

Si Photonics

Lecture 12 : Semiconductor Lasers

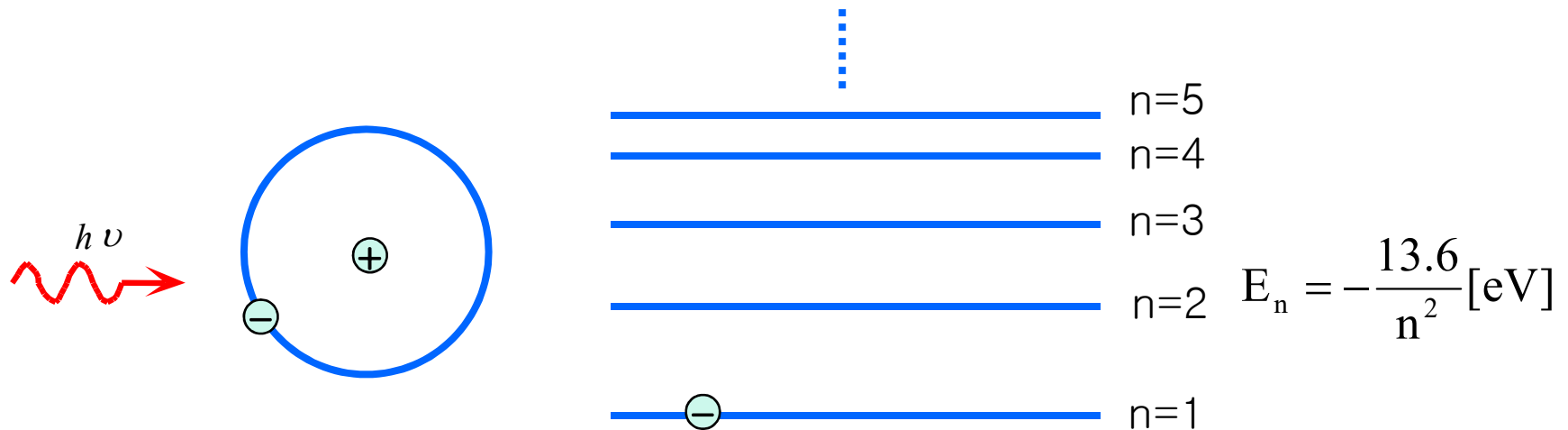
Woo-Young Choi

**Dept. of Electrical and Electronic Engineering
Yonsei University**

Lecture 12: Semiconductor Lasers

What happens when photons interact with a matter whose electron transition energies are compatible with photon energies?

Example: Electron energy levels in an hydrogen atom



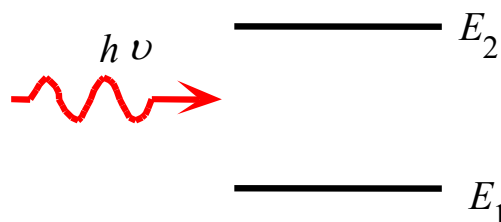
According to QM, energy levels inside an atom are quantized

What happens when $h\nu = E_n - E_m$?

Lecture 12: Semiconductor Lasers

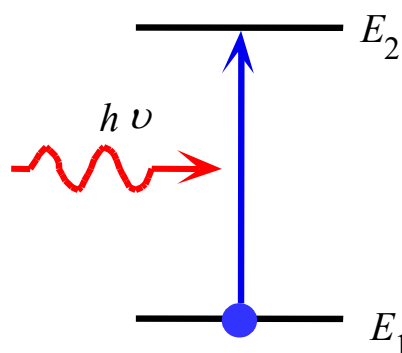
Consider for simplicity only two energy levels: ground state and first excited state

Assume $h\nu = E_2 - E_1$

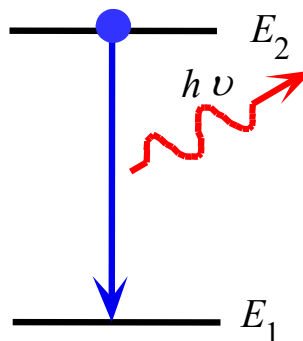


→ Three interaction processes are possible

Absorption

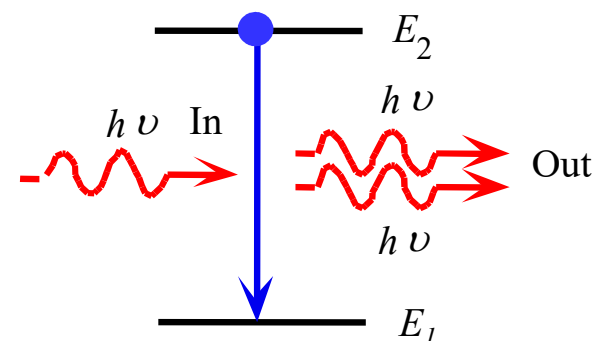


Spontaneous Emission



output photons are
“random” except energy

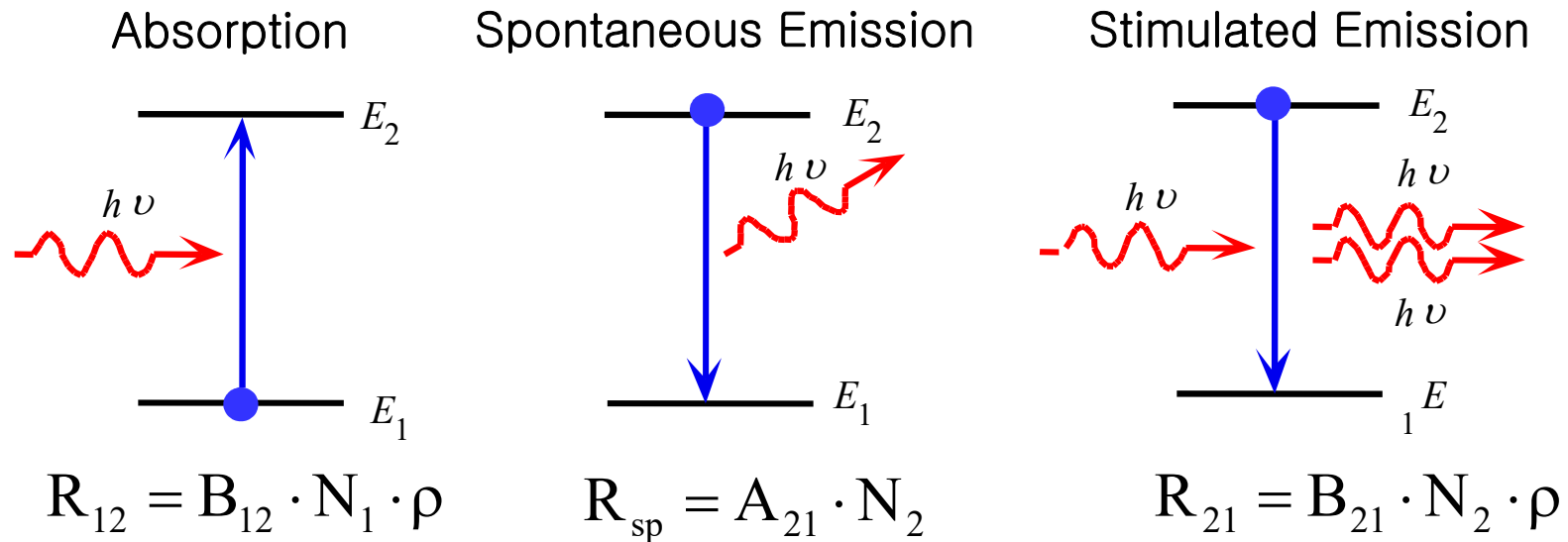
Stimulated Emission



output photons are “identical”
to input photons: amplification

Lecture 12: Semiconductor Lasers

Determine the rate for each process: How many per unit volume per second



ρ : photon density (spectral photon energy density)

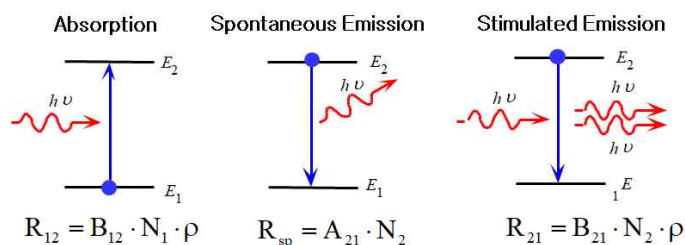
$N_{1,2}$: electron density at $E_{1,2}$

B_{12}, B_{sp}, B_{21} : constants

Lecture 12: Semiconductor Lasers

What happens at equilibrium?

No net change of N_1 , N_2 , ρ



$$R_{12} = R_{sp} + R_{21}$$

$$B_{12} \cdot N_1 \cdot \rho = A_{21} \cdot N_2 + B_{21} \cdot N_2 \cdot \rho$$

$$\rho = \frac{A_{21}/B_{12}}{\frac{N_1}{N_2} - \frac{B_{21}}{B_{12}}}$$

From another branch of physics
(statistical mechanics),

Electron distribution at equilibrium

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right)$$

$$\therefore \rho(E_2 - E_1) = \frac{A_{21}/B_{12}}{e^{\left(\frac{E_2 - E_1}{kT}\right)} - \frac{B_{21}}{B_{12}}}$$

Lecture 12: Semiconductor Lasers

From Planck law for black-body radiation
(photon distribution at equilibrium)

$$\rho(E_2 - E_1) = \frac{A_{21} / B_{12}}{e^{\left(\frac{E_2 - E_1}{kT}\right)} - \frac{B_{21}}{B_{12}}}$$

$$\rho(h\nu) = \frac{8\pi h\nu^3}{c^3 \left[\exp\left(\frac{h\nu}{kT}\right) - 1 \right]}$$

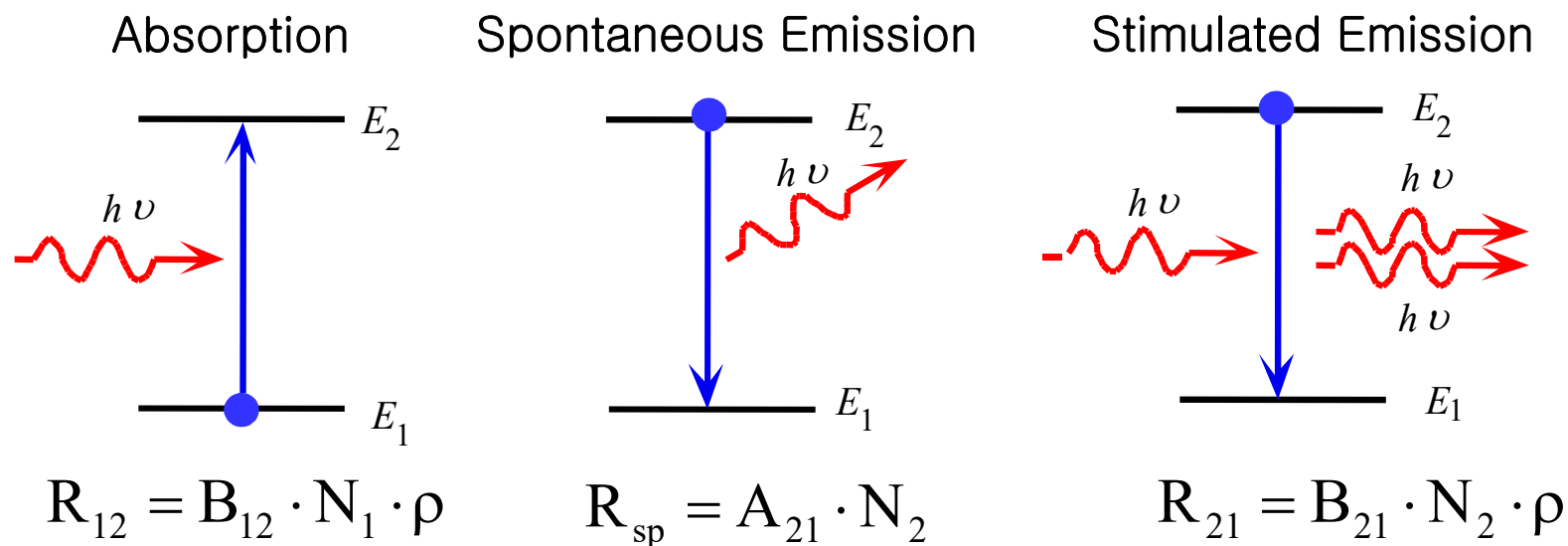
For $h\nu = E_2 - E_1$, two expressions should be identical

$$\frac{B_{21}}{B_{12}} = 1$$

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h\nu^3}{c^3}$$

➔ Einstein's A, B constants

Lecture 12: Semiconductor Lasers



Interpretations:

$$\frac{B_{21}}{B_{12}} = 1$$

– Absorption and simulated emission have the same coefficient

$$R_{21} - R_{12} = B \cdot \rho \cdot (N_2 - N_1)$$

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h \nu^3}{c^3}$$

– Spontaneous emission and stimulated emission are intrinsically related

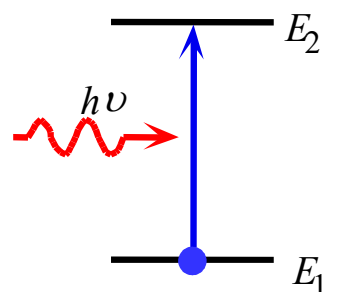
➔ Spontaneous emission is simulated emission due to *vacuum fluctuation* (QM interpretation of EM waves)

Lecture 12: Semiconductor Lasers

Which process is dominant at equilibrium?

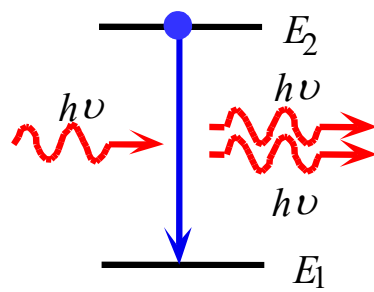
Stimulated emission vs. absorption

Absorption



$$R_{12} = B \cdot N_1 \cdot \rho$$

Stimulated Emission



$$R_{21} = B \cdot N_2 \cdot \rho$$

$$\frac{R_{21}}{R_{12}} = \frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right)$$

$$kT \sim 26meV \text{ (T=300K)}$$

For example, $\lambda = 1.55\mu m$ $E_{photon} = h\nu \simeq \frac{1.24}{\lambda[\mu m]} eV = \frac{1.24}{1.55} eV = 0.8eV$

$$\frac{R_{21}}{R_{12}} = \exp\left(-\frac{0.8eV}{0.026eV}\right) \sim 4.3 \times 10^{-14}$$

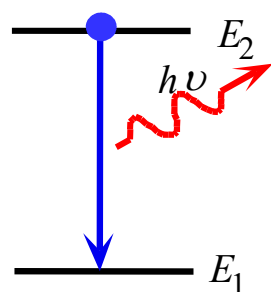
Almost all incident photons are absorbed at equilibrium

Lecture 12: Semiconductor Lasers

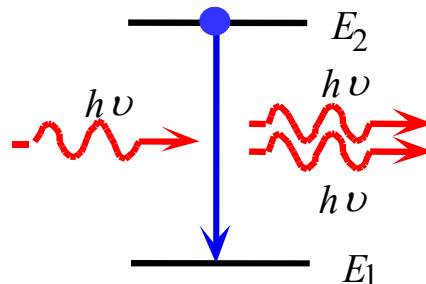
Which process is dominant at equilibrium?

Stimulated emission vs. spontaneous emission

Spontaneous Emission Stimulated Emission



$$R_{sp} = A_{21} \cdot N_2$$



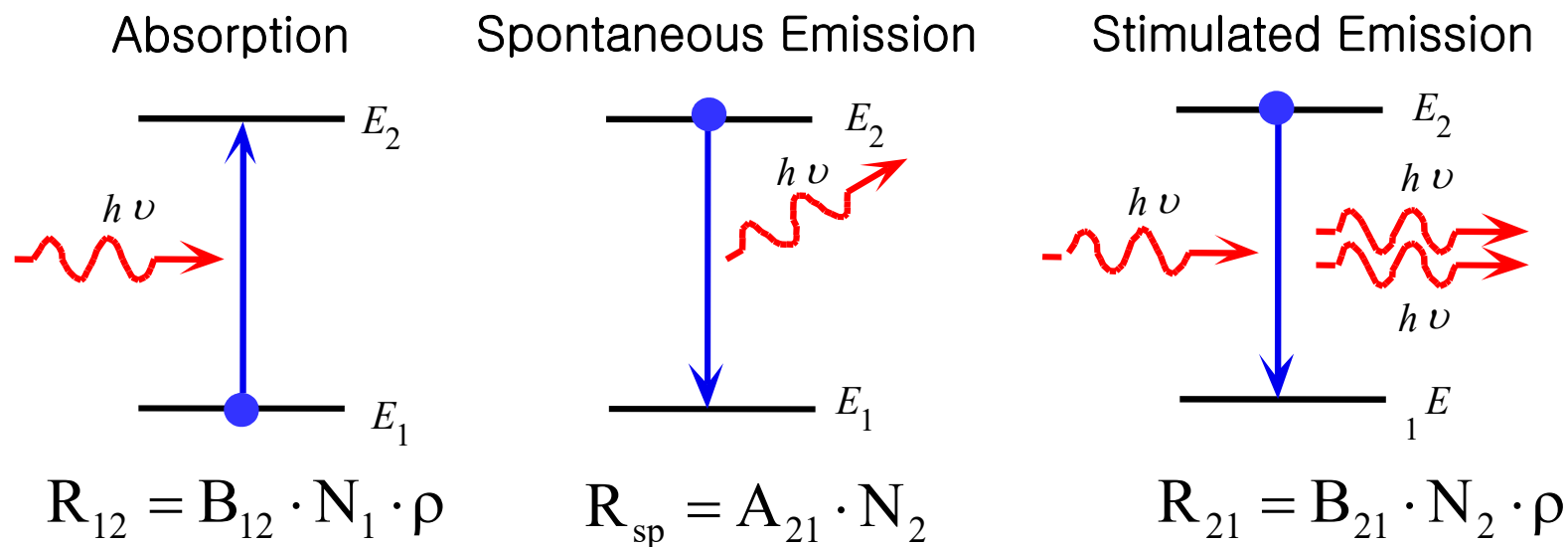
$$R_{21} = B_{21} \cdot N_2 \cdot \rho$$

$$\frac{R_{21}}{R_{sp}} = \frac{B_{21}N_2\rho}{A_{21}N_2} = \frac{c^3}{8\pi h\nu^3} \frac{8\pi h\nu^3}{c^3 \left[\exp\left(\frac{h\nu}{kT}\right) - 1 \right]} = \frac{1}{\exp\left(\frac{E_2 - E_1}{kT}\right) - 1} = \frac{1}{\exp\left(\frac{0.8eV}{0.026eV}\right) - 1} \ll 1$$

Almost all photon emission at equilibrium is due to spontaneous emission

Lecture 12: Semiconductor Lasers

How can we induce stimulated emission?



Make N_2 larger than N_1 : Break equilibrium by pumping carriers into E_2

$N_2 = N_1$: transparent

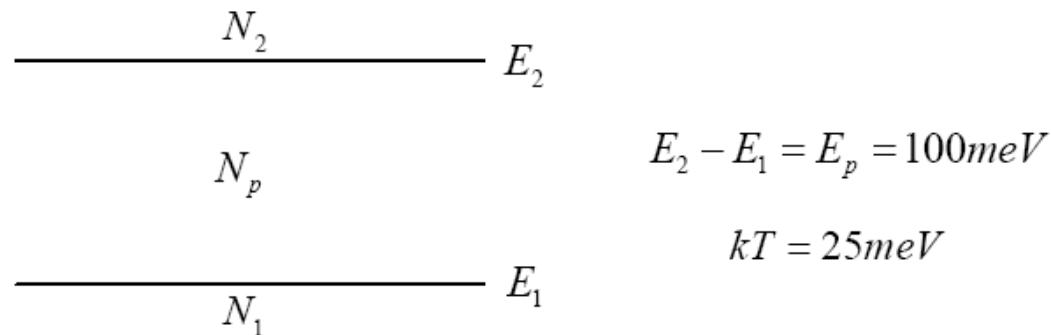
$N_2 > N_1$: population inversion → Optical gain

Lecture 12: Semiconductor Lasers

Homework:

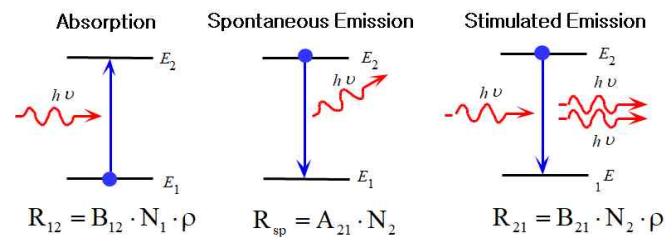
A material with two energy levels and photons are at the equilibrium state as shown below. The photon energy, E_p , is equal to $E_2 - E_1 = 100\text{meV}$. Use $kT = 25\text{meV}$.

- (a) What is the expression for the stimulated emission rate?
- (b) Determine the numerical value of N_1/N_2 , the ratio between electron densities at E_2 and E_1 .
- (c) What is the percentage of photons that are due to stimulated emission?
- (d) Electron are excited from E_1 to E_2 by optical pumping. If the total density of electrons in the material is N , what should be N_2 in order to reach the transparency condition?



Lecture 12: Semiconductor Lasers

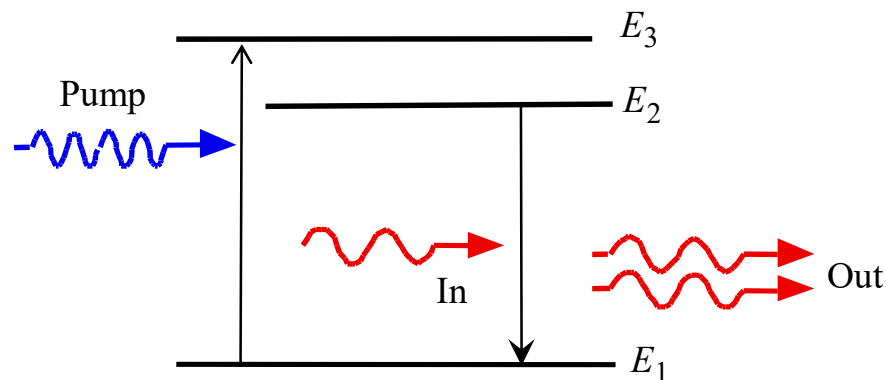
Optical pumping



2-level system not practical

Pump signal has the same wavelength as input/output signal

Consider 3-level systems



Requirements:

- (1) carriers at E_3 quickly come down to E_2
- (2) Carriers at E_2 radiatively come down to E_1

Lecture 12: Semiconductor Lasers

Optical gain materials for 3-level systems

Erbium



One of rare earth metals

1/IA

2

18/VIIA

Periodic Table

1998 Dr. Michael Blaber

1
H
1.008

2
He
4.003

3
Li
6.941

4
Be
9.012

5
B
10.81

6
C
12.01

7
N
14.01

8
O
16.00

9
F
19.00

10
Ne
20.18

11
Na
22.99

12
Mg
24.30

13
Al
26.98

14
Si
28.09

15
P
30.97

16
S
32.07

17
Cl
35.05

18
Ar
39.95

19
K
39.10

20
Ca
40.08

21
Sc
44.96

22
Ti
47.87

23
V
50.94

24
Cr
52.00

25
Mn
54.94

26
Fe
55.85

27
Co
58.93

28
Ni
58.69

29
Cu
63.55

30
Zn
65.39

31
Ga
69.72

32
Ge
72.61

33
As
74.92

34
Se
78.96

35
Br
79.90

36
Kr
83.80

37
Rb
85.47

38
Sr
87.62

39
Y
88.91

40
Zr
91.22

41
Nb
92.91

42
Mo
95.94

43
Tc
98.91

44
Ru
101.1

45
Rh
102.9

46
Pd
106.4

47
Ag
107.9

48
Cd
112.4

49
In
114.8

50
Sn
118.7

51
Sb
121.8

52
Te
127.6

53
I
126.9

54
Xe
131.3

55
Cs
132.9

56
Ba
137.3

57
La
138.9

58
Ce
140.1

59
Pr
140.9

60
Nd
144.2

61
Pm
144.9

62
Sm
150.4

63
Eu
152.0

64
Gd
157.2

65
Tb
158.9

66
Dy
162.5

67
Ho
164.9

68
Er
167.3

69
Tm
168.9

70
Yb
173.0

71
Lu
175.0

87
Fr
223.0

88
Ra
226.0

89
Ac
227.0

90
Th
232.0

91
Pa
231.0

92
U
238.0

93
Np
237.0

94
Pu
239.1

95
Am
241.1

96
Cm
244.1

97
Bk
247.1

98
Cf
251.1

99
Es
252.1

100
Fm
257.1

101
Md
258.1

102
No
259.1

103
Lr
262.1

s

d

f

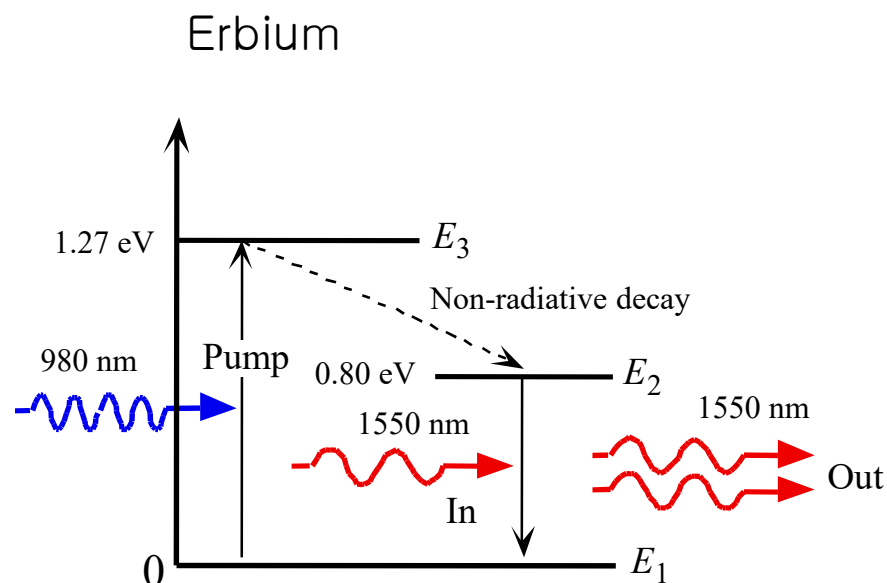
Lanthanides

Actinides

Er

Scandium	Aerospace components, aluminum alloys
Yttrium	Lasers, TV and computer displays, microwave filters
Lanthanum	Oil refining, hybrid-car batteries, camera lenses
Cerium	Catalytic converters, oil refining, glass-lens production
Praseodymium	Aircraft engines, carbon arc lights
Neodymium	Computer hard drives, cell phones, high-power magnets
Promethium	Portable x-ray machines, nuclear batteries
Samarium	High-power magnets, ethanol, PCB cleansers
Europium	TV and computer displays, lasers, optical electronics
Gadolinium	Cancer therapy, MRI contrast agent
Terbium	Solid-state electronics, sonar systems
Dysprosium	Lasers, nuclear-reactor control rods, high-power magnets
Holmium	High-power magnets, lasers
Erbium	Fiber optics, nuclear-reactor control rods
Thulium	X-ray machines, superconductors
Ytterbium	Portable x-ray machines, lasers
Lutetium	Chemical processing, LED lightbulbs

Lecture 12: Semiconductor Lasers

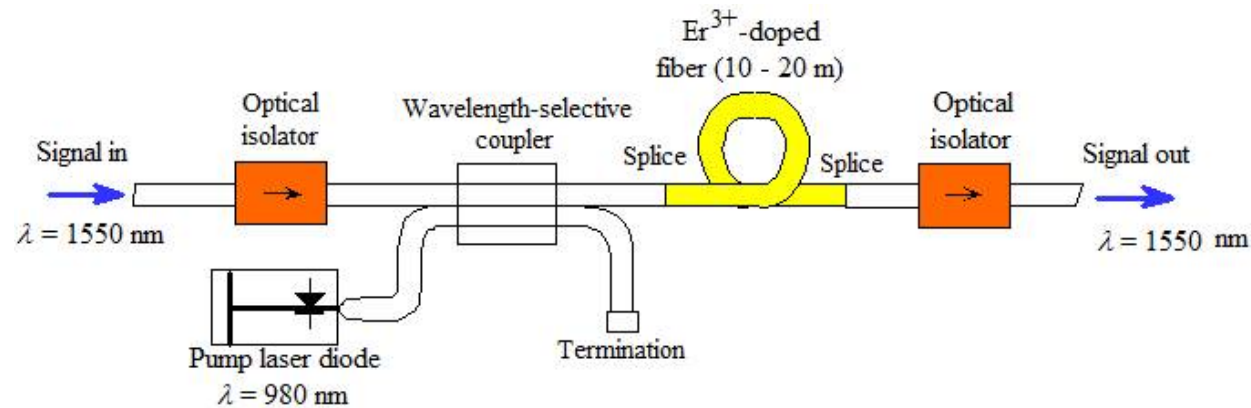


- Pump light ($\lambda=980\text{nm}$) absorbed generating carriers at E_3
- Carriers at E_3 rapidly transfer to E_2 building up N_2
- When $N_2 > N_1$ (population inversion), stimulated emission $>$ absorption for 1550nm light

- Very useful for optical communication applications
 - High-power semiconductor lasers easily available for 980nm pumping source
 - Er can be easily added to core of Silica fiber
- ➔ EDF (Er-Doped Fiber)

Lecture 12: Semiconductor Lasers

EDFA: Er-Doped Fiber Amplifier

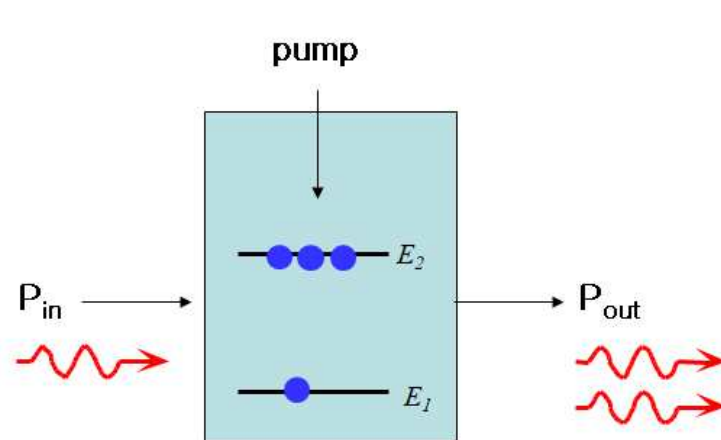


A key component for long-distance optical communication systems

Roughly, one EDFA for every $\sim 100\text{km}$ fiber

Lecture 12: Semiconductor Lasers

Optical Amplifier



$$P_{out} = G P_{in}$$

$$= \exp(gz) P_{in}$$

$$E_{out} = \exp(-jkz) E_{in}$$

$$k = \beta - j\alpha,$$

$$E_{out} = \exp(-j\beta z) \exp(-\alpha z) E_{in}$$

$$P_{out} = \exp(-2\alpha z) P_{in}$$

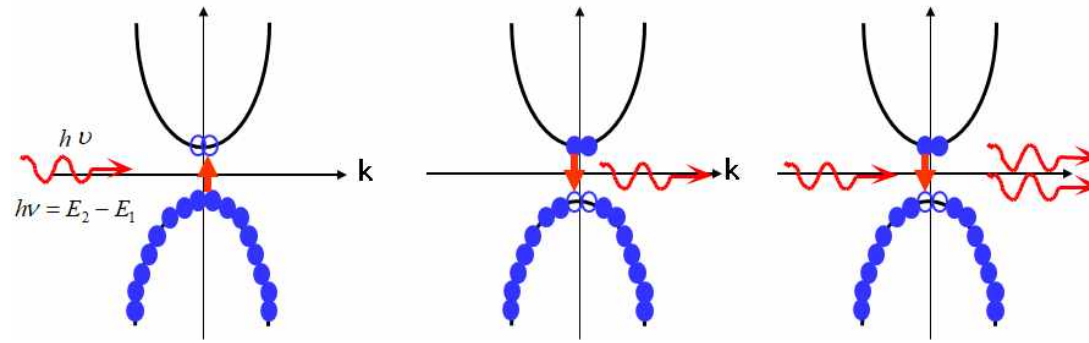
$$g = -2\alpha$$

Spontaneous emission?

➔ Noise

Lecture 12: Semiconductor Lasers

Semiconductors with direct bandgap



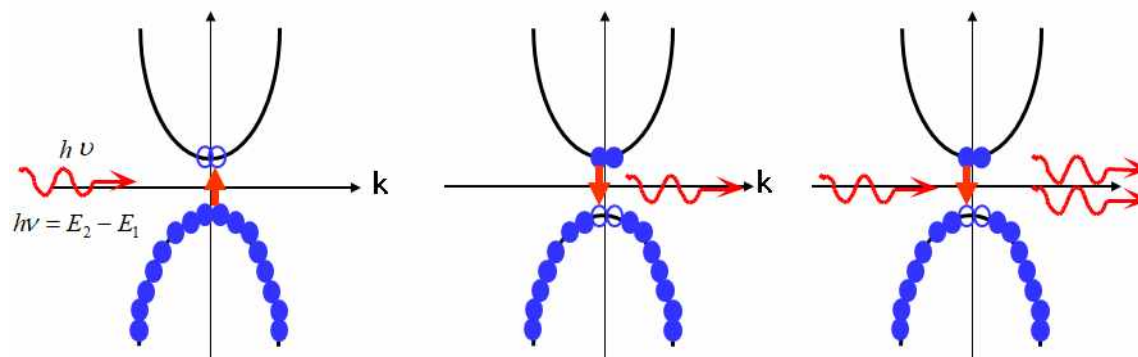
(E-k diagrams: Dispersion diagram for electron waves)

- Continuous energy levels across the band gap

Conduction band: more holes than electrons

Valence band: more electrons than holes

Lecture 12: Semiconductor Lasers

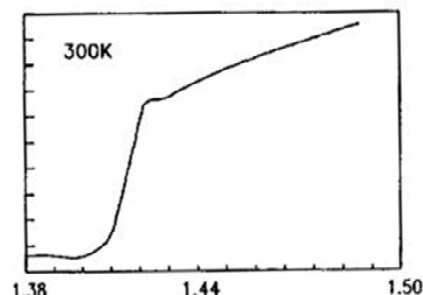


Interaction with photons if $h\nu > E_g$
 Continuous spectrum for absorption and emission

For GaAs

13/IIIA	14/IVA	15/VA	16/VIA
5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00
13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.07
31 Ga 68.72	32 Ge 72.61	33 As 74.92	34 Se 78.96
49 In 114.8	50 Sn 118.7	51 Sb 121.8	52 Te 127.6

Absorption Spectrum



$$R_{12}(h\nu = E_2 - E_1) = B_{12} \cdot N_1(E_1) \cdot P_2(E_2) \cdot \rho(h\nu)$$

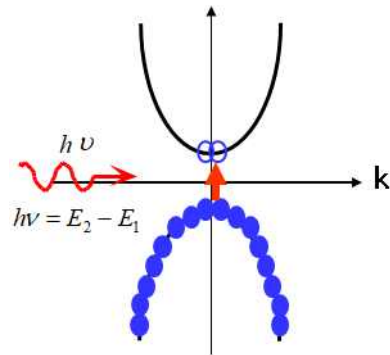
More holes as E_2 becomes larger

More electrons as E_1 becomes smaller

→ More absorption
 for larger photon energies

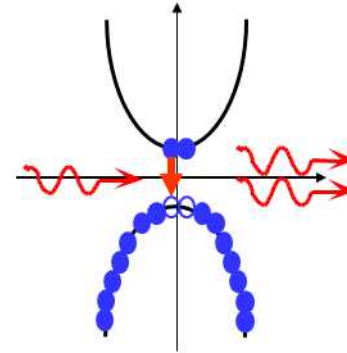
Lecture 12: Semiconductor Lasers

Absorption



$$R_{12}(h\nu) = B_{12} \cdot N_1(E_1) \cdot P_2(E_2) \cdot \rho(h\nu)$$

Stimulated Emission



Spontaneous
emission
not considered

$$R_{21}(h\nu) = B_{21} \cdot N_2(E_2) \cdot P_1(E_1) \cdot \rho(h\nu)$$

Which is larger? $N_1(E_1) \cdot P_2(E_2)$ or $N_2(E_2) \cdot P_1(E_1)$?

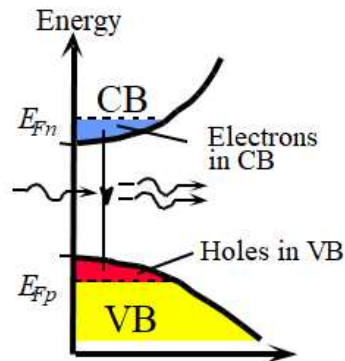
Depends how much pumping and photon energy ($E_2 - E_1$)

→ Electrons in conduction
band, holes in the valence
band by pumping

Lecture 12: Semiconductor Lasers

Optical Gain for semiconductor

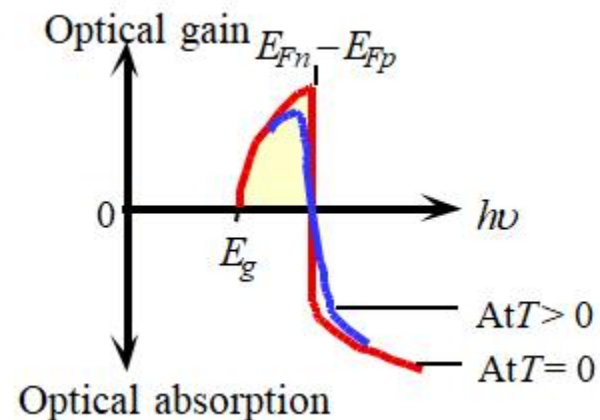
$$g(E_2 - E_1) \sim N_2(E_2) \cdot P_1(E_1) - N_1(E_1) \cdot P_2(E_2) = g_c(E_2)f(E_2)g_v(E_1)[1 - f(E_1)] - g_v(E_1)f(E_1)g_c(E_2)[1 - f(E_2)]$$



$g_c(E)$: Density of states for conduction band at E

$g_v(E)$: Density of states for valence band at E

$f(E)$: Fermi factor at E

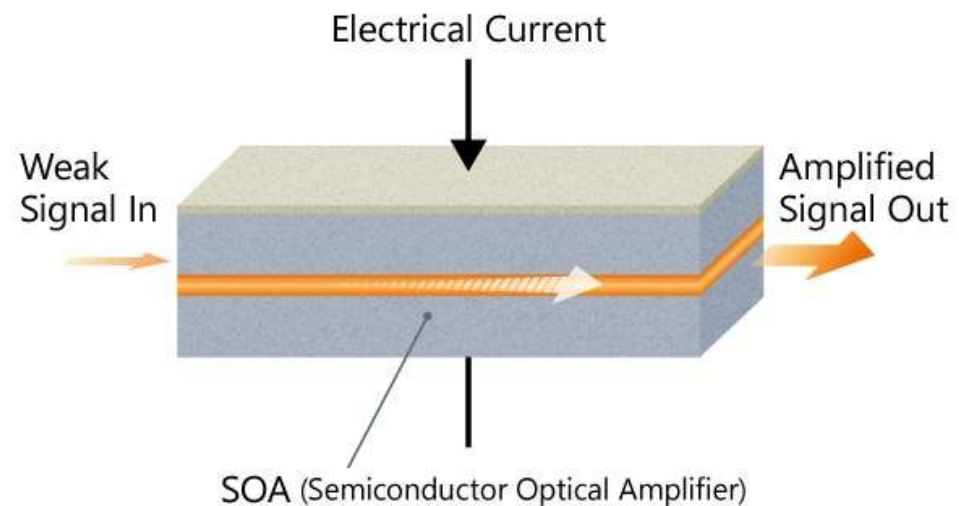
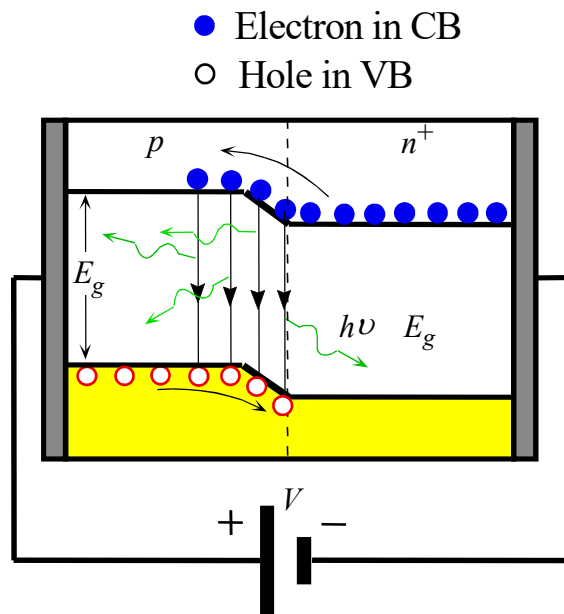


- No interaction for $E_p < E_g$
- For $E_2 - E_1 > E_g$, gain increases up to around $h\nu = E_{Fn} - E_{Fp}$
- Gain < 0 for $h\nu > E_{Fn} - E_{Fp}$
- Sharper transition at lower T

Lecture 12: Semiconductor Lasers

How to electrically pump electrons and holes?

Forward-biased PN junction



Lecture 12: Semiconductor Lasers

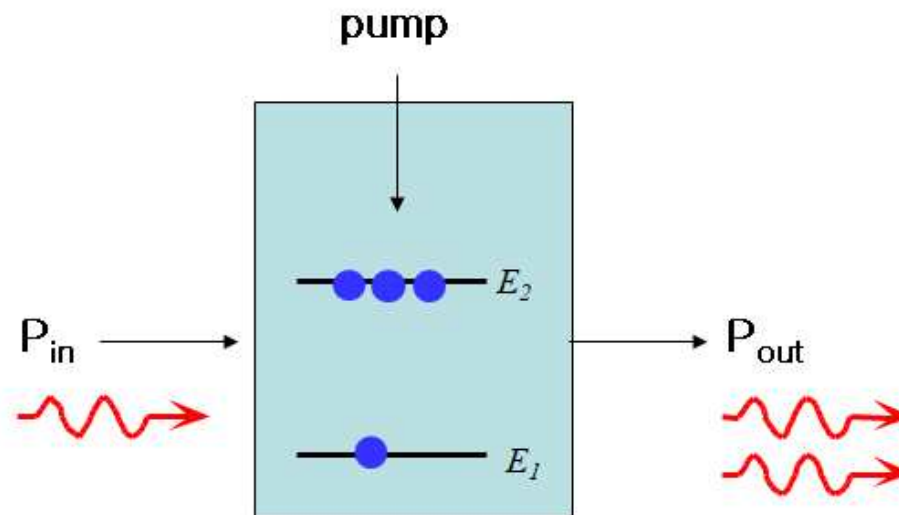
Homework #1:

Assume the optical gain coefficient in semiconductor is given as $g = a(N - N_0)$ [1/cm], where $a = 10^{-17} \text{cm}^2$, $N_0 = 10^{18} \text{cm}^{-3}$ for $\lambda = 1 \mu\text{m}$.

If 0.5cm long semiconductor optical amplifier (SOA) is made up of above semiconductor, what is the required carrier density in order to achieve SOA power gain of 20dB for $\lambda = 1 \mu\text{m}$ input signal?

Lecture 12: Semiconductor Lasers

Optical Amplifier



Light source based on stimulated emission?

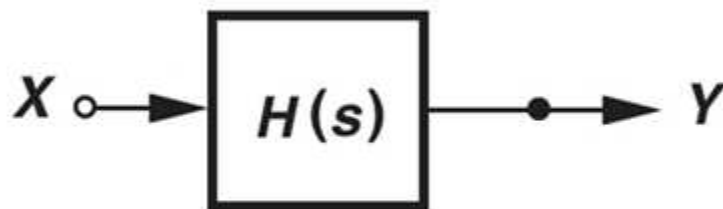
Laser (Light Amplification by Stimulated Emission of Radiation)

→ Optical oscillator

Lecture 12: Semiconductor Lasers

In Electronic circuits

Amplifier

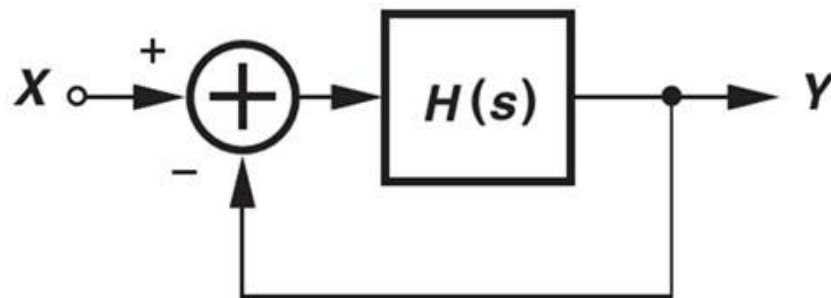


$$Y = (X - Y)H(s)$$

$$Y(1 + H(s)) = XH(s)$$

$$\frac{Y}{X} = \frac{H(s)}{1 + H(s)}$$

With Feedback



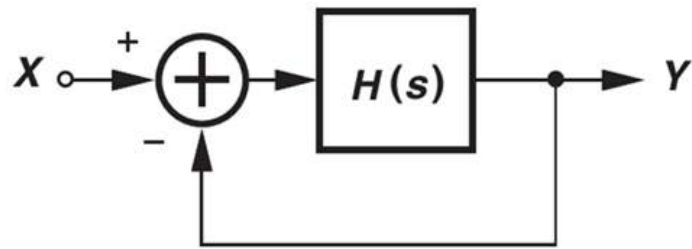
If $H(s) = -1$, Output without input

→ Oscillation

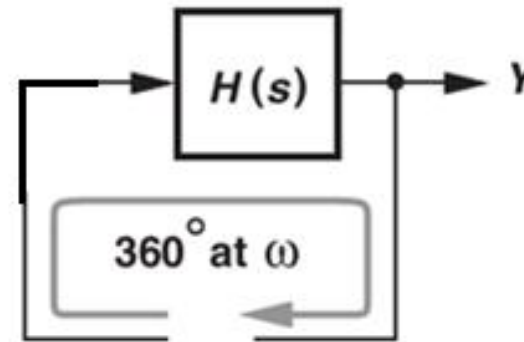
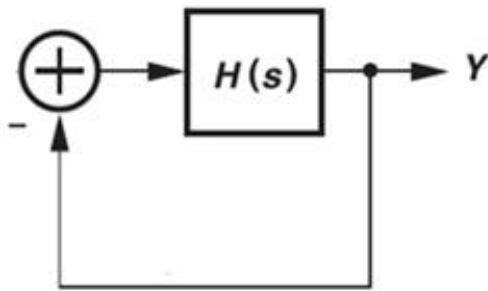
$$|H(j\omega)| = 1 \text{ and } \angle H(j\omega) = 180^\circ$$

(Barkhausen oscillation condition)

Lecture 12: Semiconductor Lasers



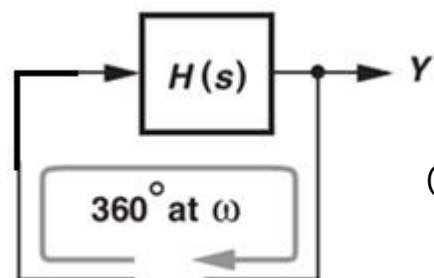
$$|H(j\omega)| = 1 \text{ and } \angle H(j\omega) = 180^\circ$$



Oscillator: Amplifier with feedback

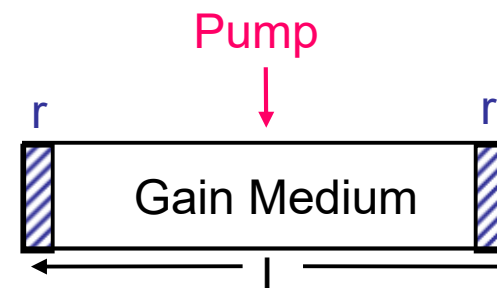
In-phase and the same magnitude after one round trip

Lecture 12: Semiconductor Lasers



LASER:

Optical Amplifier + Mirrors



In gain medium: $k = nk_0 + j\frac{g}{2}$

Initially E_0

After one round trip $E_0 \cdot e^{-jkL} \cdot r \cdot e^{-jkL} \cdot r = E_0 \quad r^2 \cdot e^{-j2kL} = 1$

$$r^2 \cdot e^{-j2(nk_0 + j\frac{g}{2})L} = 1 \quad r^2 \cdot e^{gL} e^{-j2nk_0L} = 1$$

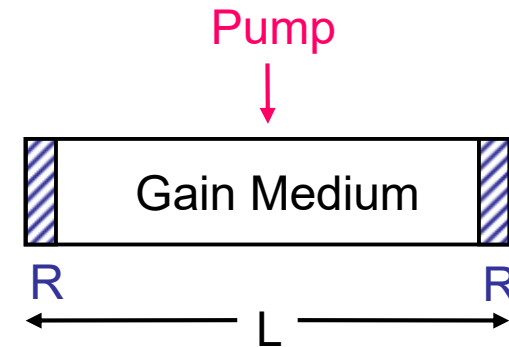
(Magnitude = 1, Phase = $2m\pi$) $r^2 \cdot e^{gL} = 1 \quad 2nk_0L = 2m\pi$

What provides initial E_0 ? Spontaneous emission (Noise)

Lecture 12: Semiconductor Lasers

$$r^2 \cdot e^{gL} = 1 \quad 2nk_0L = 2m\pi$$

$$g = \frac{1}{L} \ln \frac{1}{R} \quad g_{th} \text{ (threshold gain)} = \alpha_m \text{ (mirror loss)}$$



➔ Minimum gain required for lasing (mirror loss compensation)

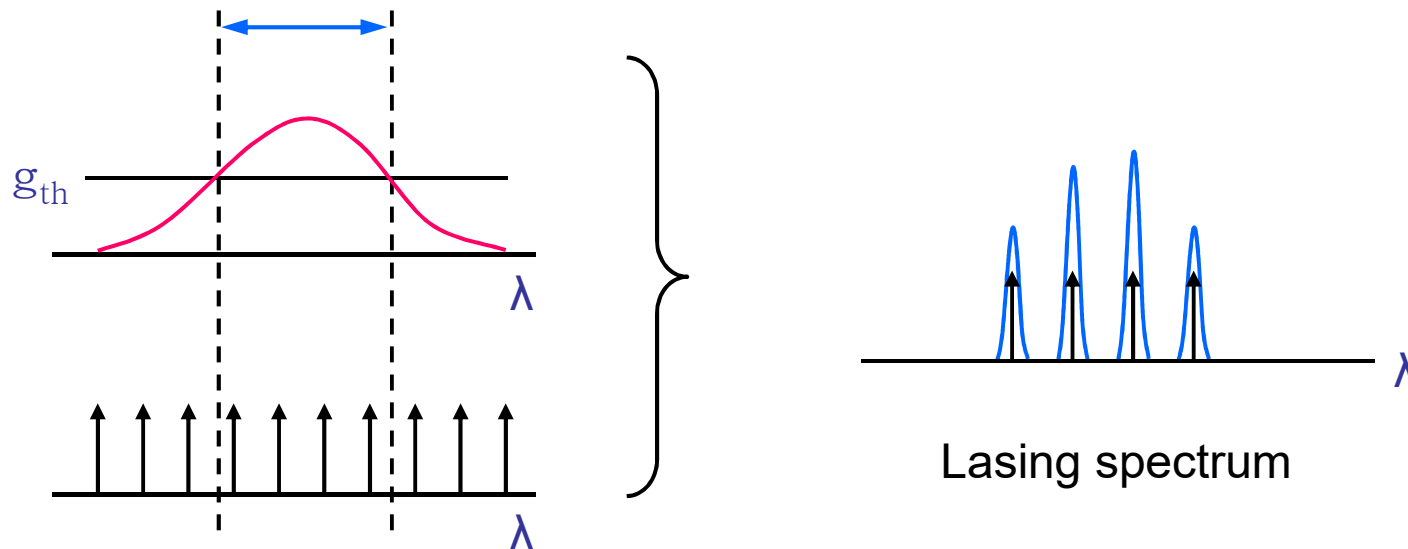
$$2nk_0L = 2m\pi \quad 2n \frac{2\pi}{\lambda} L = 2m\pi \Rightarrow \frac{\lambda}{n} = \frac{2L}{m} \text{ or } L = m \frac{\lambda}{2n}$$

Cavity length should be integer multiples of half wavelength ➔ lasing mode

Lecture 12: Semiconductor Lasers

Two conditions for lasing: (1) $g_{\text{th}} = \frac{1}{L} \ln \frac{1}{R}$ and (2) $\frac{\lambda}{n} = \frac{2L}{m}$

Gain is function of pumping and λ



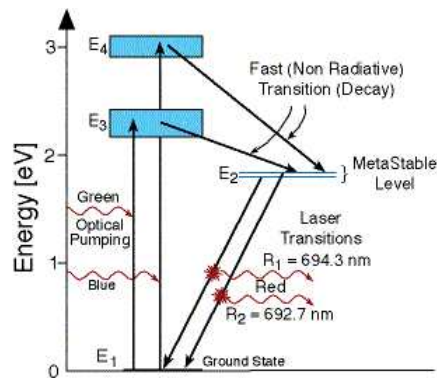
Lasing modes has non-zero linewidth due to various linewidth-broadening effects

Lecture 12: Semiconductor Lasers

Any optical gain material with mirrors can be a laser

First laser demonstrated by Maiman in 1960 at Hughes Aircraft Company

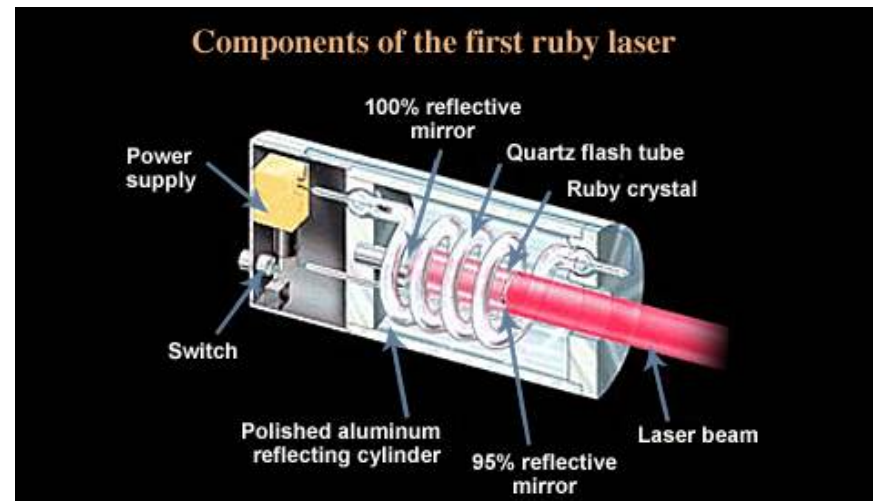
Optical Gain Material: Cr in Al_2O_3



Pump: Xenon flash lamp



Ted Maiman
(1927–2007)



Lecture 12: Semiconductor Lasers

1964 Nobel Prize in Physics for invention of laser



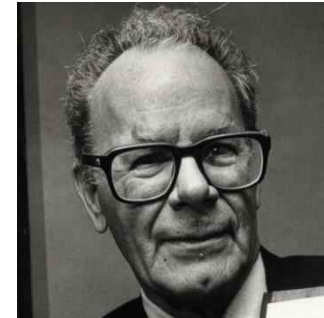
Charles Townes
(1915–2015)
(1/2)



Nikolay Basov
(1922–2001)
(1/4)



Aleksandr Prokhorov
(1916–2002)
(1/4)



Gordon Gould
(1920–2005)

30-year battle
for laser patent

Lecture 12: Semiconductor Lasers

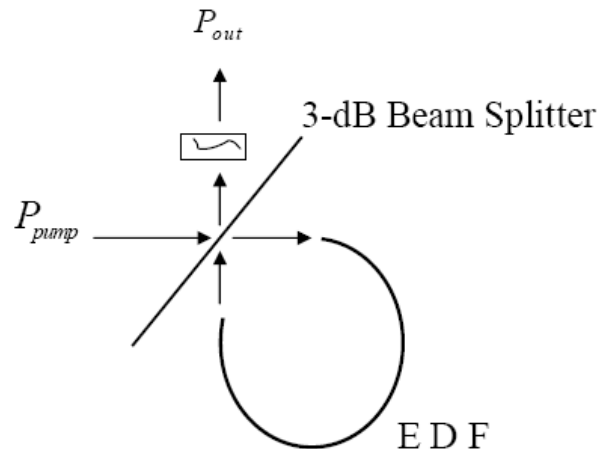
Homework #2

A fiber ring laser lasing around $1.55\mu\text{m}$ is realized with a piece of Er-doped fiber (EDF) and a 3-dB beam splitter as shown below. The 3-dB beam splitter divides the input power into two equal output powers. Assume all the pump power transmitted by the beam splitter is absorbed by EDF and the resulting excited carriers are uniformly distributed within EDF. Also assume the reflected pump power is filtered out by an optical filter so that only the laser output is present at the output. Values of parameters that are needed to solve this problem are given below.

l (EDF length): 1m

$\lambda_{\text{pump}} = 0.98\mu\text{m}$

n_{eff} (at $1.55\mu\text{m}$) = 1.55

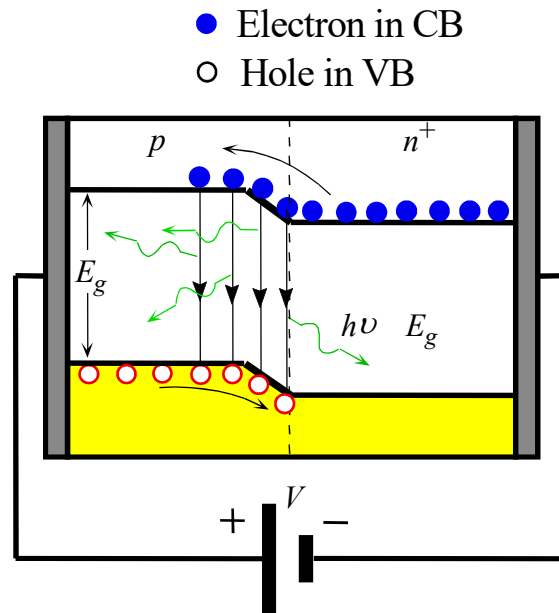


(a) What is the threshold gain of the laser in $1/\text{m}$?

(b) The laser produces multi-mode lasing spectrum. What is the mode separation in wavelength at around $1.55\mu\text{m}$?

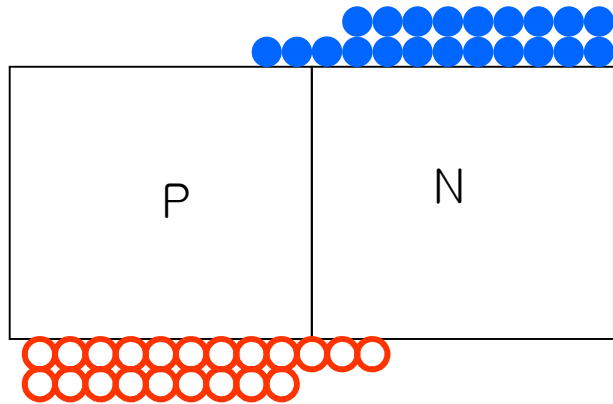
Lecture 12: Semiconductor Lasers

Forward-biased PN junction for electrical pumping



But not very efficient

Lecture 12: Semiconductor Lasers

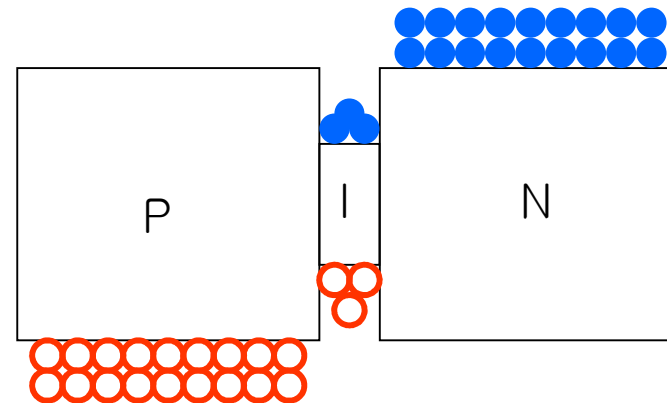


$$\frac{R_{21}(h\nu)}{R_{12}(h\nu)} = \frac{N_2(E_2) \cdot P_1(E_1)}{N_1(E_1) \cdot P_2(E_2)} > 1$$

But injected carriers *diffuse*

Efficient carrier confinement

→ Bandgap engineering

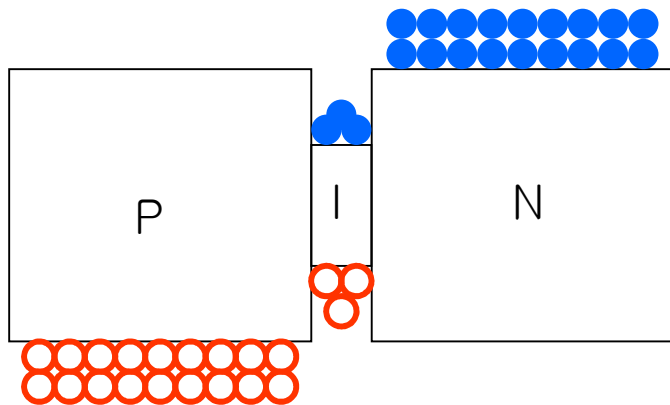


Double heterojunction
with larger E_g for P, N regions

Active region very thin and undoped (I)

→ Much larger injected carrier *density*

Lecture 12: Semiconductor Lasers



Double heterojunction

The Nobel Prize in Physics 2000



Zhores I. Alferov
Prize share: 1/4

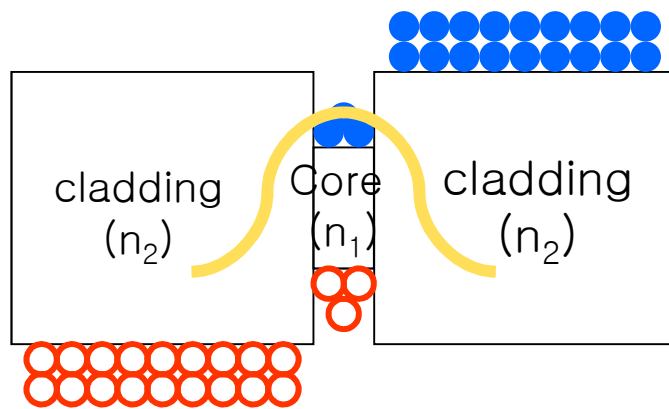


Herbert Kroemer
Prize share: 1/4

The Nobel Prize in Physics 2000 was awarded to Zhores I. Alferov and Herbert Kroemer *"for developing semiconductor heterostructures used in high-speed- and optoelectronics"*

Lecture 12: Semiconductor Lasers

Double heterojunction provides efficient photon confinement as well



Smaller E_g materials have larger n ($n_1 > n_2$)

→ Dielectric waveguide!

=> More photons interacting with injected electrons and holes in the active region

Portion of photons interacting with injected carriers?

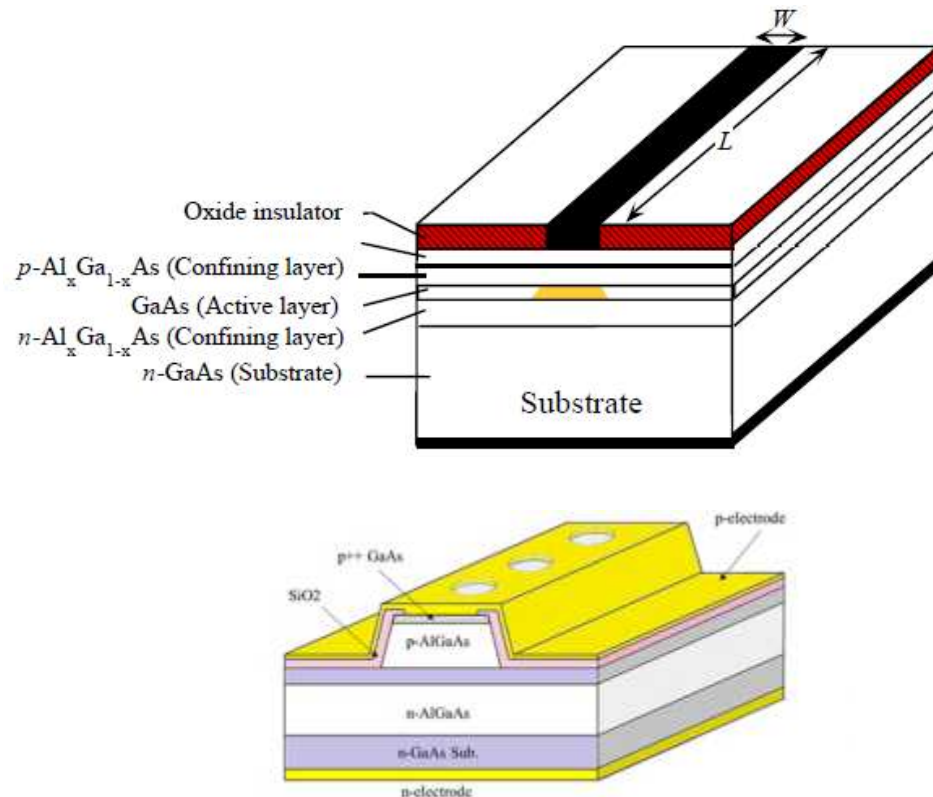
→ Use Γ

$$g_{\text{th}} = \frac{1}{L} \ln \frac{1}{R} = \alpha_m \quad \Rightarrow \quad \Gamma g_{\text{th}} = \alpha_m$$

$$\frac{\lambda}{n} = \frac{2L}{m} \quad \Rightarrow \quad \frac{\lambda}{n_{\text{eff}}} = \frac{2L}{m};$$

Lecture 12: Semiconductor Lasers

Semiconductor Laser: PIN Heterojunction + Mirrors (Cleaved Facets)
(Laser Diode) → Fabry–Perot laser diode



Electrically, PIN diode

Optically, 2-D dielectric waveguide

Vertical waveguide:

Heterojunction

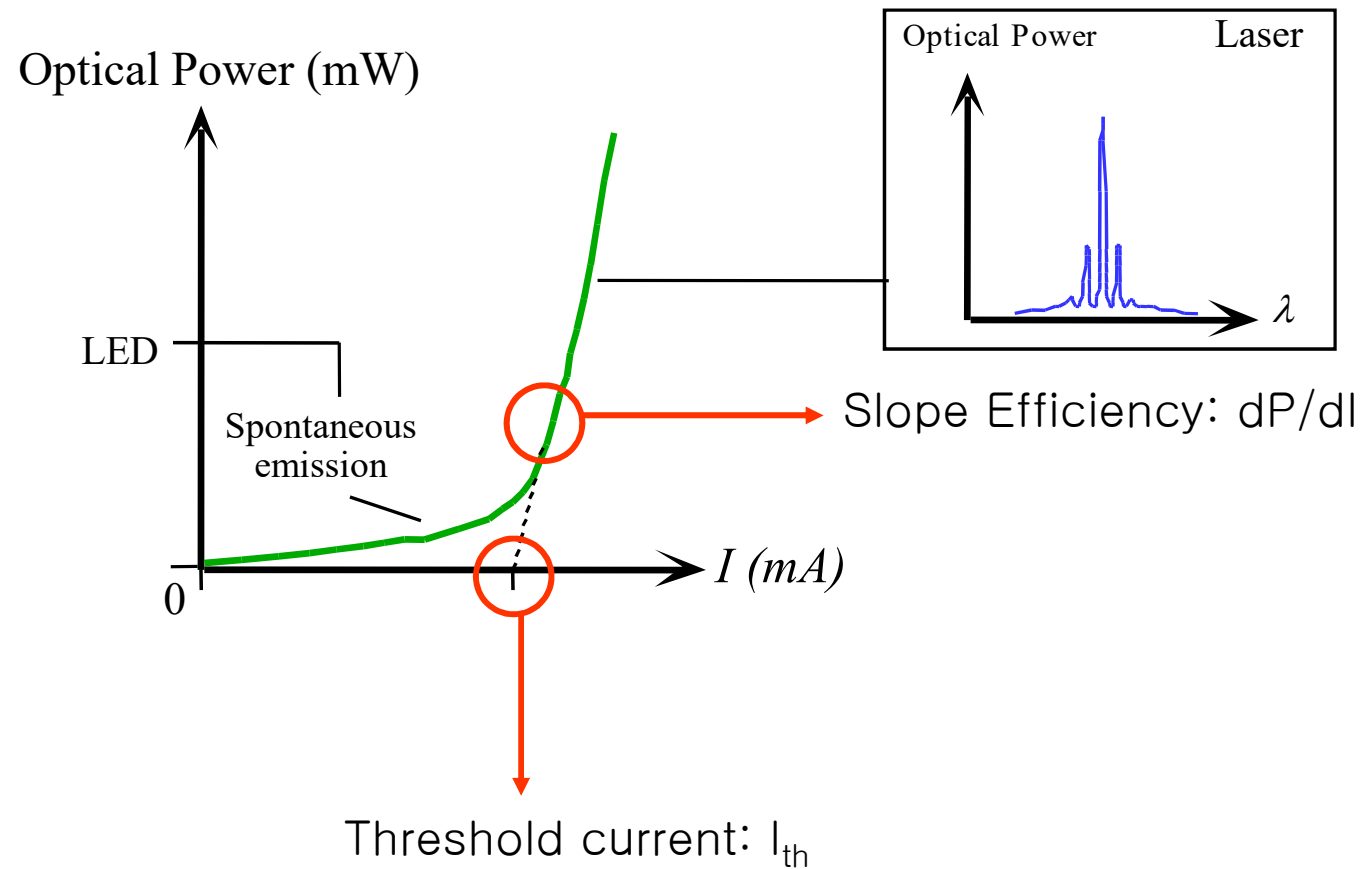
Lateral waveguide ?

Ridge waveguide

→ Single waveguide mode

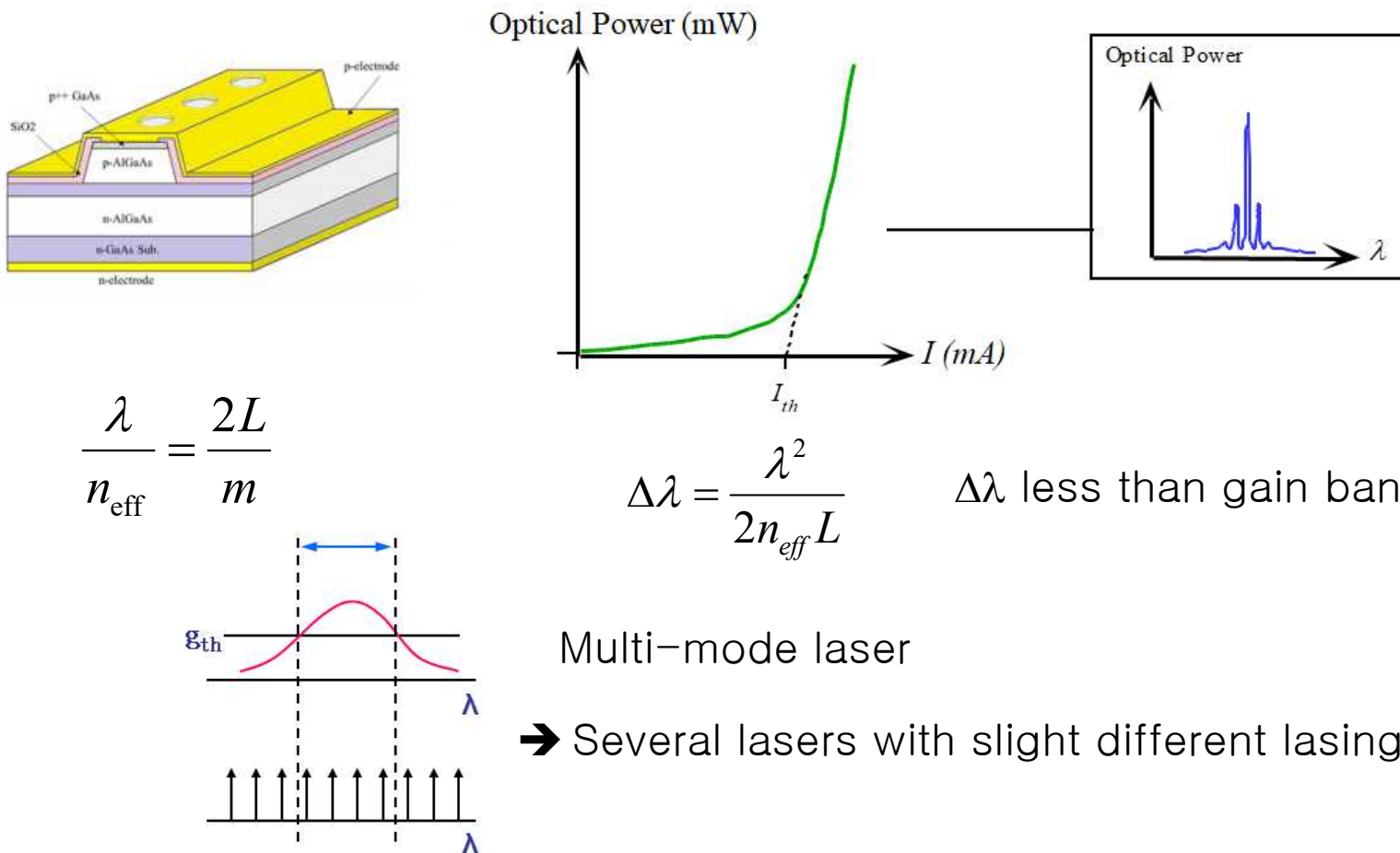
But multiple lasing mode

Lecture 12: Semiconductor Lasers



Lecture 12: Semiconductor Lasers

F-P semiconductor laser usually has multiple lasing modes

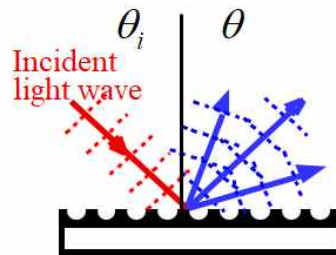


Lecture 12: Semiconductor Lasers

How to make single lasing-mode semiconductor laser?

Use λ -selective mirror: Diffraction Grating

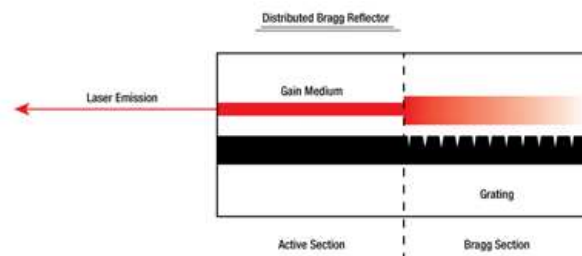
Remember



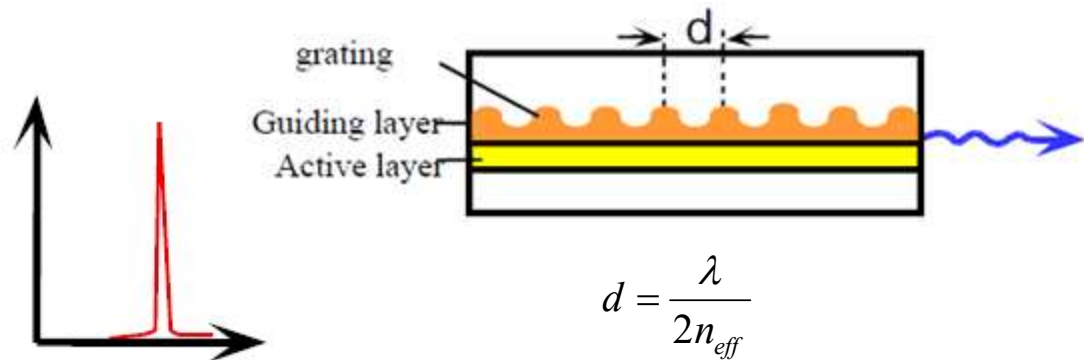
$$d(\sin \theta - \sin \theta_i) = m \cdot \lambda$$

For mirror, $\theta_i = 90^\circ$ and $\theta = -90^\circ$, $d = m \frac{\lambda}{2}$

Distributed Bragg Reflector (DBR) Laser



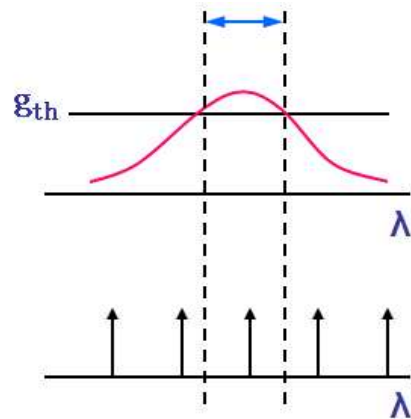
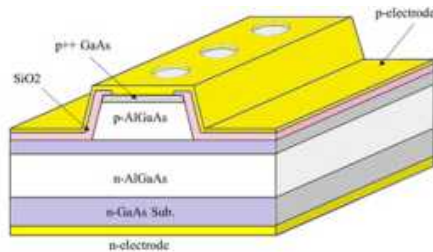
Distributed Feedback (DFB) Laser



$$d = \frac{\lambda}{2n_{eff}}$$

Lecture 12: Semiconductor Lasers

Another approach: Make L very small so that $\Delta\lambda$ larger than gain bandwidth



$$\Delta\lambda = \frac{\lambda^2}{2n_{eff}L}$$

gain bandwidth in the order of 10nm

$$\lambda : 1.5\mu\text{m}$$

$$n_{eff} : 3.5$$

$$L \sim 30\mu\text{m}$$

Fabrication not easy

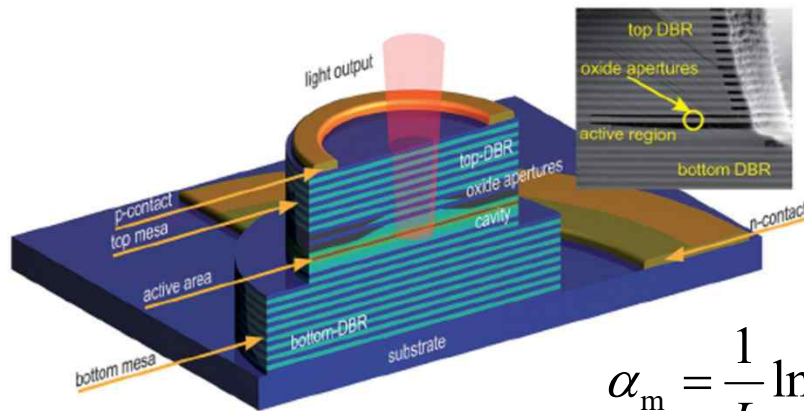
$$\alpha_m = \frac{1}{L} \ln \frac{1}{R}$$

Large mirror loss
resulting in large I_{th}

➔ Short cavity with very large reflectivity?

Lecture 12: Semiconductor Lasers

Solution: Very short cavity *vertical* lasers with very high reflectivity mirrors (VCSEL: Vertical Cavity Surface Emitting Laser)



$$\alpha_m = \frac{1}{L} \ln \frac{1}{R}$$

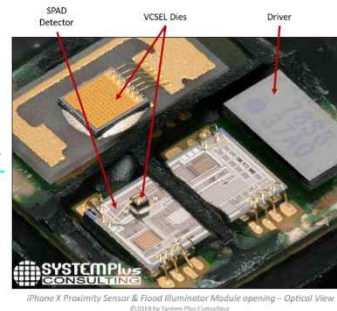
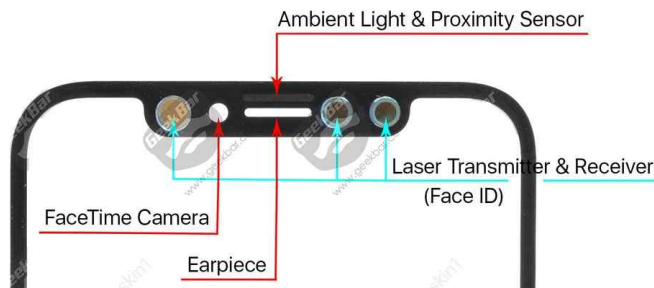
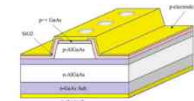
Electrically, vertical PIN

Optically, vertical cavity

Top and bottom DBR mirrors with very high reflectivity

Precise vertical layer deposition can be easily done in semiconductor fabrication

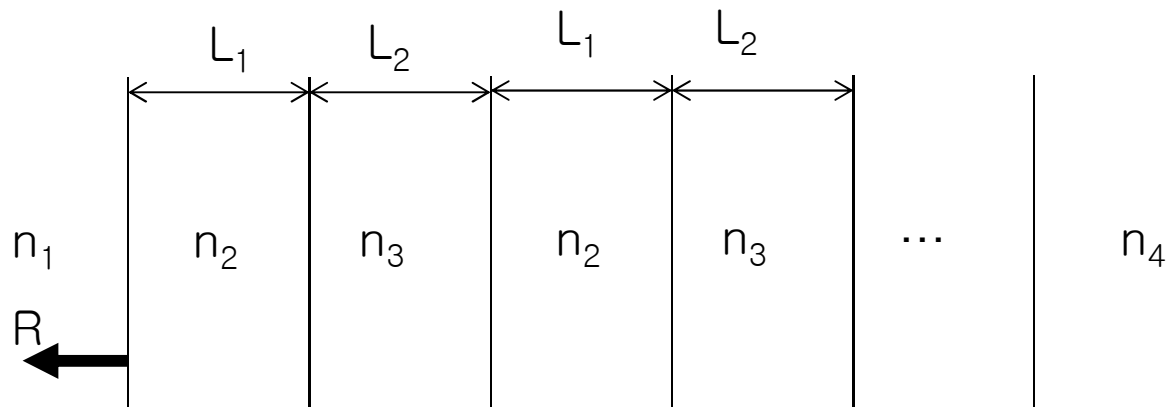
→ Very small and cost-effective (typically ~850nm)



Lecture 12: Semiconductor Lasers

Distributed Bragg Reflector (DBR) mirror

Repeat the quarter-wavelength pair m times.



$$R = \left(\frac{n_1 - \left(\frac{n_2}{n_3} \right)^{2m} n_4}{n_1 + \left(\frac{n_2}{n_3} \right)^{2m} n_4} \right)^2$$

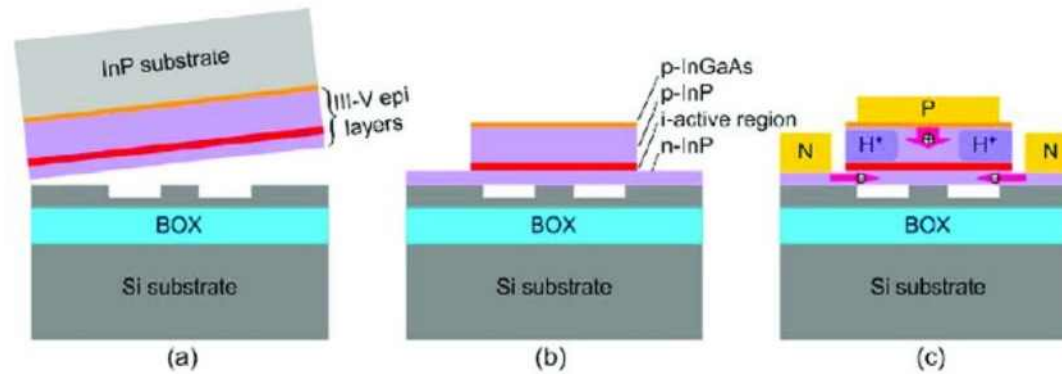
If $n_2 > n_3$, $R \sim \left(\frac{-(n_2/n_3)^{2m} n_4}{+(n_2/n_3)^{2m} n_4} \right)^2 = 1$

If $n_2 < n_3$, $R \sim \left(\frac{n_1}{n_1} \right)^2 = 1$

Lecture 12: Semiconductor Lasers

Hybrid Si Laser

- Wafer bonding between III–V wafers for gain and SOI wafer



Lecture 12: Semiconductor Lasers

Homework #3

Consider a laser made up of a gain material and two external mirrors as shown below. The external mirrors have reflectivity of 0.3. The end facets of the gain material have anti-reflection coatings so that their reflectivities are zero. The gain of the material is a function of wavelength and injected carrier density: $g(\lambda, n) = a(n - n_0) - b(\lambda - \lambda_0)^2$, where, $a = 2.4 \times 10^{-17} \text{ cm}^2$, $n_0 = 1 \times 10^{18} / \text{cm}^3$, and $b = 4800 / \text{cm} \cdot \mu\text{m}^2$, and $\lambda_0 = 1.0 \mu\text{m}$. The reflective index of the material is 3 and there is no internal loss. Use $\Gamma = 1$.

- (a) Determine the resonance condition for the lasing wavelength?
- (b) At what wavelength in μm does the first lasing mode appear?
- (c) What is the threshold gain in cm^{-1} for the first lasing mode?
- (d) An optical amplifier is made by removing two mirrors. If the injected carrier density is twice of the threshold carrier density, what is the output optical power for input light at $1.0 \mu\text{m}$?

