}} \quad T = \left| \frac{E_{out}}{E_{in}} \right|^2 = \frac{(\alpha - \gamma)^2}{(1 - \alpha\gamma)^2}$$

$T=0$ if $\gamma=\alpha$; critical coupling

$\gamma<\alpha$; over coupling

$\gamma>\alpha$; under coupling

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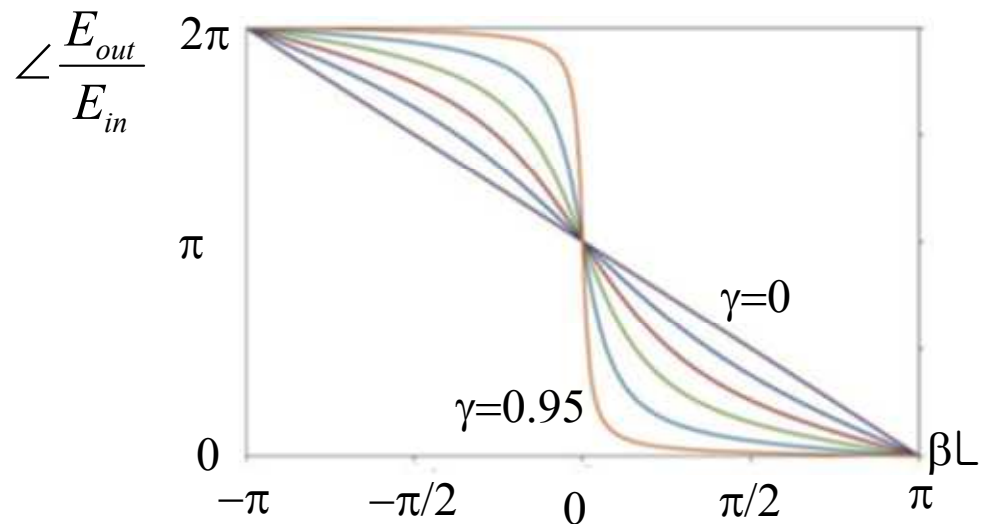
What good is this?

$$\text{If } \alpha=1, \quad \frac{E_{out}}{E_{in}} = \frac{\gamma - e^{-j\beta L}}{1 - \gamma e^{-j\beta L}} \quad T = \frac{\alpha^2 + \gamma^2 - 2\alpha\gamma \cos(\beta L)}{1 + (\alpha\gamma)^2 - 2\alpha\gamma \cos(\beta L)} = 1$$

Phase-domain filter

➔ Optical All-Pass Filter

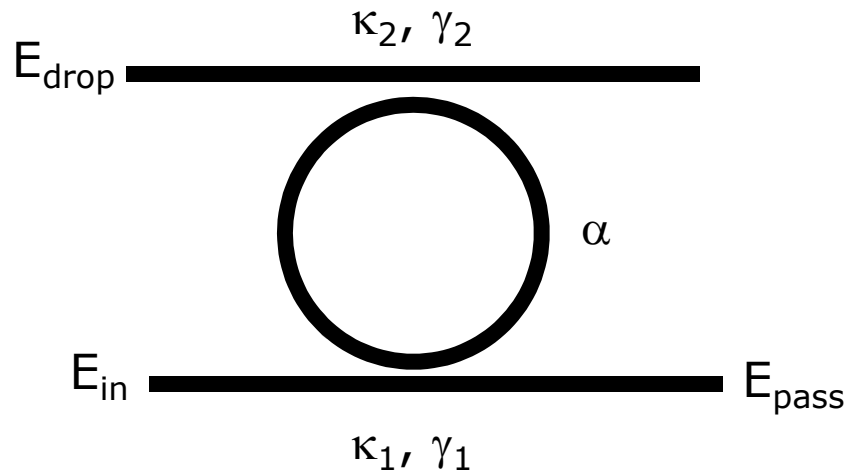
Optical Delay Line



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Homework #2

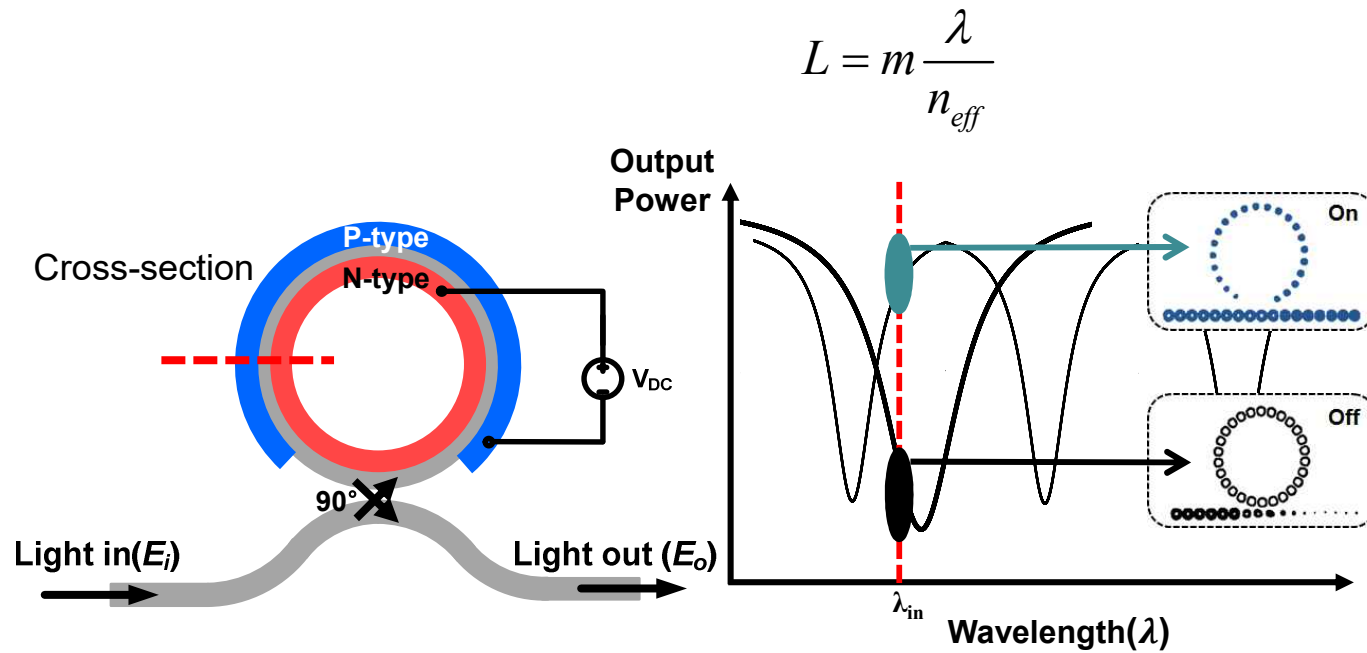
Consider a ring interferometer consisting of two directional couplers, top and bottom, shown below.



- (a) Determine the expression for $E_{\text{pass}}/E_{\text{in}}$
- (b) What is the FSR in terms of wavelength for this interferometer?
- (c) What is the FSR of this interferometer if n_{eff} changes with the wavelength, or $dn_{\text{eff}}/d\lambda$ is not zero?

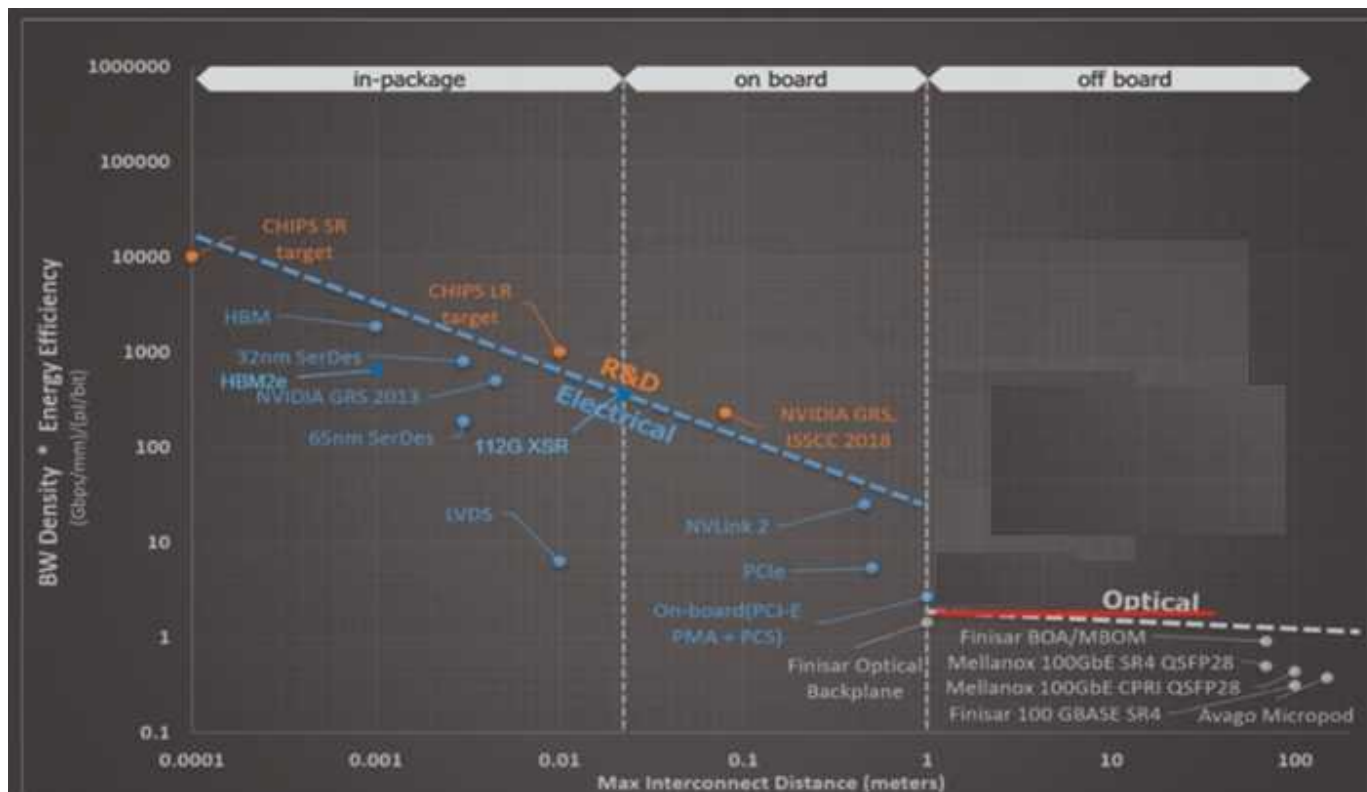
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Si intensity modulator based on ring interferometer?



Lecture 11: Si Waveguide Devices

Interconnect Figure of Merit



(Darpa)

Si photonics is expected to close the gap and improve overall performance