Si Photonics

Lecture 3: Reflection From Perfect Conductor

Woo-Young Choi

Dept. of Electrical and Electronic Engineering
Yonsei University

Lecture 3: Reflection From Perfect Conductor

$$\nabla \cdot \overline{D} = \rho$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} ==> -j\omega \overline{B}$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} ==> \overline{J} + j\omega \overline{D}$$

$$\overline{D} = \varepsilon \overline{E}$$

$$\overline{B} = \mu \overline{H}$$

For EM wave equation derivation

$\overline{J} = 0$ assumed

What if E-field generates currents?

$\overline{J} = \sigma \cdot \overline{E}$, $\sigma \neq 0$

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$$\nabla \cdot \overline{D} = \rho$$

$$\nabla \times \overline{E} = -j\omega \overline{B}$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{H} = \overline{J} + j\omega \overline{D}$$

$$\overline{D} = \varepsilon \overline{E}$$

$$\overline{B} = \mu \overline{H}$$

$$\overline{J} = \sigma \cdot \overline{E}, \ \sigma \neq 0$$

$$\nabla \times \overline{H} = \sigma \overline{E} + j\omega\varepsilon \overline{E} = j\omega(\varepsilon - j\frac{\sigma}{\omega})\overline{E}$$

$$= j\omega\varepsilon_c \overline{E}$$

➔ Complex permittivity $\varepsilon_c = \varepsilon - j\dfrac{\sigma}{\omega}$

Wave number k?

$$k = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu\varepsilon}\left(1 - j\frac{\sigma}{\varepsilon\omega}\right)^{\frac{1}{2}} = \beta - j\alpha,$$

Consider plane wave solution:

$$\overline{E} = \overline{x}E_0 e^{-jkz} = \overline{x}E_0 e^{-j(\beta - j\alpha)z} = \overline{x}E_0 e^{-j\beta z} e^{-\alpha z}$$

Exponential Decay!

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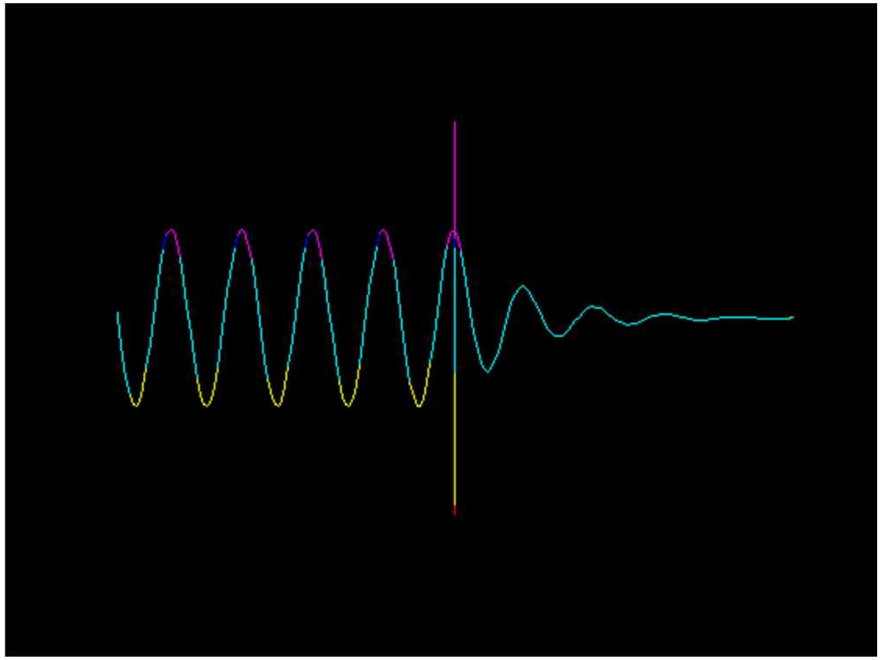
$$\overline{E} = \overline{x} E_0 e^{-j\beta z} e^{-\alpha z}$$



Why decay?

α : attenuation constant
➔ loss

β : phase constant

How much does the wave penetrate into the medium?

$$\delta = \frac{1}{\alpha}$$ Penetration depth, Or Skin depth

What if σ is infinitely large?

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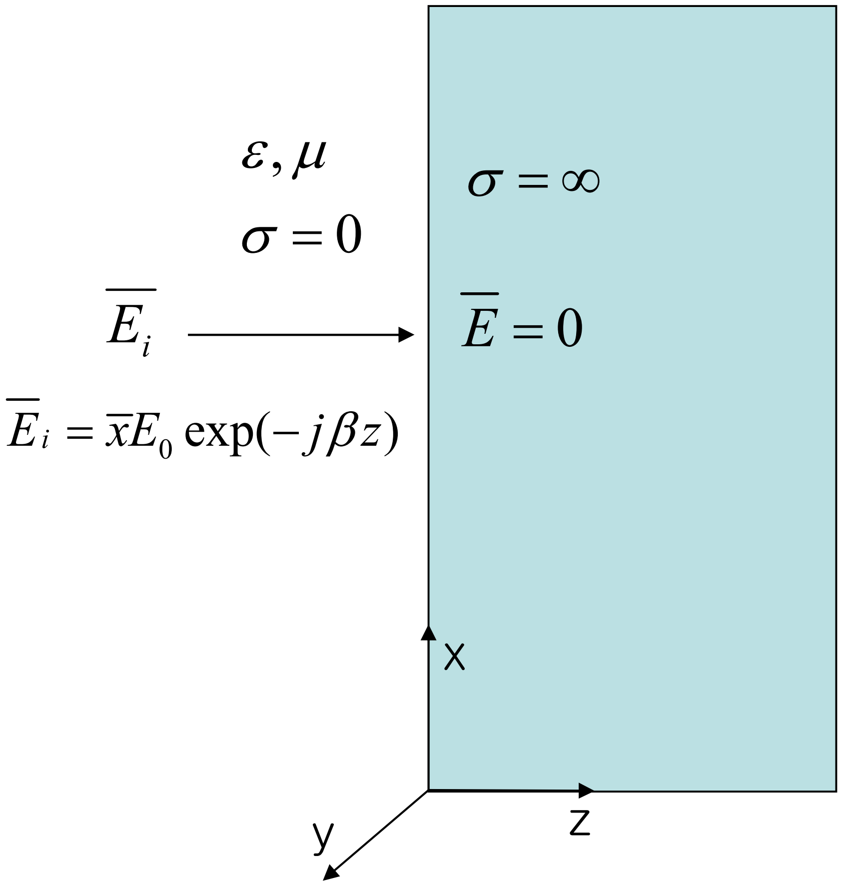
Consider EM waves normally incident at a perfect conductor at z=0



No penetration (skin depth = 0)

➔ Reflection

Reflected E-field?

$$\overline{E}_r = ?$$

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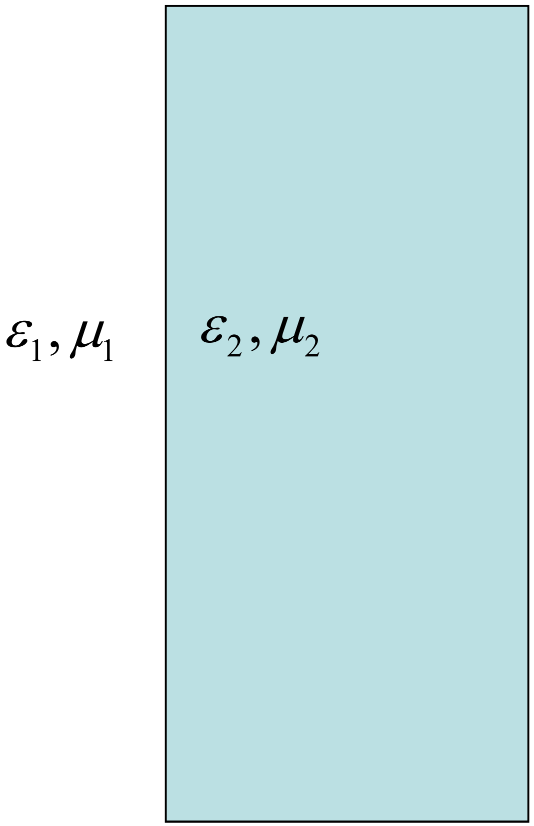
✓ Boundary Conditions: Constraints on E,H fields at a boundary

➔ Each of Maxwell's Equations provides one constraint on E or H



$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \qquad E_{1,t} = E_{2,t}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} \qquad \overline{a_n} \times (\overline{H}_2 - \overline{H}_1) = \overline{J}_s$$

$$\nabla \cdot \overline{D} = \rho \qquad \varepsilon_2 E_{2,n} - \varepsilon_1 E_{1,n} = \rho_s$$

$$\nabla \cdot \overline{B} = 0 \qquad \mu_2 H_{2,n} - \mu_1 H_{1,n} = 0$$

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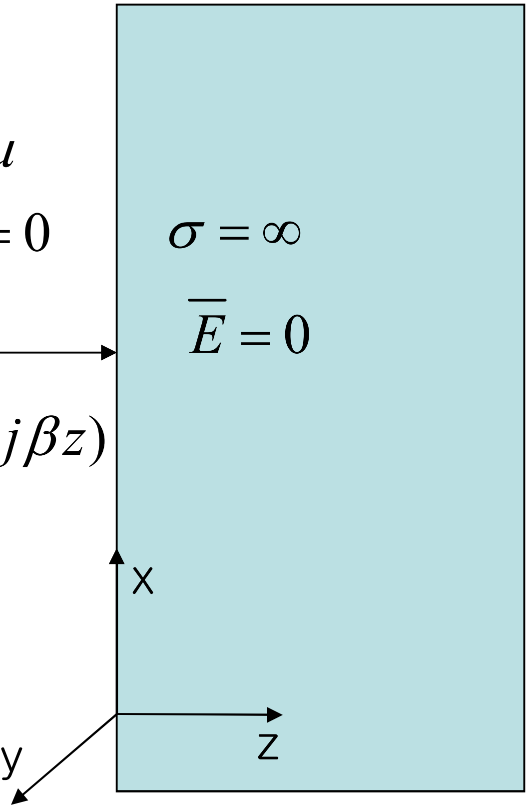
ε, μ

$\sigma = 0$

$\overline{E}_i = \overline{x}E_0 \exp(-j\beta z)$



$\overline{E}_r = ?$ $\quad E_{1,t} = E_{2,t}$

At z = 0, $\quad E_{2,t} = 0$

$$\overline{E}_i(z=0) + \overline{E}_r(z=0) = 0$$

$$\overline{E}_r = -\overline{x}E_0 \exp(j\beta z)$$

$$\overline{E}_{total}(z) = \overline{x}E_0 \exp(-j\beta z) - \overline{x}E_0 \exp(j\beta z) = \overline{x}E_0(-2j)\sin(\beta z)$$

$$\overline{E}_{total}(z,t) = \mathrm{Re}\left[\overline{x}E_0(-2j)\sin(\beta z)\exp(j\omega t)\right] = \overline{x}E_0 2\sin(\beta z)\sin(\omega t)$$

➔ Standing Wave!

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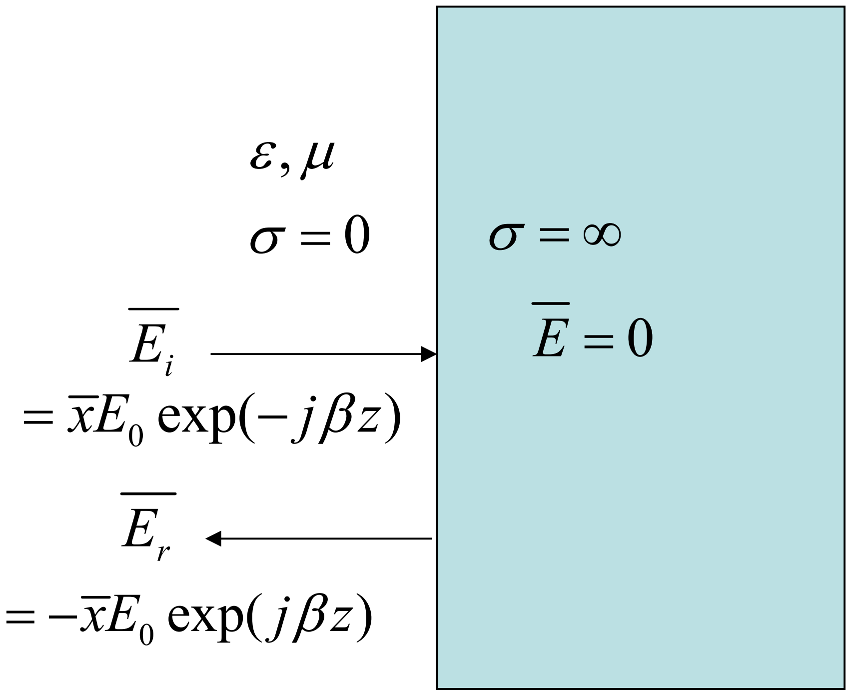

H-field for z<0?

$$\overline{H}_i = \overline{y} \frac{E_0}{\eta} \exp(-j\beta z)$$

$$\overline{H}_r = \overline{y} \frac{E_0}{\eta} \exp(j\beta z)$$

$$\overline{H}_{total}(z) = \overline{y} \frac{E_0}{\eta} \exp(-j\beta z) + \overline{y} \frac{E_0}{\eta} \exp(j\beta z) = \overline{y} \frac{2E_0}{\eta} \cos(\beta z)$$

$$\overline{H}_{total}(z,t) = \overline{y} \frac{2E_0}{\eta} \cos(\beta z) \cos(\omega t)$$

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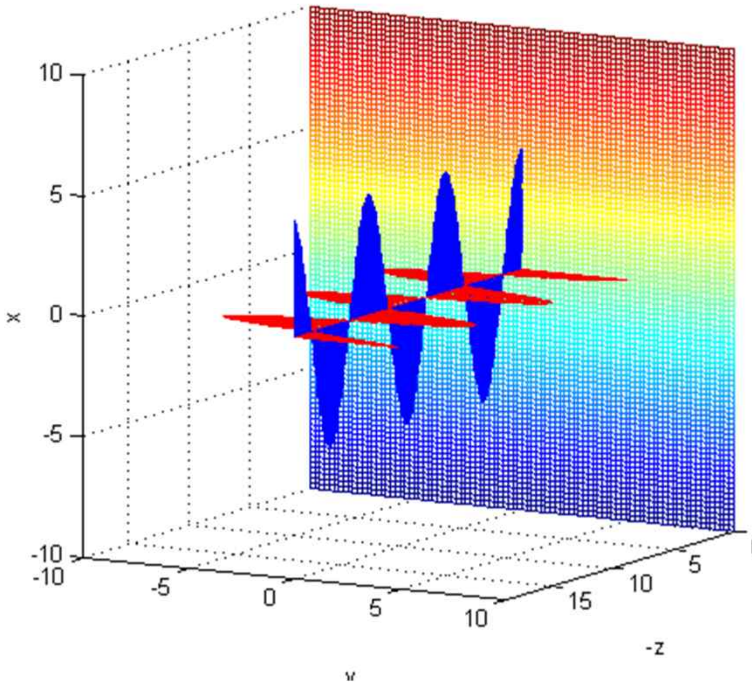
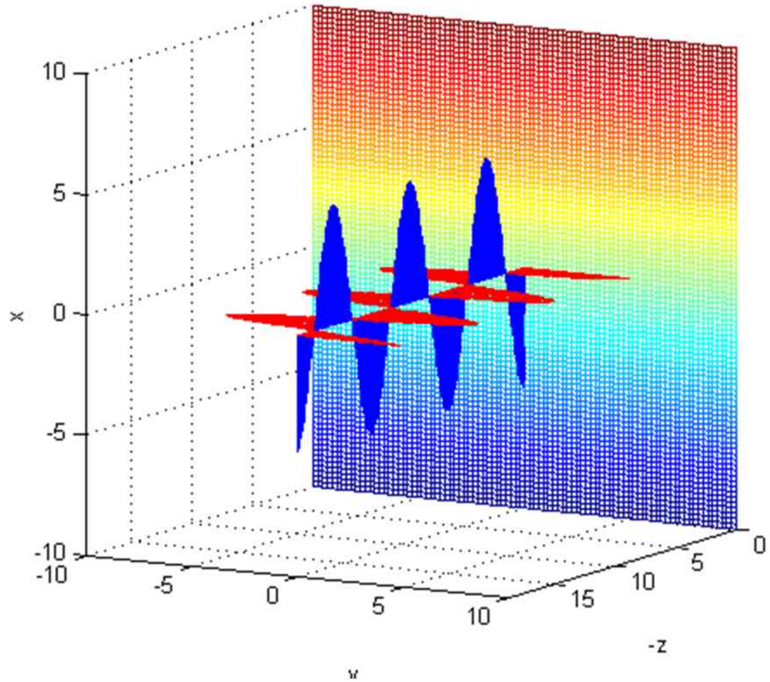
Incident Wave

$$\overline{E}_i = \overline{x}E_0 \exp(-j\beta z)$$



Reflected Wave

$$\overline{E}_r = -\overline{x}E_0 \exp(j\beta z)$$



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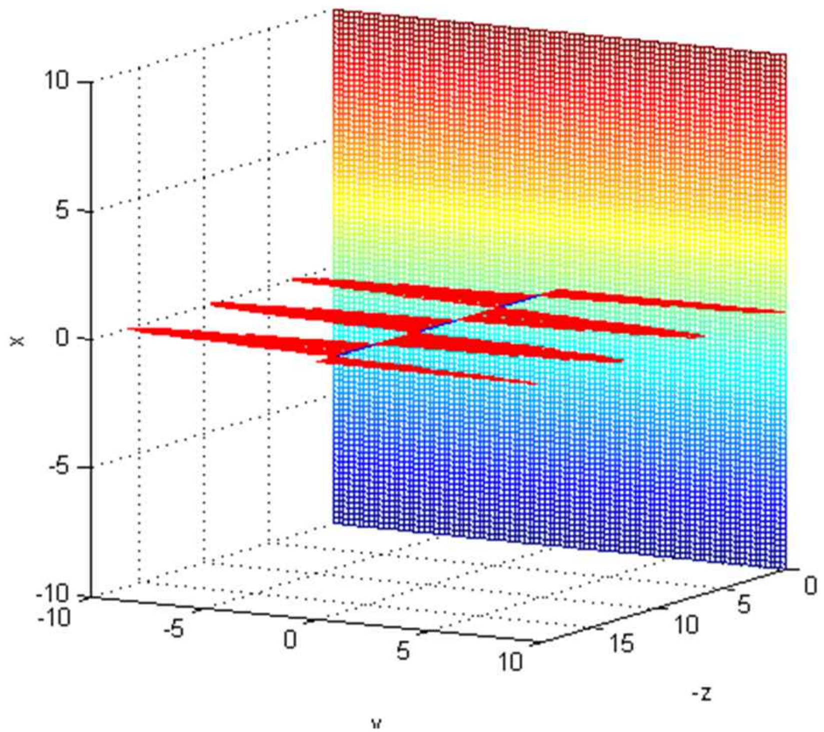
$$\bar{E}_{total}(z,t) = \bar{x}E_0 2\sin(\beta z)\sin(\omega t) \qquad \bar{H}_{total}(z,t) = \bar{y}\frac{2E_0}{\eta}\cos(\beta z)\cos(\omega t)$$



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B.C. at z=0?

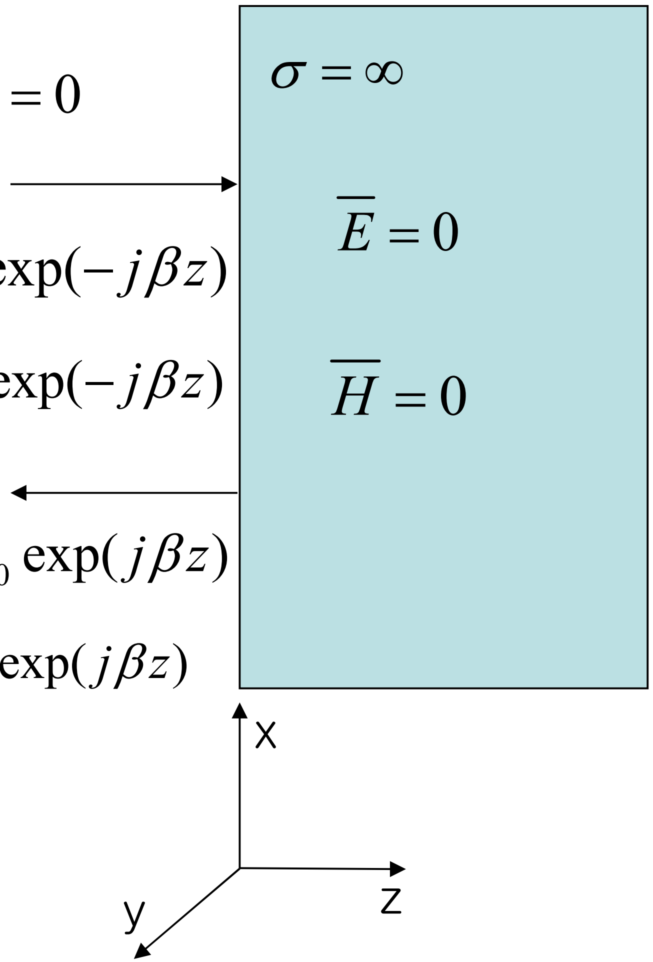
$$\varepsilon, \mu, \sigma = 0$$

$$\overline{E_i} = \overline{x} E_0 \exp(-j\beta z)$$

$$\overline{H}_i = \overline{y} \frac{E_0}{\eta} \exp(-j\beta z)$$

$$\overline{E_r} = -\overline{x} E_0 \exp(j\beta z)$$

$$\overline{H}_r = \overline{y} \frac{E_0}{\eta} \exp(j\beta z)$$



$$\sigma = \infty$$

$$\overline{E} = 0$$

$$\overline{H} = 0$$

$$E_{1,t} = E_{2,t} \qquad \varepsilon_2 E_{2,n} - \varepsilon_1 E_{1,n} = \rho_s$$

$$\rho_s = 0$$

$$\overline{a_n} \times (\overline{H}_2 - \overline{H}_1) = \overline{J}_s \qquad \mu_2 H_{2,n} - \mu_1 H_{1,n} = 0$$

$$\overline{z} \times \left(0 - \overline{y} \frac{2E_0}{\eta_1} \right) = \overline{J}_s$$

$$\overline{J}_s = \overline{x} \frac{2E_0}{\eta}$$

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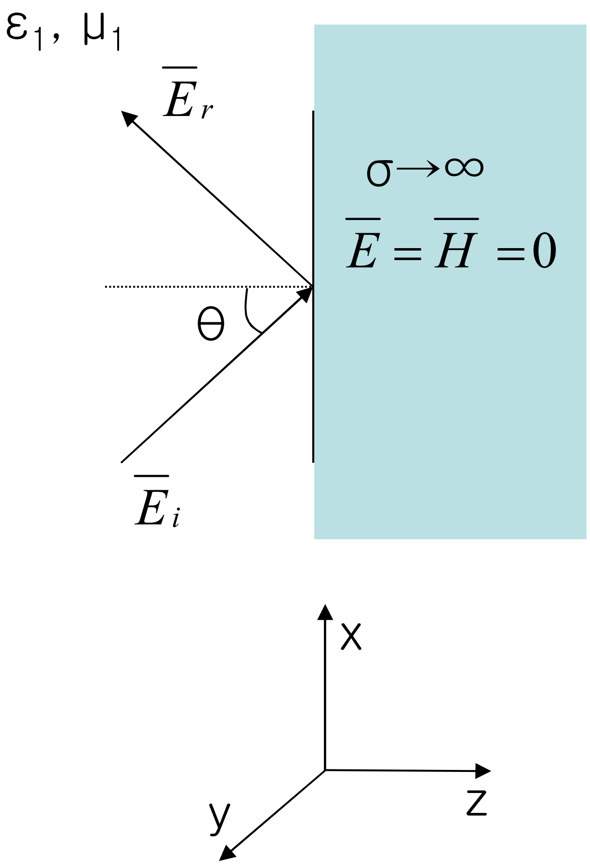

$$\overline{E}_i = \overline{y} E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\beta_x = k \sin\theta, \quad k = \omega\sqrt{\mu_1 \varepsilon_1}$$

$$\beta_z = k \cos\theta$$

$$\overline{E}_r, \overline{H}_i, \overline{H}_r = ?$$

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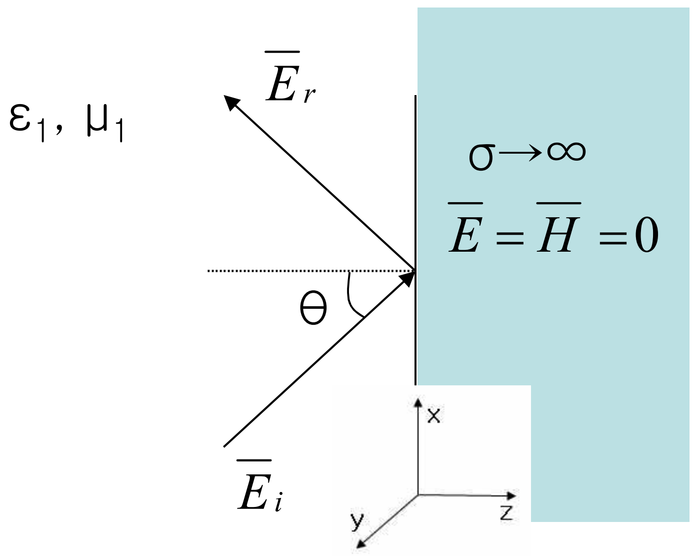

$$\overline{E}_i = \overline{y}E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\beta_x = k\sin\theta, \quad \beta_z = k\cos\theta$$

$\overline{E}_r = ?$ $$\overline{E}_r = \overline{y}E_r e^{-j\beta_{rx}x} e^{j\beta_{rz}z}$$

$$\beta_{r,x} = k\sin\theta_r, \quad \beta_{r,z} = k\cos\theta_r$$

Boundary condition at z=0

$$E_0 e^{-j\beta_x x} + E_r e^{-j\beta_{rx}x} = 0$$

$$\beta_x = \beta_{rx} \text{ and } E_0 = -E_r$$

$$\theta_i = \theta_r \qquad \beta_z = \beta_{rz}$$

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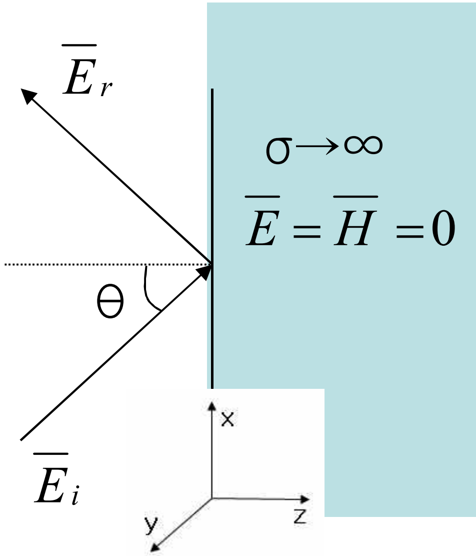
$$\overline{E}_r = -\overline{y}E_0 e^{-j\beta_x x} e^{j\beta_z z}$$

$$\beta_x = k\sin\theta$$

$$\beta_z = k\cos\theta$$



$$\overline{E}_i = \overline{y}E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\begin{aligned}\overline{E}_{total} = \overline{E}_i + \overline{E}_r &= \overline{y}E_0 e^{-j\beta_x x} e^{-j\beta_z z} - \overline{y}E_0 e^{-j\beta_x x} e^{j\beta_z z} \\ &= \overline{y}E_0 e^{-j\beta_x x}\left[e^{-j\beta_z z} - e^{j\beta_z z}\right] \\ &= \overline{y}E_0 e^{-j\beta_x x}(-2j)\sin(\beta_z z)\end{aligned}$$

x-direction: Plane wave propagation　　z-direction: Standing wave

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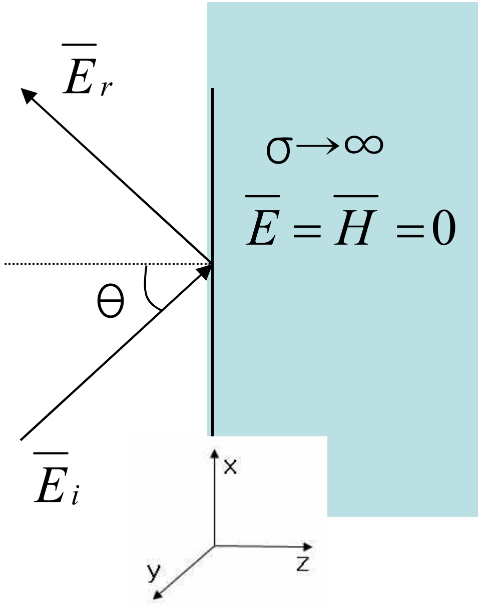

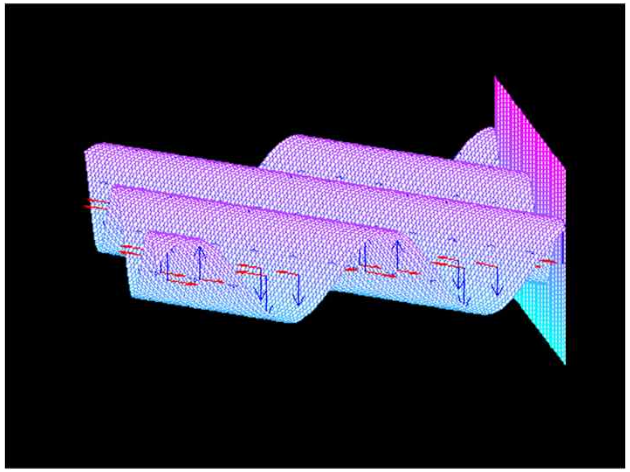

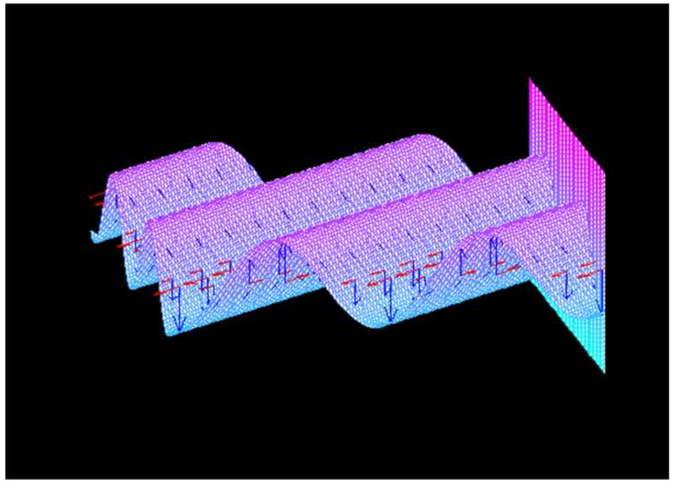

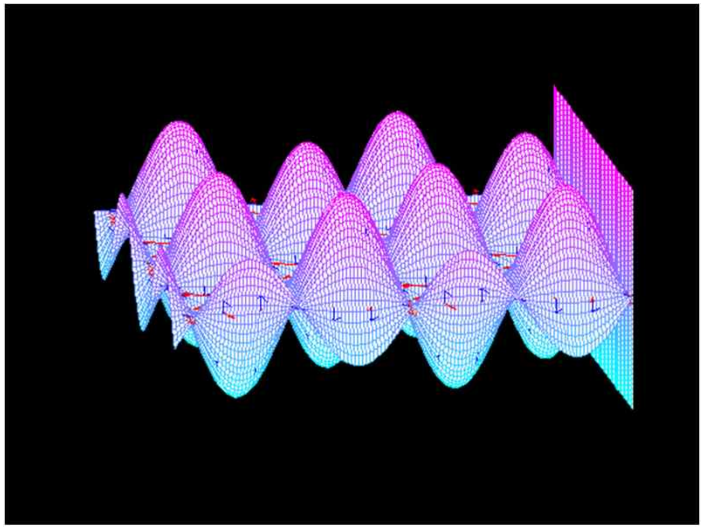

$$\overline{E}_i = \overline{y} E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\overline{E}_r = -\overline{y} E_0 e^{-j\beta_x x} e^{j\beta_z z}$$

$$\overline{E}_{total} == \overline{y} E_0 e^{-j\beta_x x} \left(-2j\right) \sin(\beta_z z)$$

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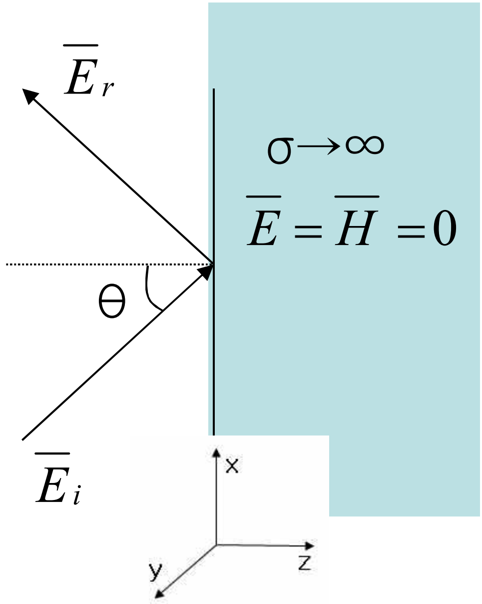

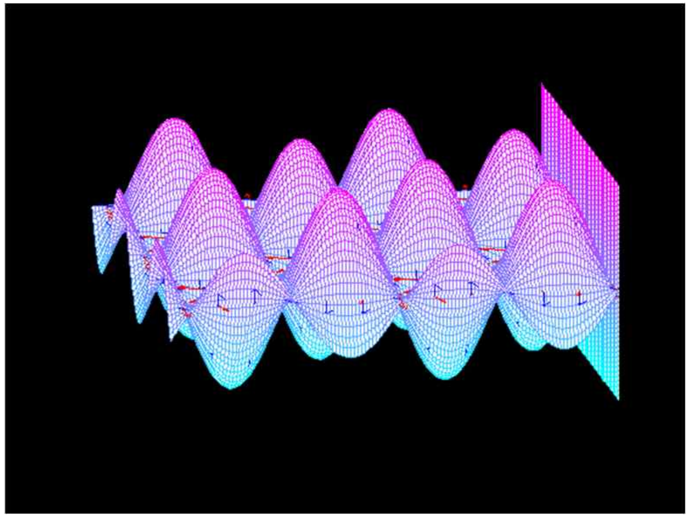

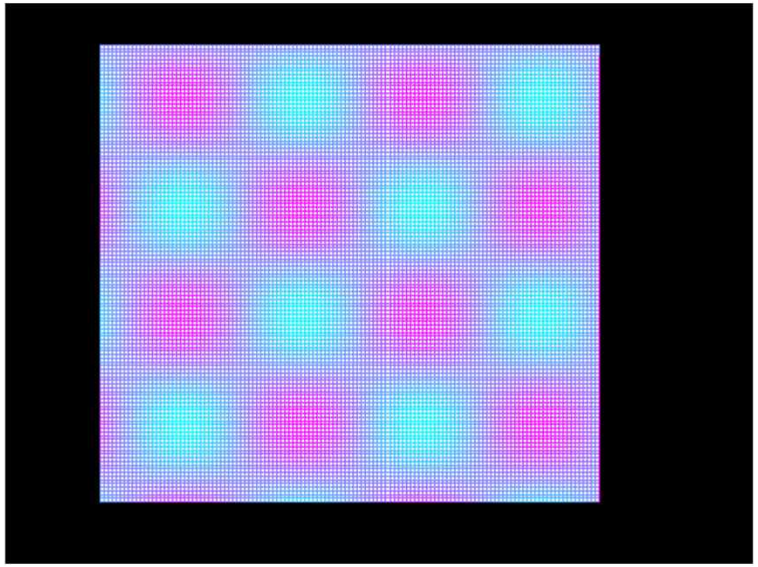

$$\overline{E}_{total} = = \overline{y} E_0 e^{-j\beta_x x} \left(-2j\right) \sin(\beta_z z)$$

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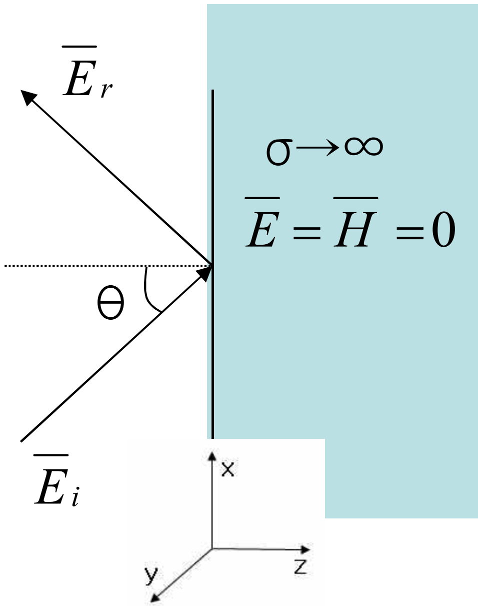

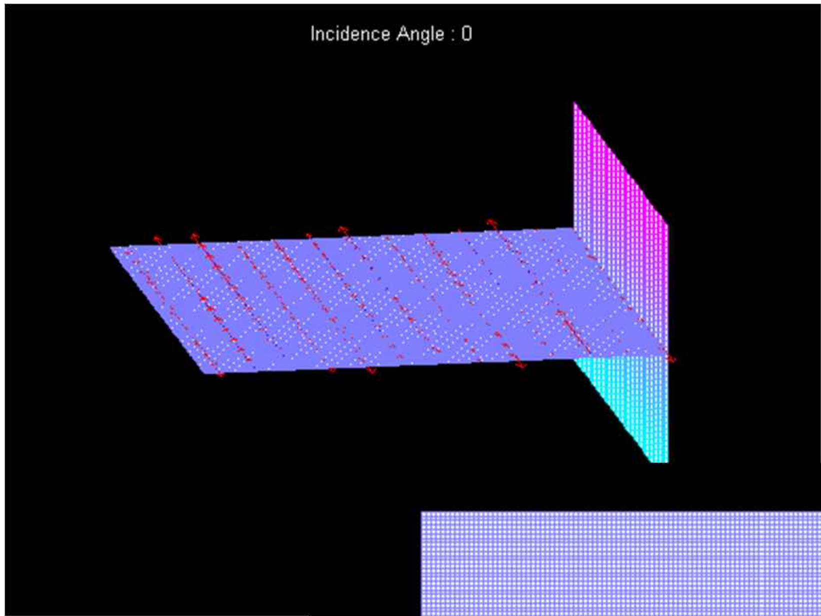

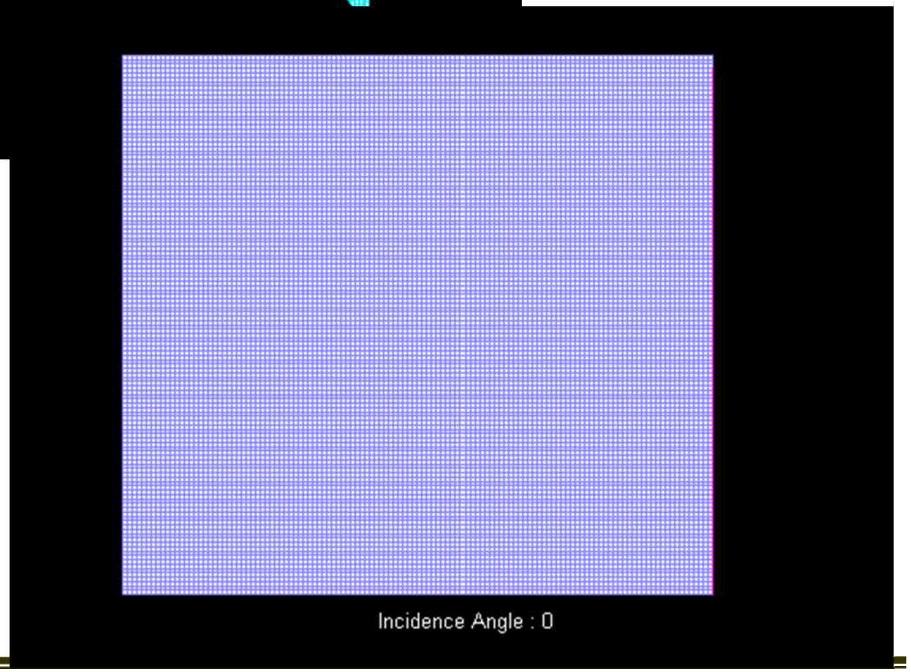

$$\overline{E}_{total} = = \overline{y} E_0 e^{-j\beta_x x}\left(-2j\right)\sin(\beta_z z)$$

$$\beta_x = k\sin\theta, \quad \beta_z = k\cos\theta$$

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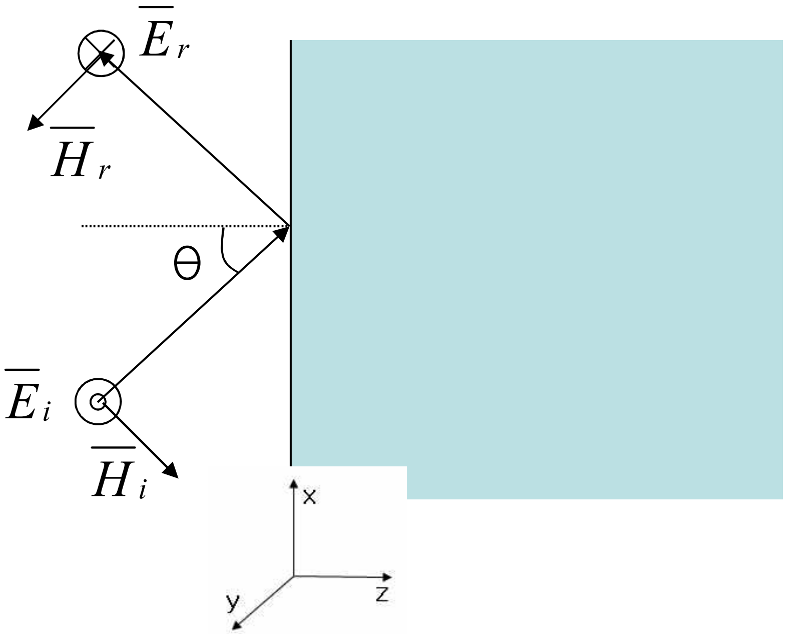
$$\overline{E}_r = -\overline{y}E_0 e^{-j\beta_x x} e^{j\beta_z z}$$



$$\overline{E}_i = \overline{y}E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

How about H-fields?

$$\overline{H}_i = \frac{1}{\eta}\left(\overline{a_k} \times \overline{E}_i\right) \qquad \overline{a_k} = \overline{x}\sin\theta + \overline{z}\cos\theta$$

$$= \frac{E_0}{\eta}\left(\overline{z}\sin\theta - \overline{x}\cos\theta\right) e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\overline{H}_r = \frac{1}{\eta}\left(\overline{a_k} \times \overline{E}_r\right) \qquad \overline{a_k} = \overline{x}\sin\theta - \overline{z}\cos\theta$$

$$\overline{H}_r = -\frac{E_0}{\eta}\left(\overline{z}\sin\theta + \overline{x}\cos\theta\right) e^{-j\beta_x x} e^{j\beta_z z}$$

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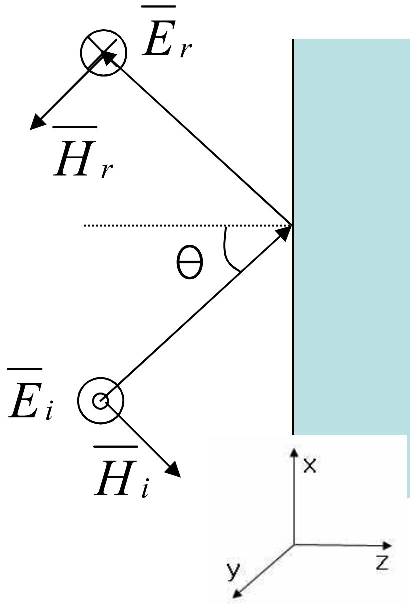

$$\overline{E}_i = \overline{y}E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\overline{E}_r = -\overline{y}E_0 e^{-j\beta_x x} e^{j\beta_z z}$$

$$\overline{E}_{total} == \overline{y}E_0 e^{-j\beta_x x}(-2j)\sin(\beta_z z)$$

$$\overline{H}_i = \frac{E_0}{\eta}\left(\overline{z}\sin\theta - \overline{x}\cos\theta\right)e^{-j\beta_x x}e^{-j\beta_z z}$$

$$\overline{H}_r = -\frac{E_0}{\eta}\left(\overline{z}\sin\theta + \overline{x}\cos\theta\right)e^{-j\beta_x x}e^{j\beta_z z}$$

$$\overline{H}_{total} = \frac{E_0}{\eta}e^{-j\beta_x x}\left[\overline{z}\sin\theta(e^{-j\beta_z z} - e^{j\beta_z z}) - \overline{x}\cos\theta(e^{-j\beta_z z} + e^{j\beta_z z})\right]$$

$$= \frac{E_0}{\eta}e^{-j\beta_x x}\left[\overline{z}\sin\theta\cdot(-2j)\sin(\beta_z z) - \overline{x}\cos\theta\cdot 2\cos(\beta_z z)\right]$$

At z=0,

$\rho_s = ?$ $\quad\overline{J}_s = ?$ $\quad\overline{a}_n \times (\overline{H}_2 - \overline{H}_1) = \overline{J}_s$

$\overline{a}_n = \overline{z}$ $\quad\overline{H}_2 = 0$ $\quad\overline{J}_s = \overline{z} \times (-\overline{H}_1) = \overline{y}\frac{2E_0}{\eta}\cos\theta e^{-j\beta_x x}$

Propagation of current densities into x-direction

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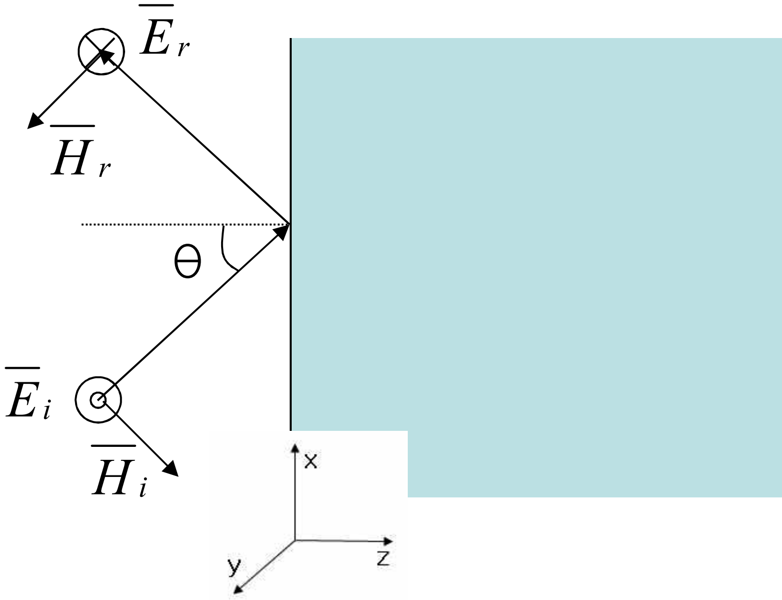

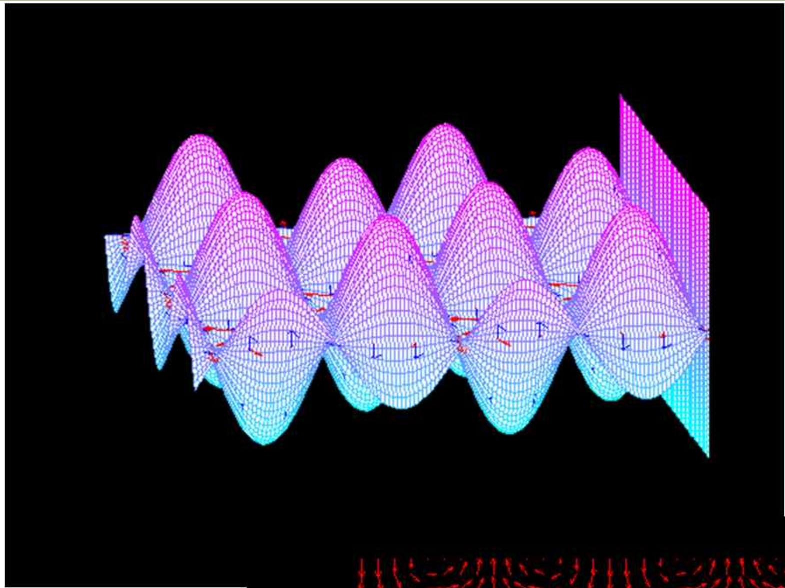

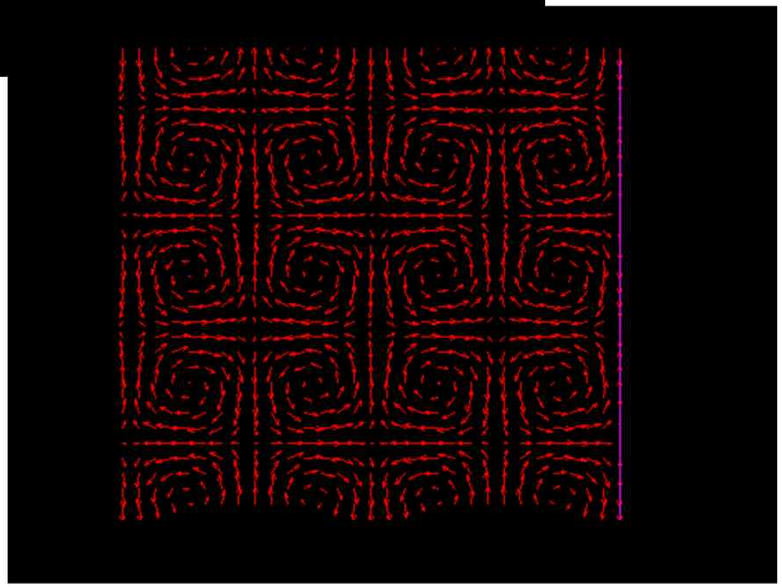

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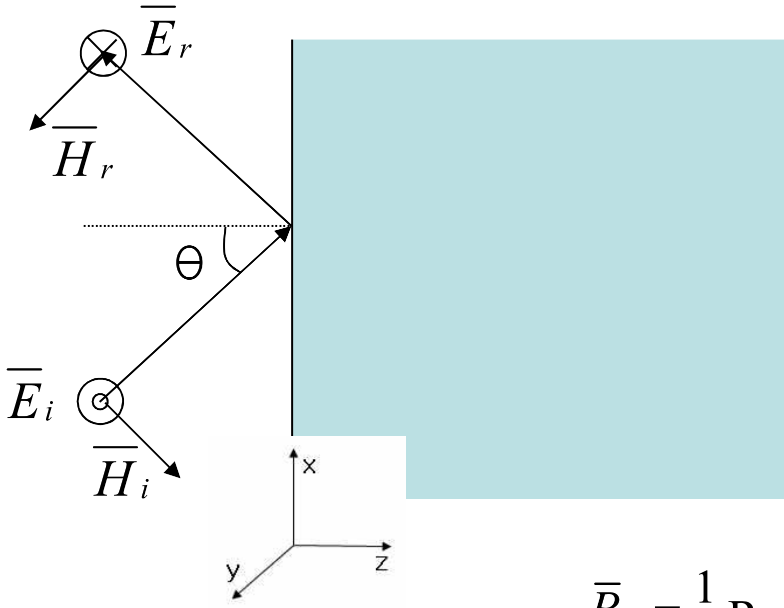

$$\overline{E}_{total} == \overline{y}E_0 e^{-j\beta_x x}\left(-2j\right)\sin(\beta_z z)$$

$$\overline{H}_{total} = \frac{E_0}{\eta} e^{-j\beta_x x}\left[\overline{z}\sin\theta\cdot(-2j)\sin(\beta_z z) - \overline{x}\cos\theta\cdot 2\cos(\beta_z z)\right]$$

$$\overline{H}^*_{total} = \frac{E_0}{\eta} e^{j\beta_x x}\left[\overline{z}\sin\theta\cdot(2j)\sin(\beta_z z) - \overline{x}\cos\theta\cdot 2\cos(\beta_z z)\right]$$

$$\overline{P}_{av} = \frac{1}{2}\mathrm{Re}\left[\overline{E}\times\overline{H}^*\right]$$

$$= \frac{E_0{}^2}{2\eta}\mathrm{Re}\left[\overline{x}4\sin\theta\sin^2(\beta_z z) + \overline{z}(-4j)\cos\theta\sin(\beta_z z)\cos(\beta_z z)\right]$$

$$= \overline{x}\frac{2E_0{}^2}{\eta}\sin\theta\sin^2(\beta_z z)$$

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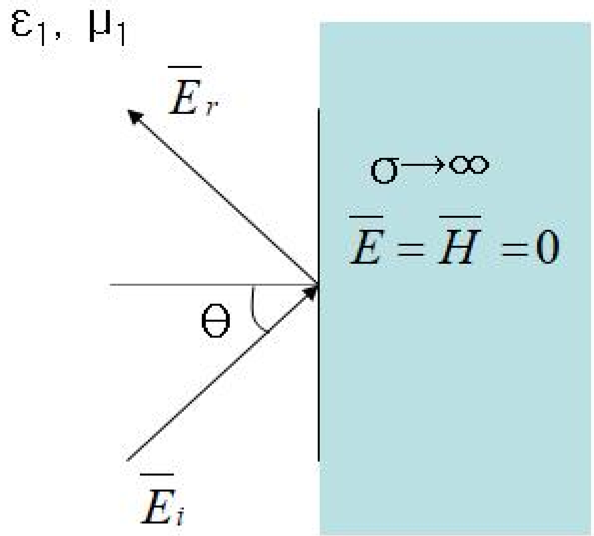
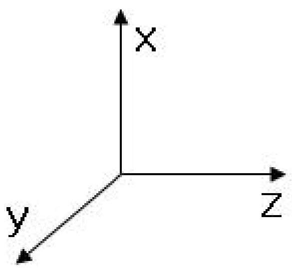




$$\overline{E}_i = \overline{y} E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

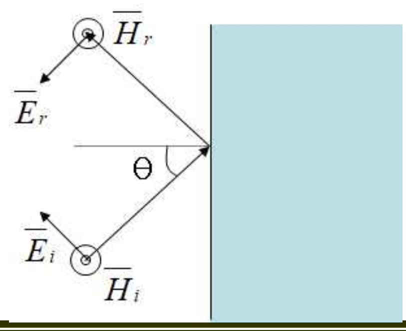
Only y−component ➔ Perpendicular polarization
(E−polarization, s−polarizat

What if E−field has x− and/or z−components?

$$\overline{E}_i = \left(\overline{x} E_x + \overline{z} E_z\right) e^{-j\beta_x x} e^{-j\beta_z z}$$

➔ Parallel polarization
(H−polarization, p−polarization)

Easier to analyze with H_i given



E_i, H_r, E_r ?

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Homework:

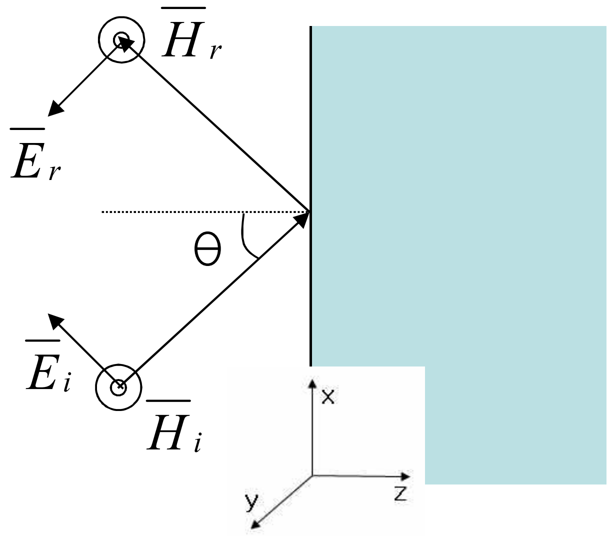
Given $\overline{H_i} = \overline{y}\frac{E_0}{\eta}e^{-j\beta_x x}e^{-j\beta_z z}$

(1) Show the following

$$\overline{E_i} = E_0\left(\overline{x}\cos\theta - \overline{z}\sin\theta\right)e^{-j\beta_x x}e^{-j\beta_z z}$$

$$\overline{H_r} = \overline{y}\frac{E_0}{\eta}e^{-j\beta_x x}e^{+j\beta_z z}$$

$$\overline{E_r} = -E_0\left(\overline{x}\cos\theta + \overline{z}\sin\theta\right)e^{-j\beta_x x}e^{+j\beta_z z}$$



(2) Determine $\overline{P}_{av}$

(3) At z=0, determine ρ_s

(4) At z=0, determine J_s