

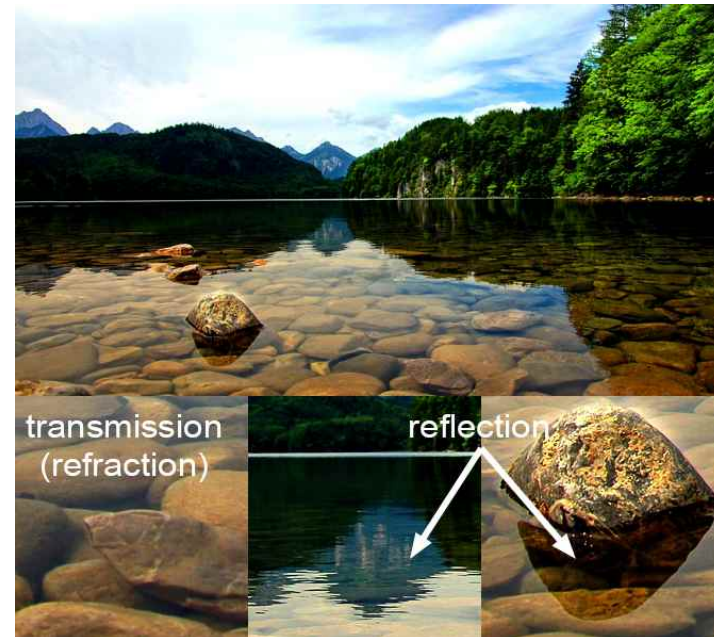
Si Photonics

Lecture 4: Reflection and Transmission at Dielectric Interface

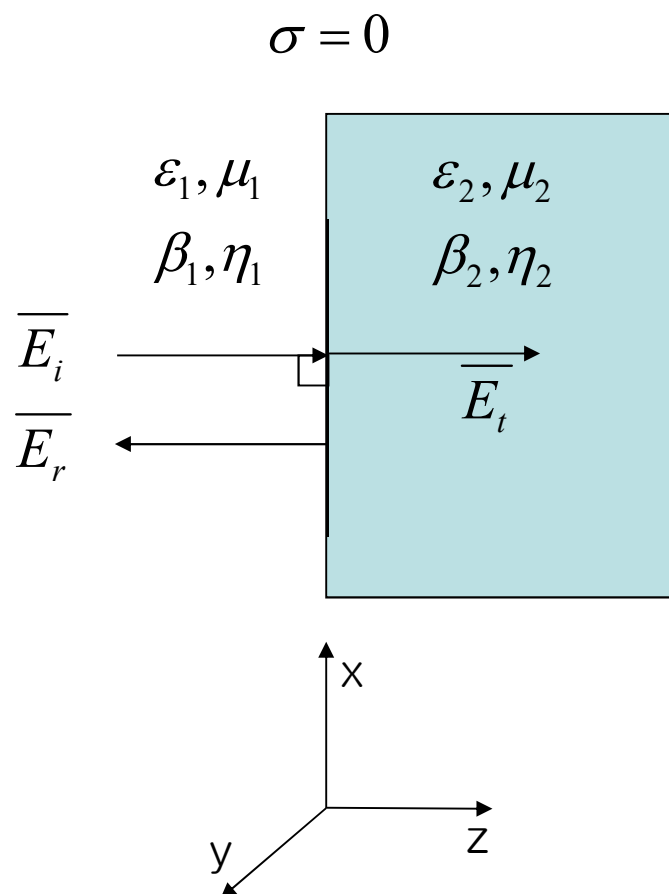
Woo-Young Choi

**Dept. of Electrical and Electronic Engineering
Yonsei University**

Lecture 4: Reflection and Transmission at Dielectric Interface



Lecture 4: Reflection and Transmission at Dielectric Interface



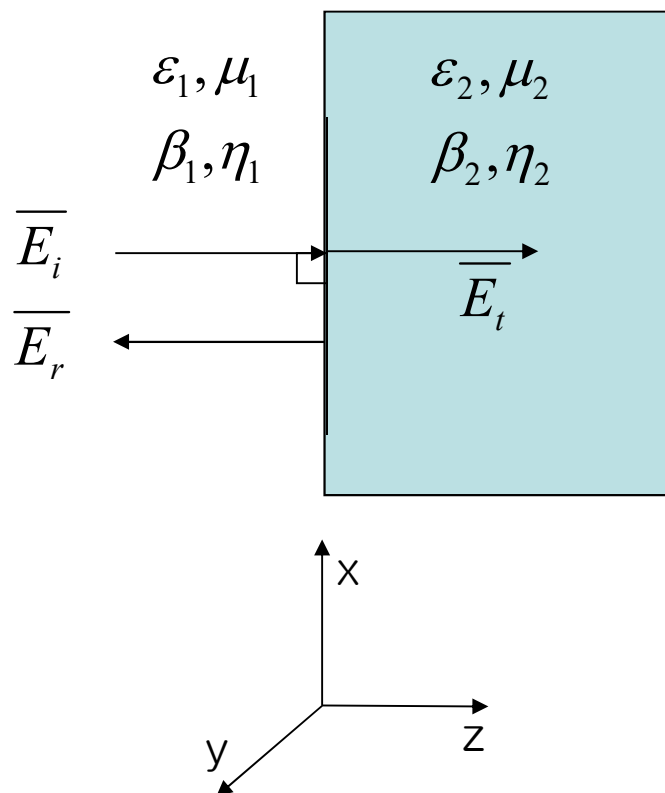
$$\overline{E}_i = \overline{y}E_0 \exp(-j\beta_1 z), \quad \overline{H}_i = -\overline{x} \frac{E_0}{\eta_1} \exp(-j\beta_1 z)$$

$$\overline{E}_r = \overline{y}E_r \exp(j\beta_1 z), \quad \overline{H}_r = \overline{x} \frac{E_r}{\eta_1} \exp(j\beta_1 z)$$

$$\overline{E}_t = \overline{y}E_t \exp(-j\beta_2 z), \quad \overline{H}_t = -\overline{x} \frac{E_t}{\eta_2} \exp(-j\beta_2 z)$$

Unknowns: E_r, E_t

Lecture 4: Reflection and Transmission at Dielectric Interface



$$\vec{E}_i = \bar{y}E_0 \exp(-j\beta_1 z), \quad \vec{H}_i = -\bar{x} \frac{E_0}{\eta_1} \exp(-j\beta_1 z)$$

$$\vec{E}_r = \bar{y}E_r \exp(j\beta_1 z), \quad \vec{H}_r = \bar{x} \frac{E_r}{\eta_1} \exp(j\beta_1 z)$$

$$\vec{E}_t = \bar{y}E_t \exp(-j\beta_2 z), \quad \vec{H}_t = -\bar{x} \frac{E_t}{\eta_2} \exp(-j\beta_2 z)$$

B.C.'s

1) E_{tan} should be continuous at $z=0$

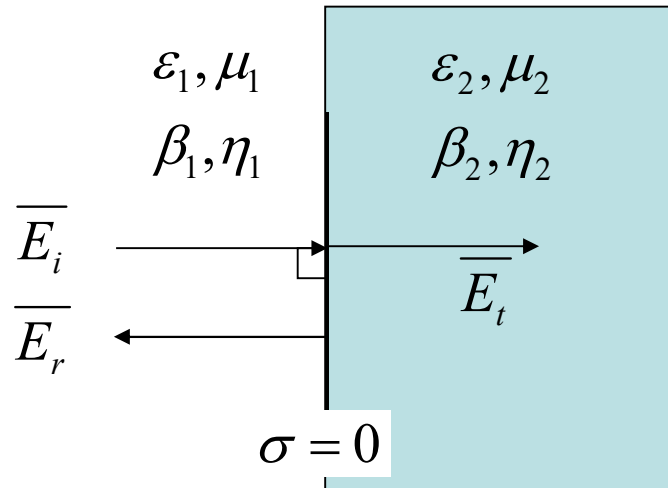
$$E_0 + E_r = E_t$$

Define $E_r = \Gamma E_0$, and $E_t = \tau E_0$

Γ : reflection coef.
 τ : transmission coef.

$$\Rightarrow 1 + \Gamma = \tau$$

Lecture 4: Reflection and Transmission at Dielectric Interface



2) H_{tan} should be continuous at $z=0$

$$\bar{a}_n \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s = 0$$

$$H_i + H_r = H_t$$

$$-\frac{E_0}{\eta_1} + \frac{E_r}{\eta_1} = -\frac{E_t}{\eta_2}$$

$$\bar{H}_i = -\bar{x} \frac{E_0}{\eta_1} \exp(-j\beta_1 z)$$

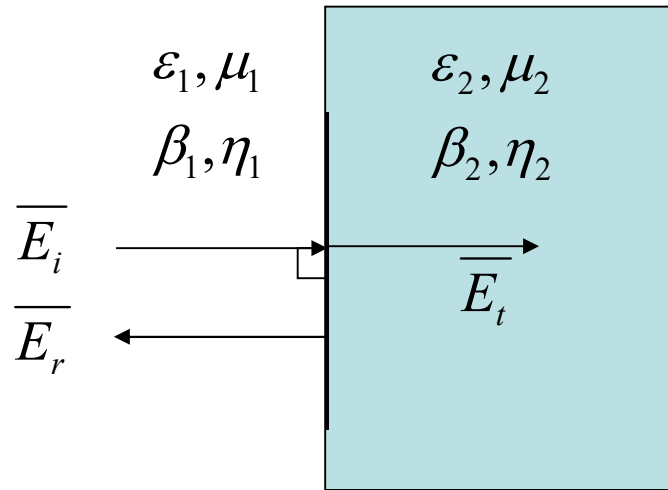
$$\bar{H}_r = \bar{x} \frac{E_r}{\eta_1} \exp(j\beta_1 z)$$

$$\bar{H}_t = -\bar{x} \frac{E_t}{\eta_2} \exp(-j\beta_2 z)$$

$$\therefore -\frac{1}{\eta_1} + \frac{\Gamma}{\eta_1} = -\frac{\tau}{\eta_2}, \quad (1 + \Gamma = \tau)$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Lecture 4: Reflection and Transmission at Dielectric Interface



$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

In dielectric materials,

$$\mu = \mu_0 \quad \varepsilon = \varepsilon_r \varepsilon_0 = n^2 \varepsilon_0$$

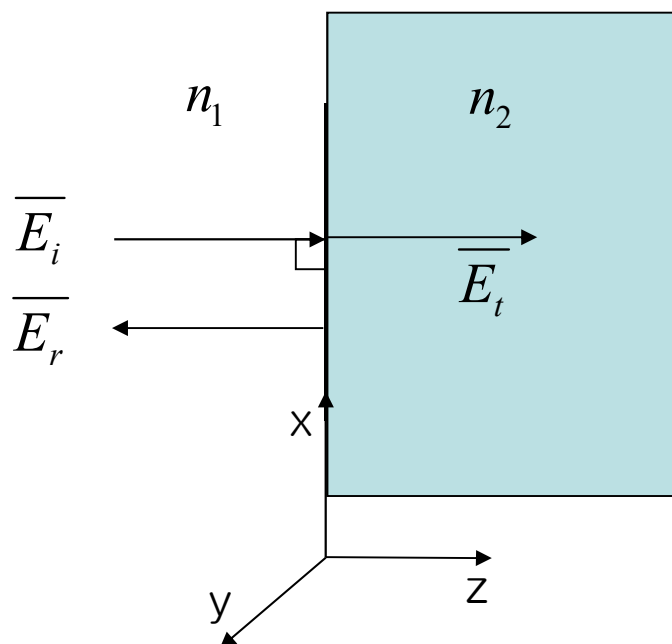
n : refractive index

$$\frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{n\sqrt{\mu_0\varepsilon_0}}$$

$$\Gamma = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}} - \sqrt{\frac{\mu_1}{\varepsilon_1}}}{\sqrt{\frac{\mu_2}{\varepsilon_2}} + \sqrt{\frac{\mu_1}{\varepsilon_1}}} = \frac{\frac{1}{n_2} \sqrt{\frac{1}{\varepsilon_0}} - \frac{1}{n_1} \sqrt{\frac{1}{\varepsilon_0}}}{\frac{1}{n_2} \sqrt{\frac{1}{\varepsilon_0}} + \frac{1}{n_1} \sqrt{\frac{1}{\varepsilon_0}}} = \frac{n_1 - n_2}{n_1 + n_2}, \quad \tau = \frac{2n_1}{n_1 + n_2}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = n\beta_0$$

Lecture 4: Reflection and Transmission at Dielectric Interface

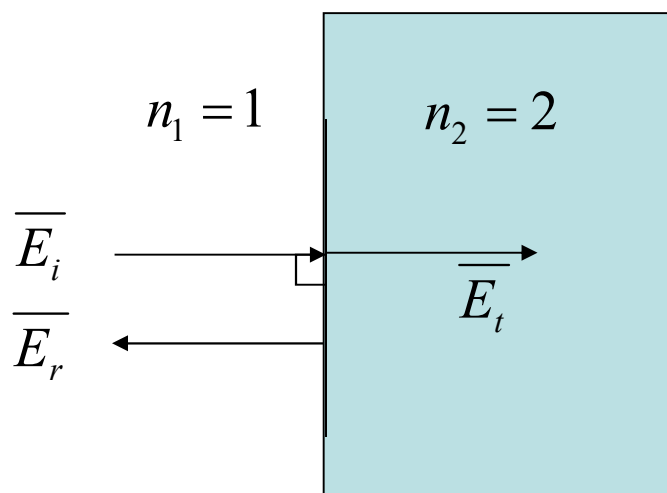


$$\Gamma = \frac{n_1 - n_2}{n_1 + n_2}, \quad \tau = \frac{2n_1}{n_1 + n_2}$$

$$\text{Max. } \Gamma : 1, \quad n_1 \gg n_2$$
$$\tau : 2$$

$$\text{Min. } \Gamma : -1, \quad n_1 \ll n_2$$
$$\tau : 0$$

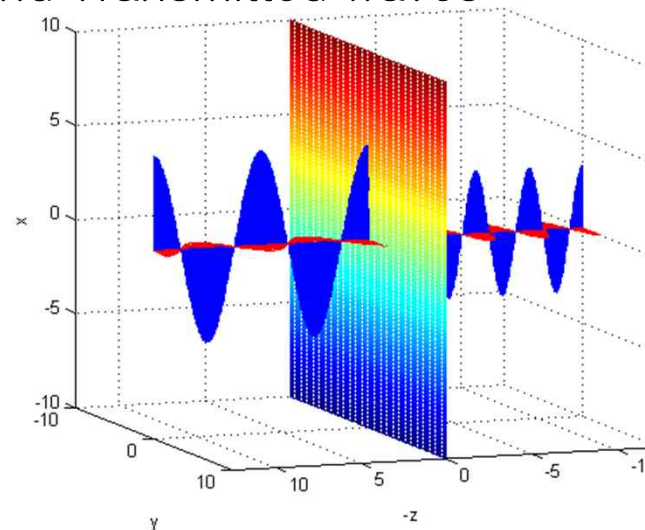
Lecture 4: Reflection and Transmission at Dielectric Interface



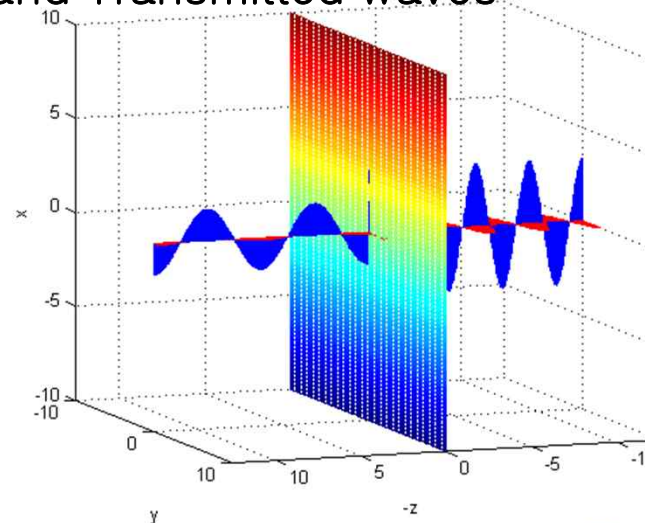
$$\Gamma = \frac{n_1 - n_2}{n_1 + n_2} = -\frac{1}{3}$$

$$\tau = \frac{2n_1}{n_1 + n_2} = \frac{2}{3}$$

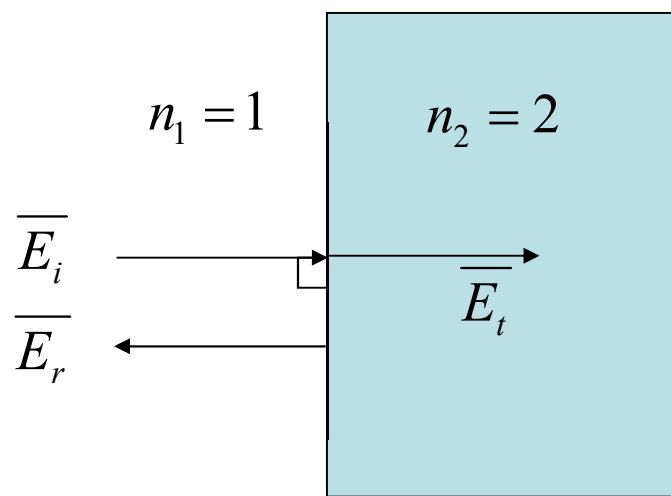
Incident and Transmitted waves



Reflected and Transmitted waves



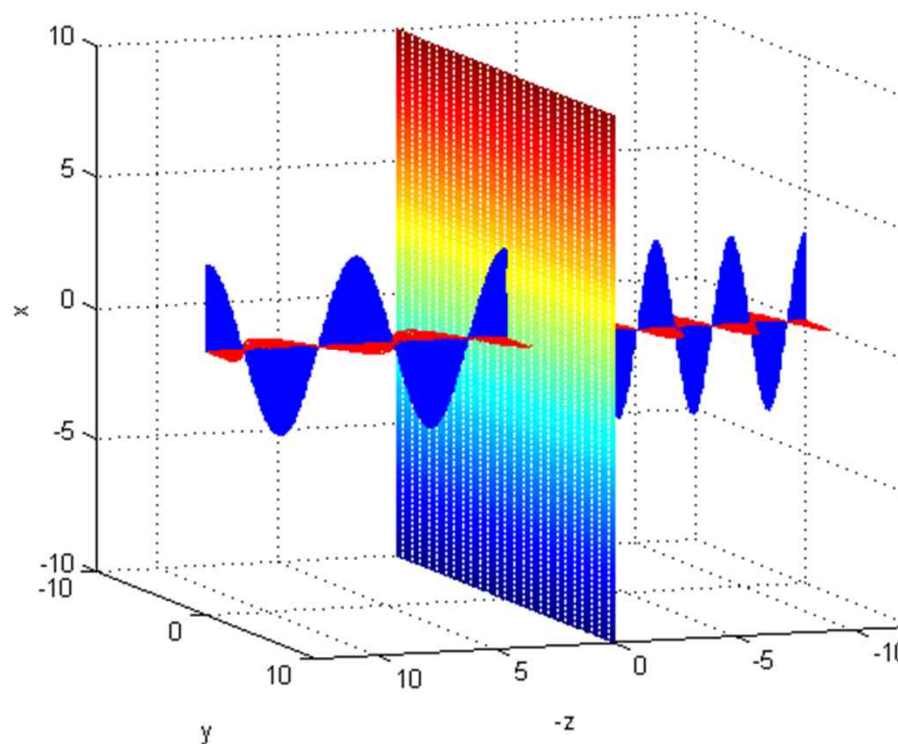
Lecture 4: Reflection and Transmission at Dielectric Interface



$$\Gamma = \frac{n_1 - n_2}{n_1 + n_2} = -\frac{1}{3}$$

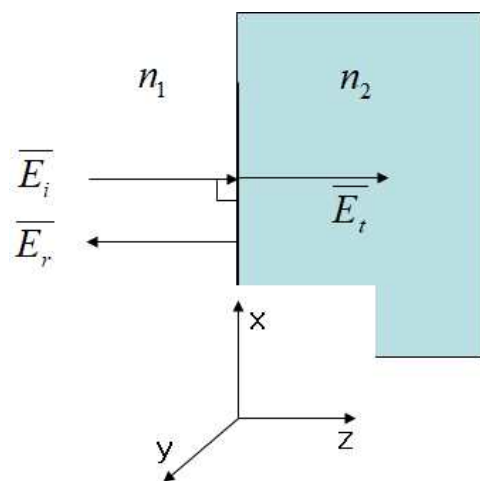
$$\tau = \frac{2n_1}{n_1 + n_2} = \frac{2}{3}$$

Total waves



Lecture 4: Reflection and Transmission at Dielectric Interface

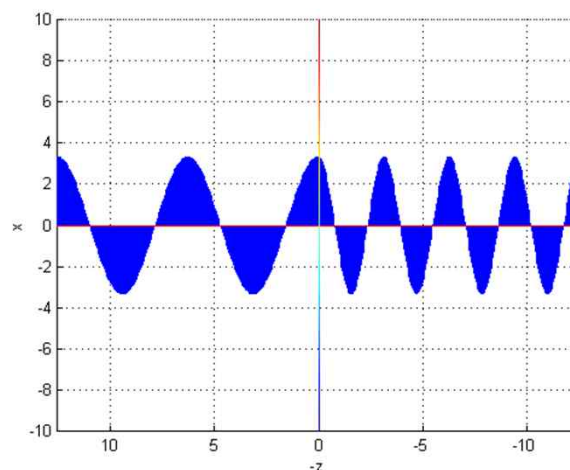
Total E-field for $z < 0$



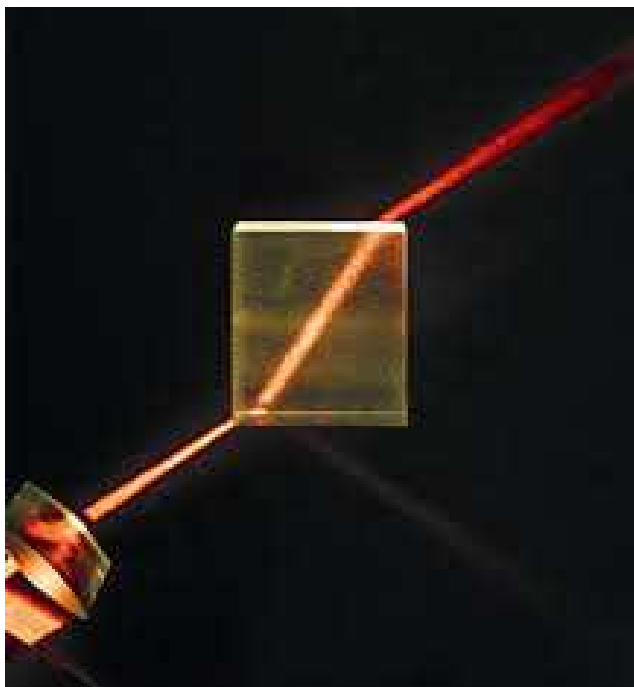
$$\begin{aligned}\bar{E}_{total}(z < 0) &= \bar{E}_i + \bar{E}_r = \bar{y}E_0(e^{-jn_1\beta_0 z} + \Gamma e^{jn_1\beta_0 z}) \\ &= \bar{y}E_0 \left\{ (1 + \Gamma)e^{-jn_1\beta_0 z} + \Gamma(e^{jn_1\beta_0 z} - e^{-jn_1\beta_0 z}) \right\} \\ &= \bar{y}E_0 \left\{ (1 + \Gamma)e^{-jn_1\beta_0 z} + 2j\Gamma \sin(n_1\beta_0 z) \right\}\end{aligned}$$

transmitting
wave

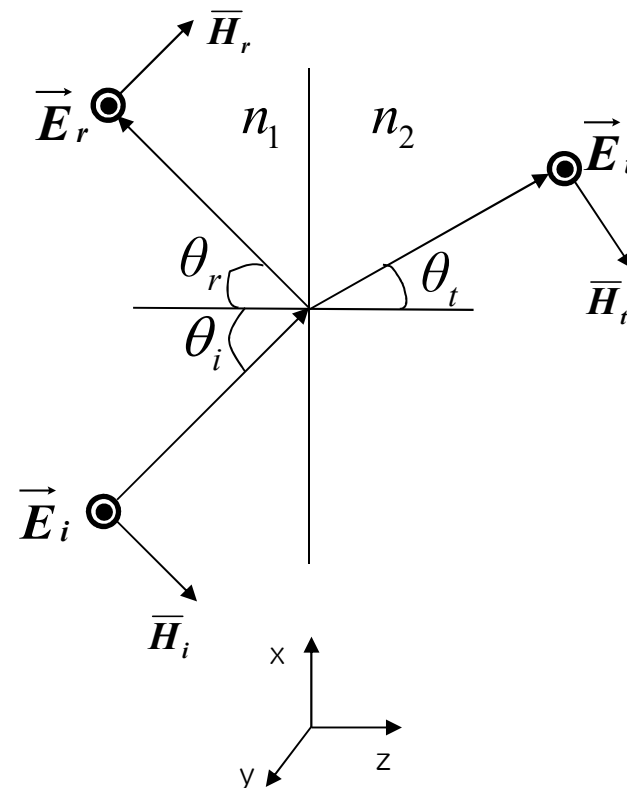
standing
wave



Lecture 4: Reflection and Transmission at Dielectric Interface

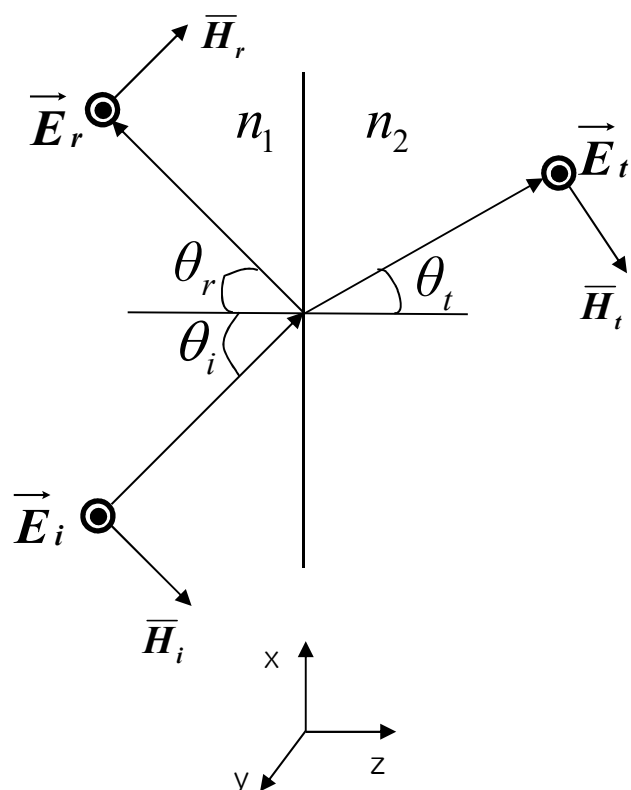


➔ Reflection, Transmission (Refraction)



(Perpendicular Polarization)

Lecture 4: Reflection and Transmission at Dielectric Interface



$$\overline{E}_i = \overline{y} E_i \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

$$\beta_{1x} = n_1 k_0 \sin \theta_i$$

$$\beta_{1z} = n_1 k_0 \cos \theta_i$$

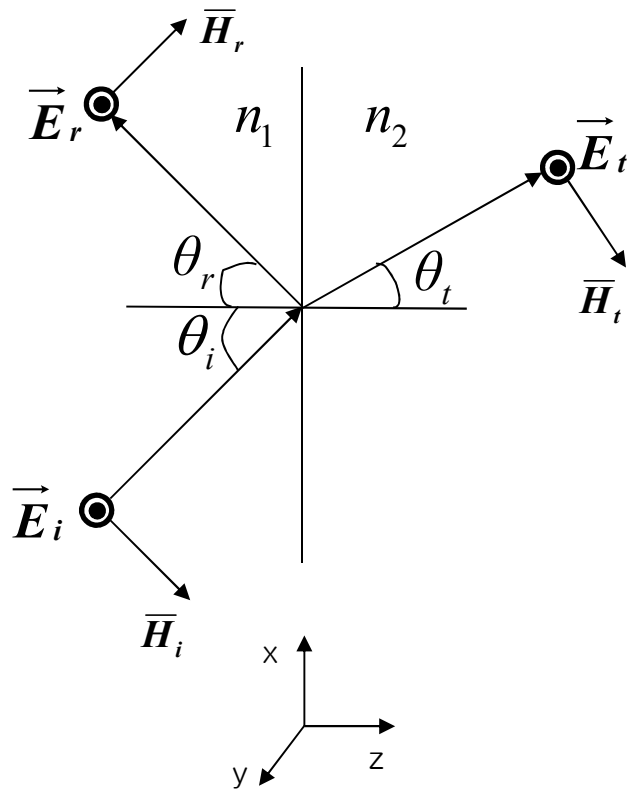
$$\overline{E}_r = \overline{y} \Gamma E_i \exp(-j\beta_{rx}x) \exp(j\beta_{rz}z)$$

Γ : reflection coefficient

$$\overline{E}_t = \overline{y} \tau E_i \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

τ : transmission coefficient

Lecture 4: Reflection and Transmission at Dielectric Interface



$$\overline{E}_i = \overline{y} E_i \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

$$\overline{H}_i = (-\overline{x} \cos \theta_i + \overline{z} \sin \theta_i) \frac{E_i}{\eta_1} \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

$$\overline{E}_r = \overline{y} \Gamma E_i \exp(-j\beta_{rx}x) \exp(j\beta_{rz}z)$$

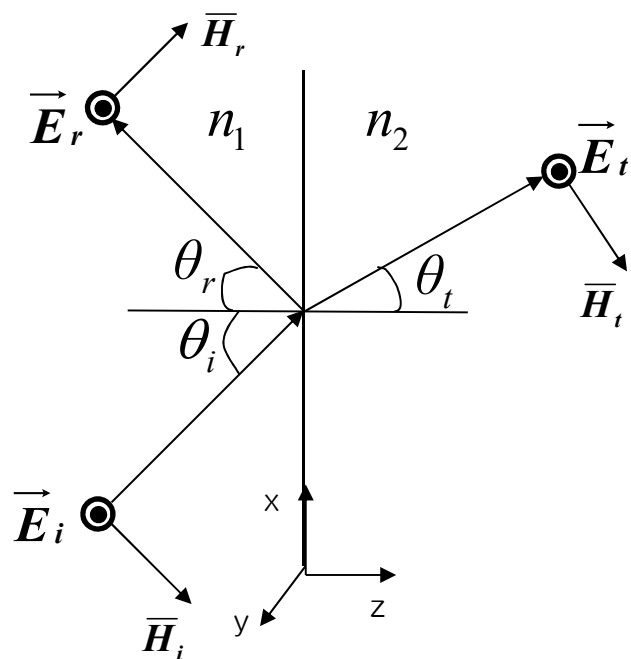
$$\overline{H}_r = (\overline{x} \cos \theta_r + \overline{z} \sin \theta_r) \frac{\Gamma E_i}{\eta_1} \exp(-j\beta_{rx}x) \exp(j\beta_{rz}z)$$

$$\overline{E}_t = \overline{y} \tau E_i \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

$$\overline{H}_t = (-\overline{x} \cos \theta_t + \overline{z} \sin \theta_t) \frac{\tau E_i}{\eta_2} \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

Unknowns: $\Gamma, \tau, \theta_r, \theta_t$

Lecture 4: Reflection and Transmission at Dielectric Interface



B.C.'s at $z=0$

$$1) \bar{E}_{\tan} \text{ continuous } (\bar{E}_i + \bar{E}_r = \bar{E}_t)$$

$$\exp(-j\beta_{1x}x) + \Gamma \exp(-j\beta_{rx}x) = \tau \exp(-j\beta_{2x}x)$$

$$\therefore (a) \beta_{1x} = \beta_{rx} = \beta_{2x} \text{ and } (b) 1 + \Gamma = \tau$$

$$\text{From } \beta_{1x} = \beta_{rx} = \beta_{2x}$$

$$n_1 k_0 \sin \theta_i = n_1 k_0 \sin \theta_r = n_2 k_0 \sin \theta_t$$

$$\theta_i = \theta_r$$

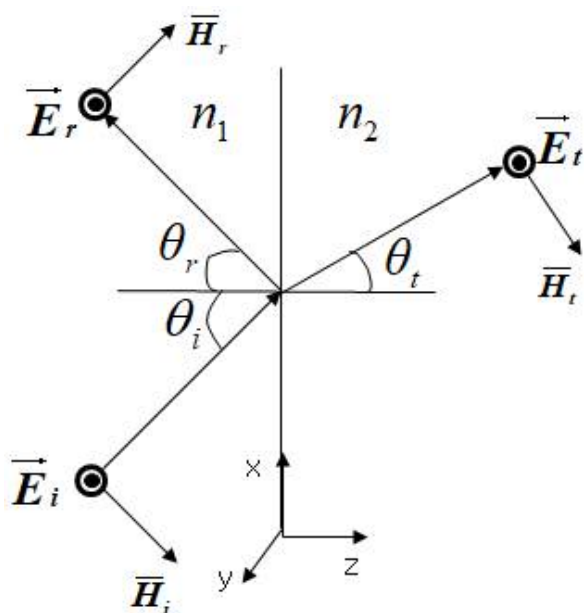
$$n_1 \sin \theta_i = n_2 \sin \theta_t \quad \text{Snell's Law}$$

$$\bar{E}_i = \bar{y} E_i \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

$$\bar{E}_r = \bar{y} \Gamma E_i \exp(-j\beta_{rx}x) \exp(j\beta_{rz}z)$$

$$\bar{E}_t = \bar{y} \tau E_i \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

Lecture 4: Reflection and Transmission at Dielectric Interface



$$\theta_i = \theta_r \quad n_1 \sin \theta_i = n_2 \sin \theta_t \quad \Gamma, \tau ?$$

B.C. on $\bar{H}$ at $z=0$

$\bar{H}_{\text{tan}}$ and $\bar{H}_n$ continuous at $z=0$

From BC on $\bar{H}_{\text{tan}}$
$$-\frac{\cos \theta_i}{\eta_1} + \frac{\cos \theta_i}{\eta_1} \Gamma = -\frac{\cos \theta_t}{\eta_2} \tau$$

With $1 + \Gamma = \tau$,
$$\Gamma_{\perp} = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t}$$

$$\tau_{\perp} = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t}$$

Remember for normal incident

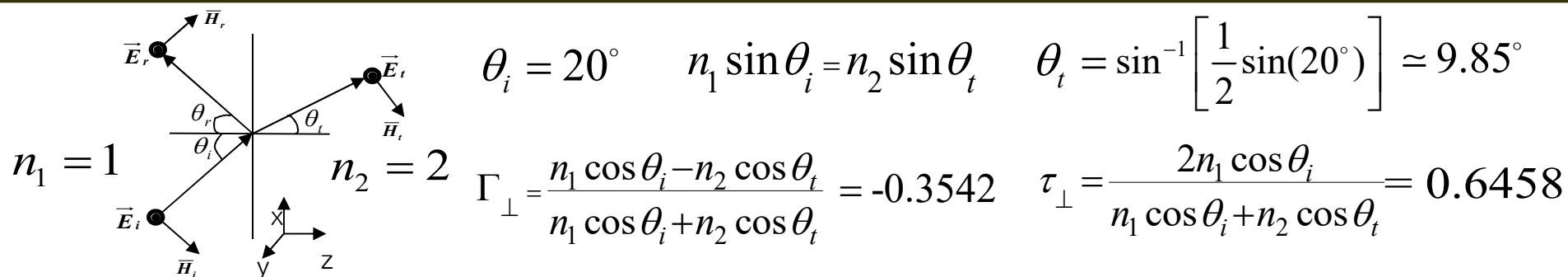
$$\Gamma = \frac{n_1 - n_2}{n_1 + n_2}, \quad \tau = \frac{2n_1}{n_1 + n_2}$$

$$\bar{H}_i = (-\bar{x} \cos \theta_i + \bar{z} \sin \theta_i) \frac{E_i}{\eta_1} \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

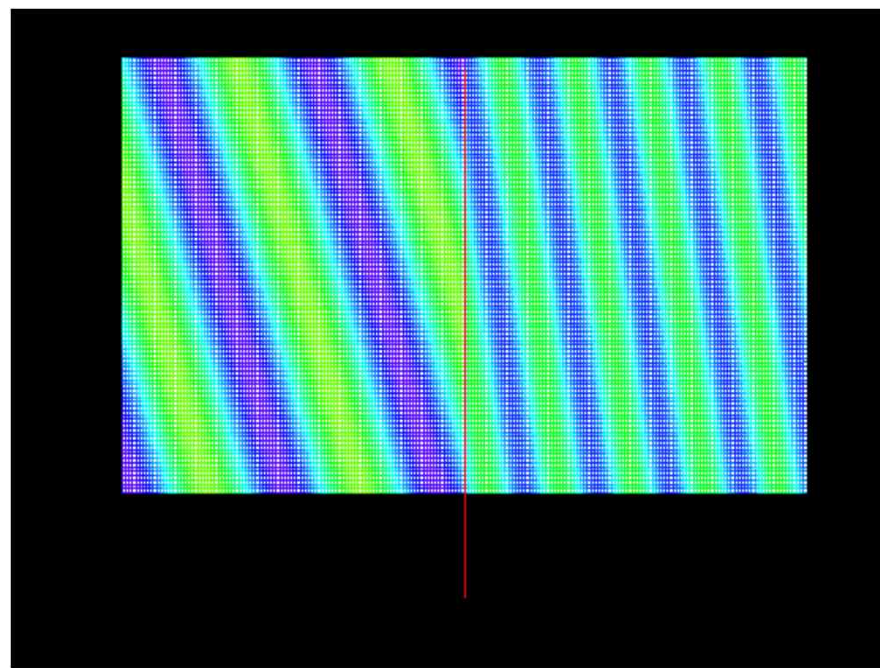
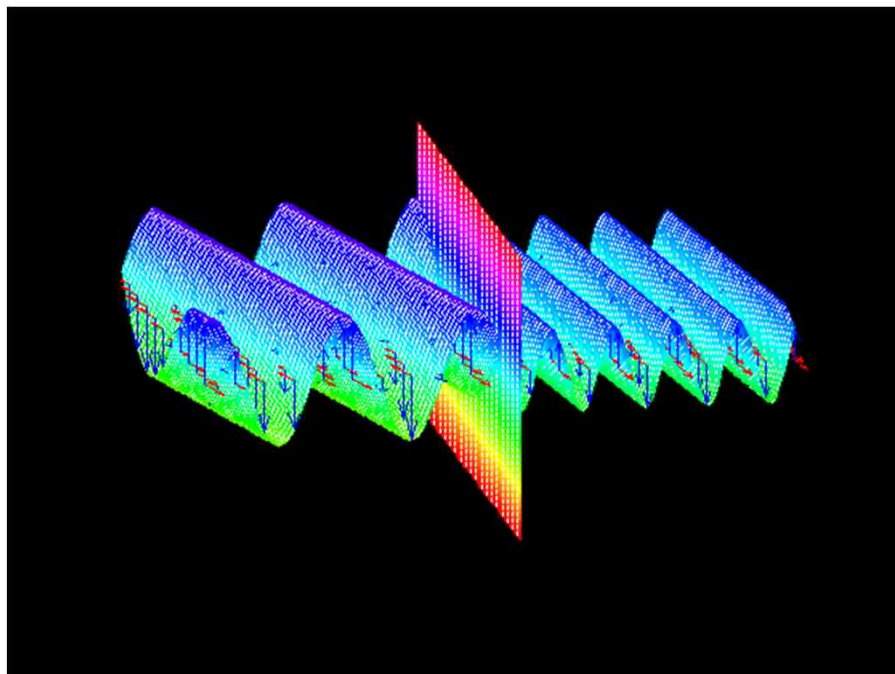
$$\bar{H}_r = (\bar{x} \cos \theta_i + \bar{z} \sin \theta_i) \frac{\Gamma E_i}{\eta_1} \exp(-j\beta_{1x}x) \exp(j\beta_{1z}z)$$

$$\bar{H}_t = (-\bar{x} \cos \theta_t + \bar{z} \sin \theta_t) \frac{\tau E_i}{\eta_2} \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

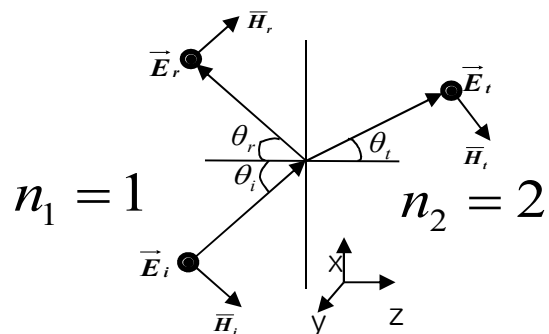
Lecture 4: Reflection and Transmission at Dielectric Interface



Incident and Transmitted Waves (Refraction)



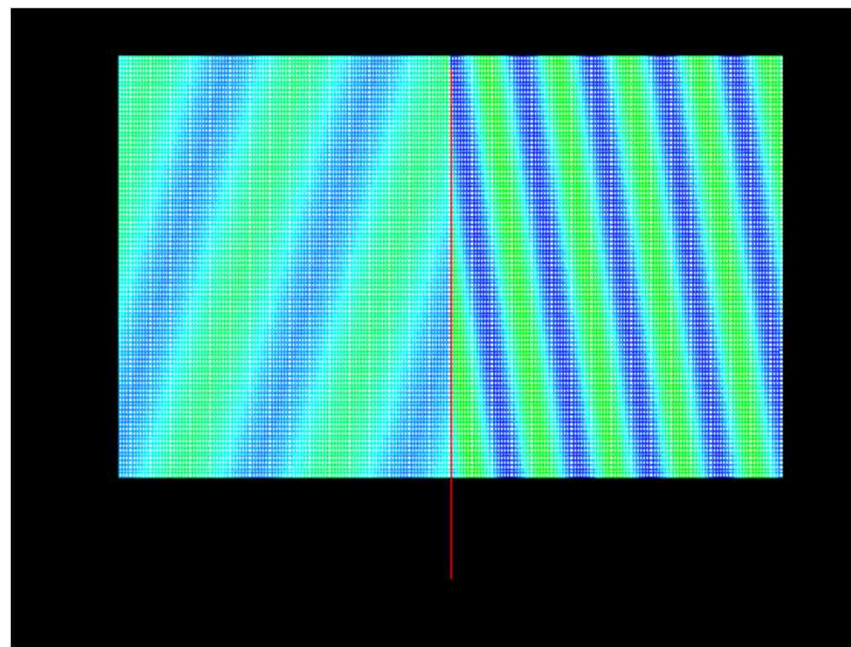
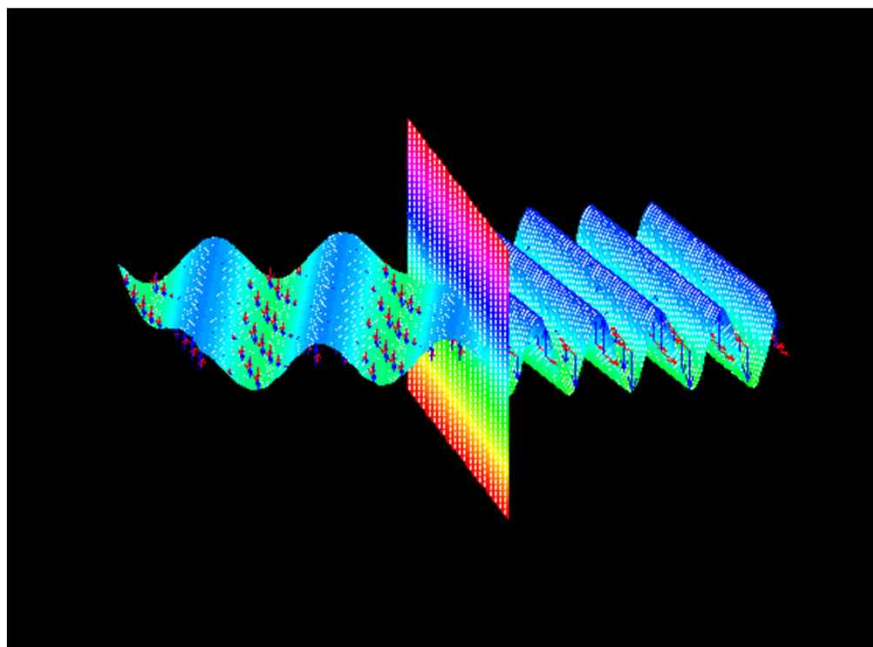
Lecture 4: Reflection and Transmission at Dielectric Interface



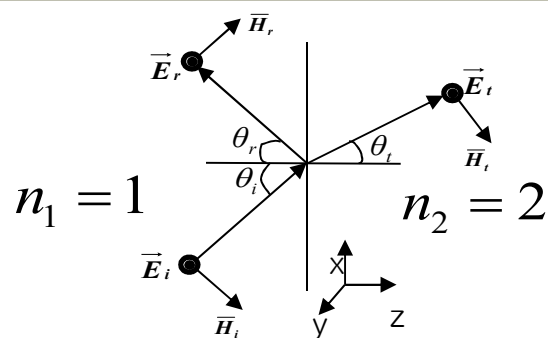
$$\text{For } \theta_i = 20^\circ \quad \theta_t \simeq 9.85^\circ$$

$$\Gamma = -0.3542, \quad \tau = 0.6458$$

Reflected and Transmitted Waves



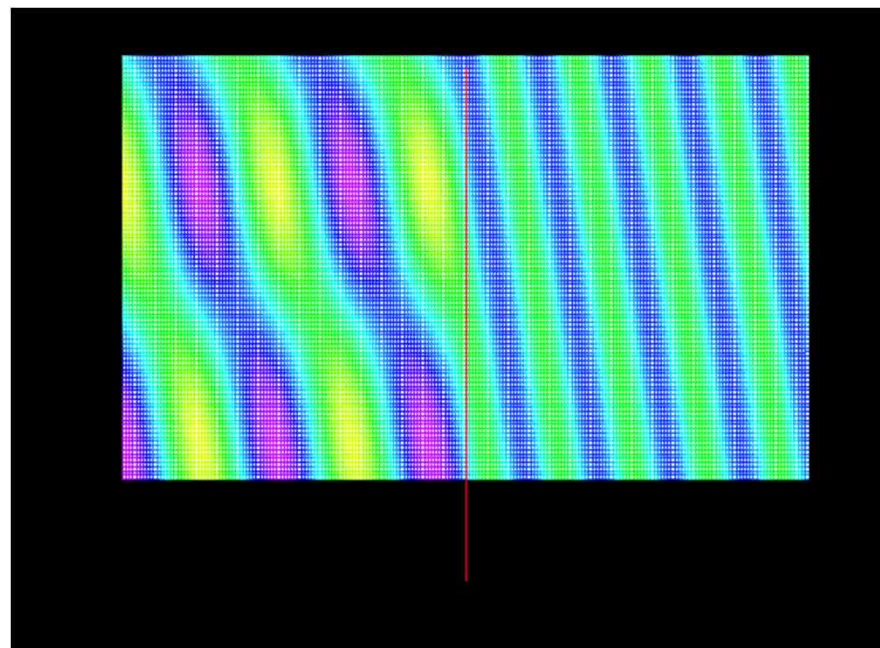
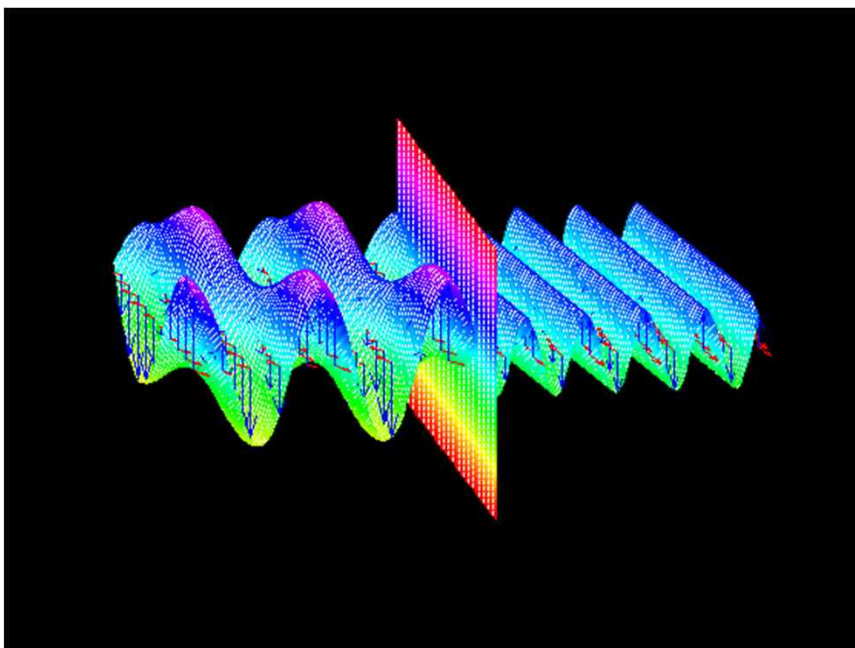
Lecture 4: Reflection and Transmission at Dielectric Interface



For $\theta_i = 20^\circ$ $\theta_t \approx 9.85^\circ$

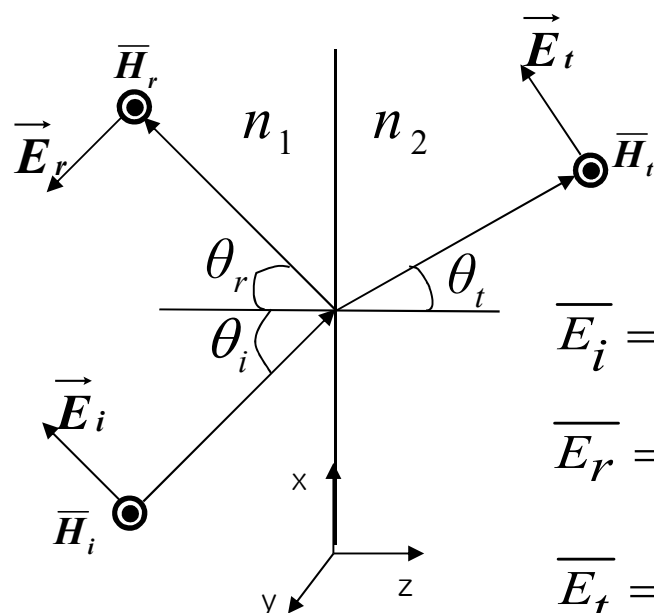
$\Gamma = -0.3542$, $\tau = 0.6458$

Total and Transmitted Waves



Lecture 4: Reflection and Transmission at Dielectric Interface

✓ Parallel Polarization



$$\overline{H}_i = \overline{y} H_i \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

$$\overline{H}_r = \overline{y} H_r \exp(-j\beta_{rx}x) \exp(j\beta_{rz}z)$$

$$\overline{H}_t = \overline{y} H_t \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

$$\overline{E}_i = (\overline{x} \cos \theta_i - \overline{z} \sin \theta_i) H_i \eta_1 \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

$$\overline{E}_r = (-\overline{x} \cos \theta_r - \overline{z} \sin \theta_r) H_r \eta_1 \exp(-j\beta_{rx}x) \exp(j\beta_{rz}z)$$

$$\overline{E}_t = (\overline{x} \cos \theta_t - \overline{z} \sin \theta_t) H_t \eta_2 \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

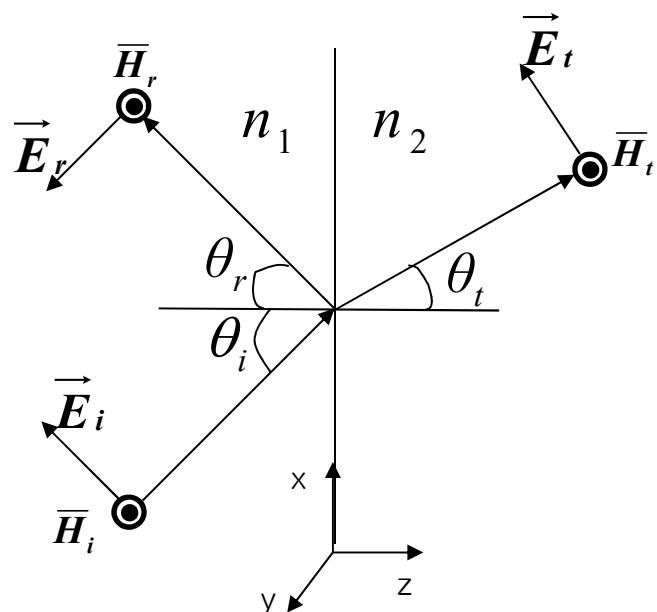
$$\frac{E_r}{E_i} = \Gamma$$

$$\frac{H_r}{H_i} = -\Gamma$$

$$\frac{E_t}{E_i} = \tau \quad \frac{H_t}{H_i} = \frac{E_t / \eta_2}{E_i / \eta_1} = \tau \frac{\eta_1}{\eta_2}$$

Lecture 4: Reflection and Transmission at Dielectric Interface

✓ Parallel Polarization



$$\frac{H_r}{H_i} = -\Gamma \quad \frac{H_t}{H_i} = \tau \frac{\eta_1}{\eta_2}$$

$$\overline{H_i} = \overline{y} H_i \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

$$\overline{H_r} = \overline{y} (-\Gamma) H_i \exp(-j\beta_{rx}x) \exp(j\beta_{rz}z)$$

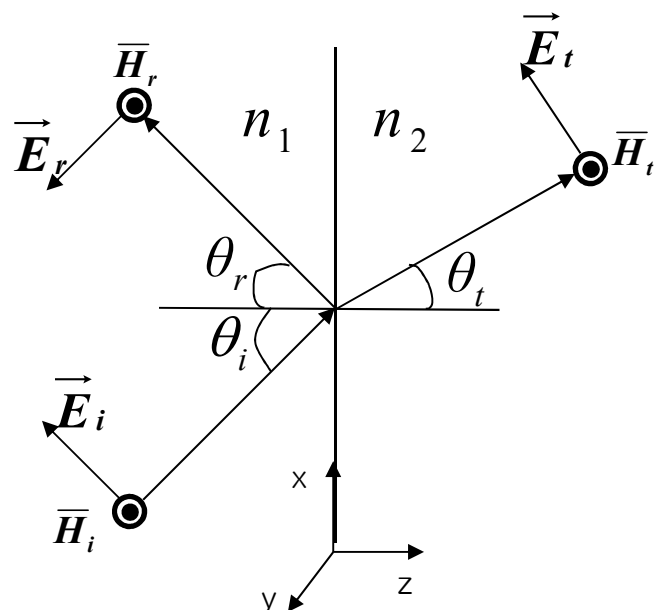
$$\overline{H_t} = \overline{y} \frac{\eta_1}{\eta_2} \tau H_i \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

$$\overline{E_i} = (\overline{x} \cos \theta_i - \overline{z} \sin \theta_i) H_i \eta_1 \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

$$\overline{E_r} = (-\overline{x} \cos \theta_r - \overline{z} \sin \theta_r) H_i \eta_1 (-\Gamma) \exp(-j\beta_{rx}x) \exp(j\beta_{rz}z)$$

$$\overline{E_t} = (\overline{x} \cos \theta_t - \overline{z} \sin \theta_t) H_i \eta_1 \tau \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

Lecture 4: Reflection and Transmission at Dielectric Interface



B.C.'s at $z=0$

1) $\bar{H}_{\tan}$ continuous ($\bar{H}_i + \bar{H}_r = \bar{H}_t$)

$$\exp(-j\beta_{1x}x) - \Gamma \exp(-j\beta_{rx}x) = \frac{\eta_1}{\eta_2} \tau \exp(-j\beta_{2x}x)$$

(a) $\beta_{1x} = \beta_{rx} = \beta_{2x}$

$$\theta_i = \theta_r$$

$n_1 \sin \theta_i = n_2 \sin \theta_t$: Snell's Law

(b) $1 - \Gamma = \frac{\eta_1}{\eta_2} \tau$

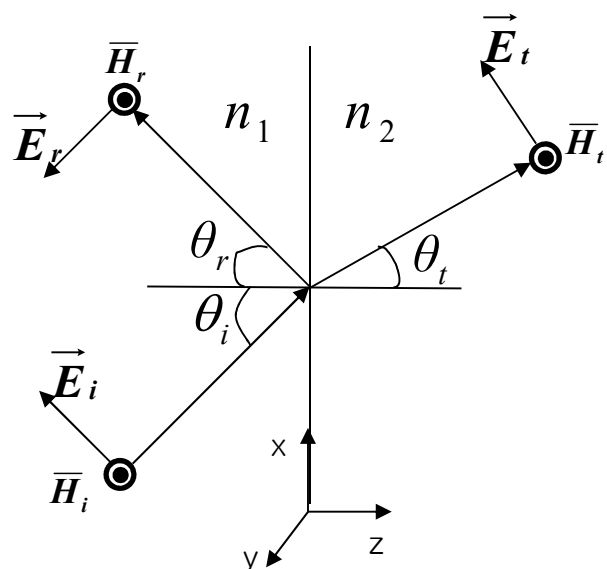
$$1 - \Gamma = \frac{n_2}{n_1} \tau$$

$$\bar{H}_i = \bar{y} H_i \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

$$\bar{H}_r = \bar{y} (-\Gamma) H_i \exp(-j\beta_{rx}x) \exp(j\beta_{rz}z)$$

$$\bar{H}_t = \bar{y} \frac{\eta_1}{\eta_2} \tau H_i \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

Lecture 4: Reflection and Transmission at Dielectric Interface



B.C.'s at $z=0$ 2) $\vec{E}_{\text{tan}}$ continuous

$$\cos \theta_i + \Gamma \cos \theta_i = \tau \cos \theta_t \quad \text{With } 1 - \Gamma = \frac{n_2}{n_1} \tau,$$

$$\Gamma_{\parallel} = \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i}, \quad \tau_{\parallel} = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i}$$

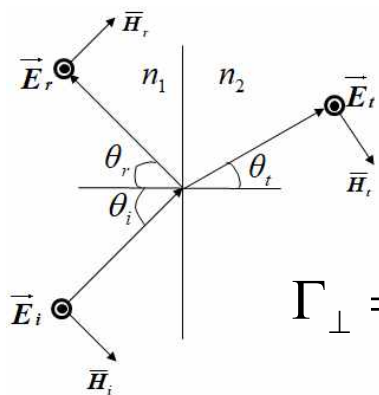
For normal incidence $\Gamma = \frac{n_1 - n_2}{n_1 + n_2}, \quad \tau = \frac{2n_1}{n_1 + n_2}$

$$\vec{E}_i = (\bar{x} \cos \theta_i - \bar{z} \sin \theta_i) H_i \eta_1 \exp(-j\beta_{1x}x) \exp(-j\beta_{1z}z)$$

$$\vec{E}_r = (-\bar{x} \cos \theta_r - \bar{z} \sin \theta_r) H_i \eta_1 (-\Gamma) \exp(-j\beta_{rx}x) \exp(j\beta_{rz}z)$$

$$\vec{E}_t = (\bar{x} \cos \theta_t - \bar{z} \sin \theta_t) H_i \eta_1 \tau \exp(-j\beta_{2x}x) \exp(-j\beta_{2z}z)$$

Lecture 4: Reflection and Transmission at Dielectric Interface

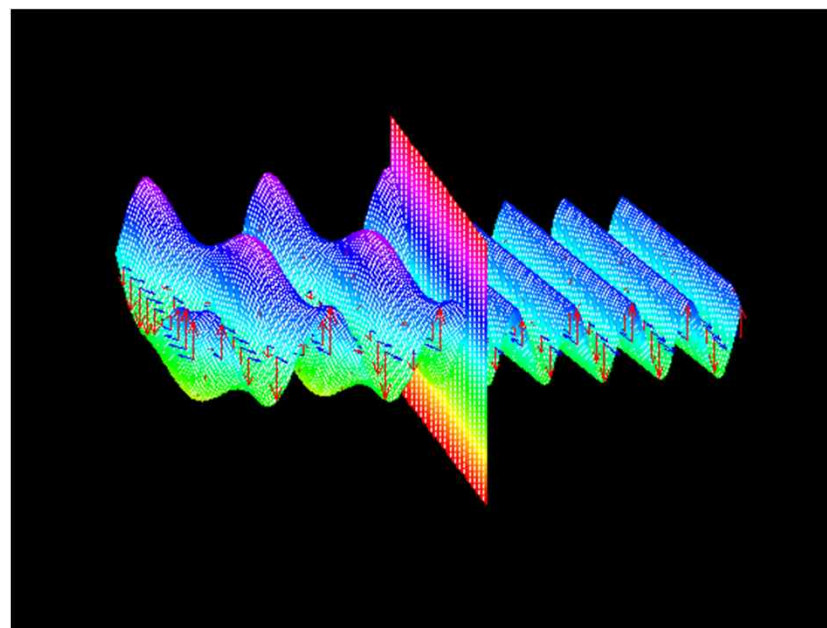
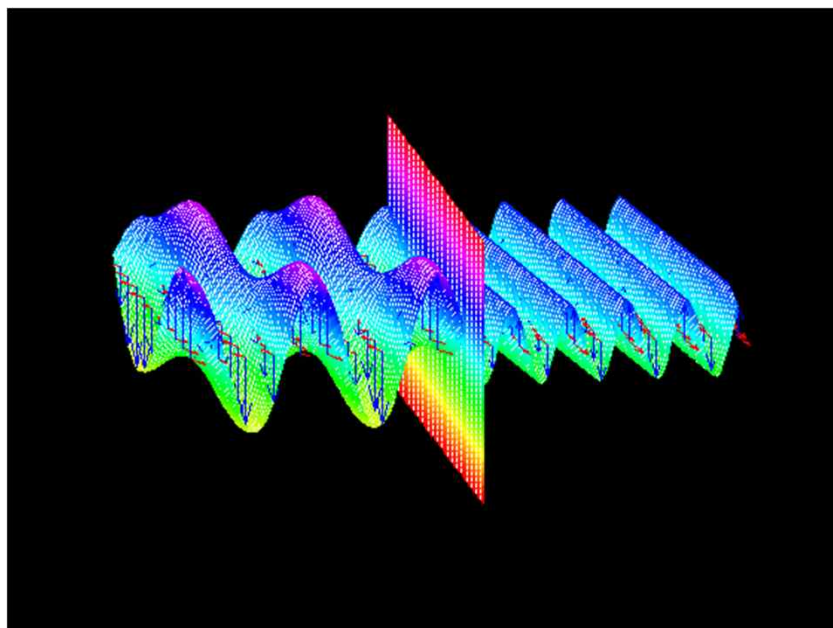
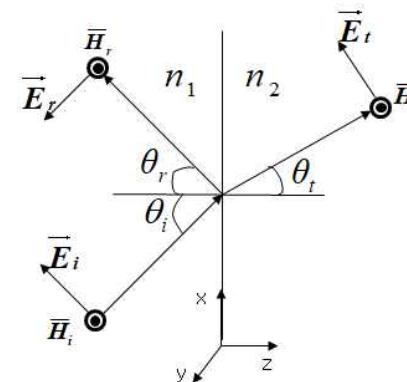


Perpendicular vs Parallel

$$n_1 = 1, n_2 = 2 \quad \theta_i = 20^\circ$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t \quad \theta_t = 9.85^\circ$$

$$\Gamma_{\perp} = -0.354, \tau_{\perp} = 0.646 \quad \Gamma_{\parallel} = -0.312, \tau_{\parallel} = 0.656$$

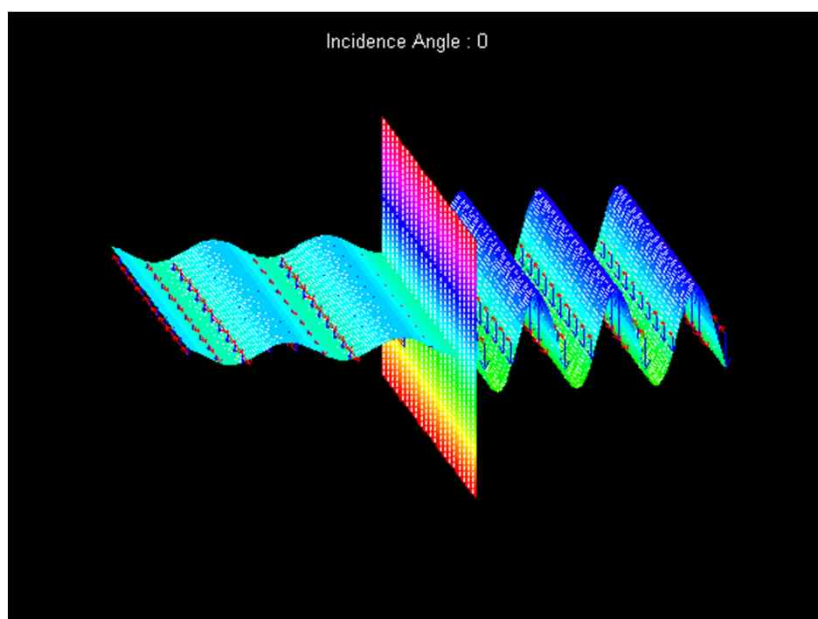


Lecture 4: Reflection and Transmission at Dielectric Interface

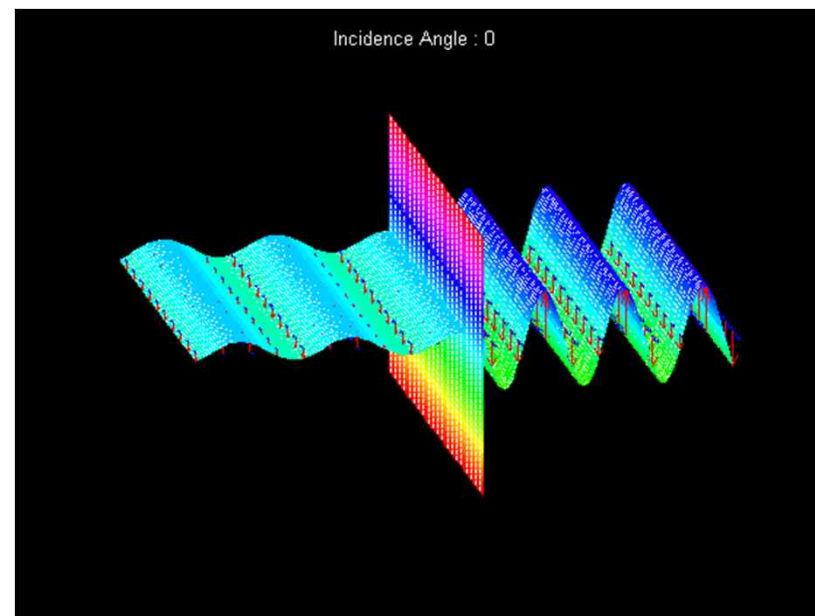
θ_i : from 0° to 90°

(Reflected and Transmitted Waves Only)

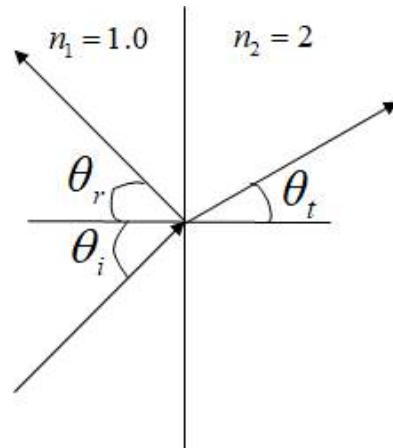
Perpendicular



Parallel

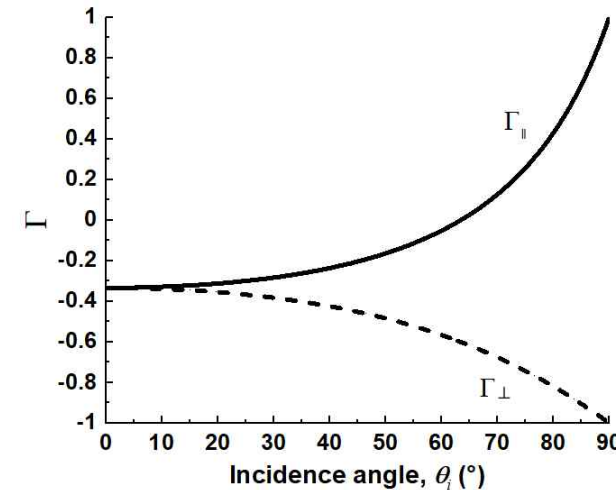


Lecture 4: Reflection and Transmission at Dielectric Interface



$$\Gamma_{\perp} = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t}$$

$$\Gamma_{\parallel} = \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i}$$



For $\Gamma_{\parallel} = \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i} = 0$ $n_2 \cos \theta_i = n_1 \cos \theta_t$ and $n_1 \sin \theta_i = n_2 \sin \theta_t$

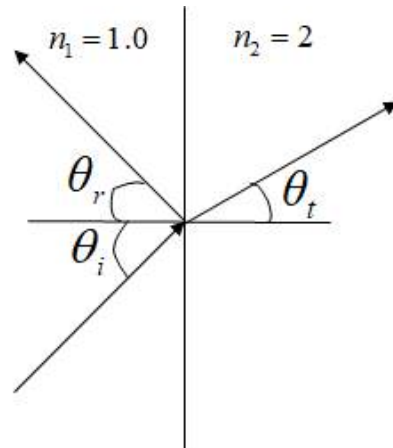
$$\sin^2 \theta_i = \frac{n_2^2}{n_1^2 + n_2^2}, \quad \cos^2 \theta_i = \frac{n_1^2}{n_1^2 + n_2^2} \quad \therefore \theta_i = \tan^{-1} \left(\frac{n_2}{n_1} \right) = \tan^{-1}(2) = 63.4^\circ$$

No reflection for parallel polarization on dielectric interface at certain incident angle

➔ Brewster angle

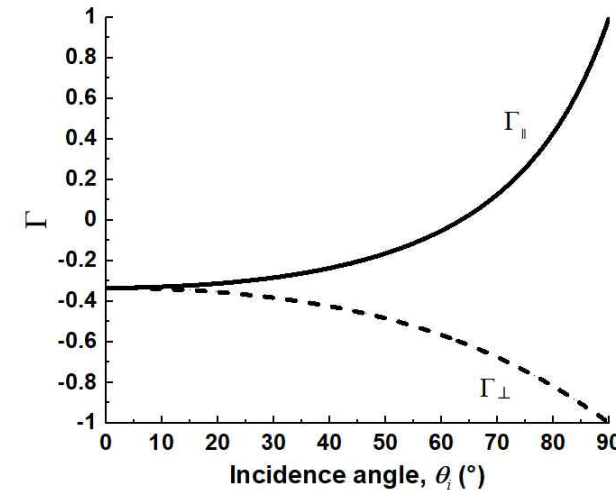
Small reflection around Brewster angle

Lecture 4: Reflection and Transmission at Dielectric Interface



$$\Gamma_{\perp} = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t}$$

$$\Gamma_{\parallel} = \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i}$$



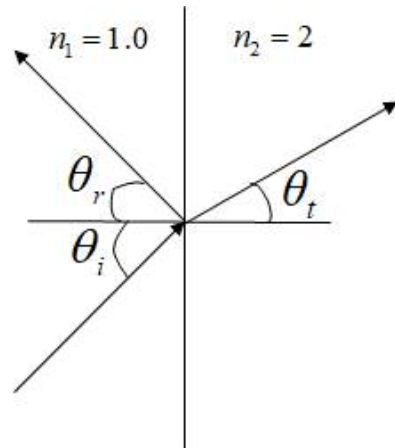
For $\Gamma_{\parallel} = \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i} = 0$ $n_2 \cos \theta_i = n_1 \cos \theta_t$ and $n_1 \sin \theta_i = n_2 \sin \theta_t$

$$\sin^2 \theta_i = \frac{n_2^2}{n_1^2 + n_2^2}, \quad \cos^2 \theta_i = \frac{n_1^2}{n_1^2 + n_2^2} \quad \therefore \theta_i = \tan^{-1} \left(\frac{n_2}{n_1} \right) = \tan^{-1}(2) = 63.4^\circ$$

No reflection for parallel polarization on dielectric interface at certain incident angle

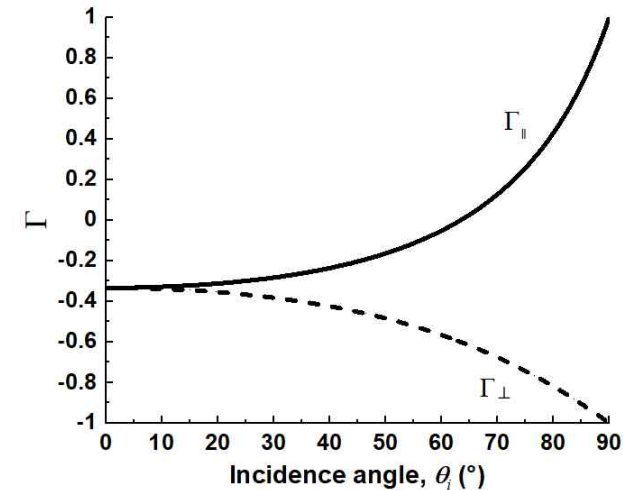
➔ Brewster angle

Lecture 4: Reflection and Transmission at Dielectric Interface



$$\Gamma_{\perp} = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t}$$

$$\Gamma_{\parallel} = \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i}$$



Brewster angle for perpendicular polarization?

$$n_1 \cos \theta_i = n_2 \cos \theta_t$$

But $n_1 \sin \theta_i = n_2 \sin \theta_t$ (Snell's Law)

$$\theta_i = \theta_t$$

(This is true only when $\mu_1 = \mu_2$)

Lecture 4: Reflection and Transmission at Dielectric Interface



- Incident sunlight is unpolarized: random E-field direction
(Both perpendicular and parallel polarization)
- At the Brewster angle, only perpendicular polarization reflected
- Polarizer can block reflected light with perpendicular polarization
- Light from inside the container can have both polarizations
- We can see inside the container

Lecture 4: Reflection and Transmission at Dielectric Interface



Filter located at the Brewster angle can almost block the reflected light from the window

→ We can see through the window



Lecture 4: Reflection and Transmission at Dielectric Interface

Polarized Sunglasses



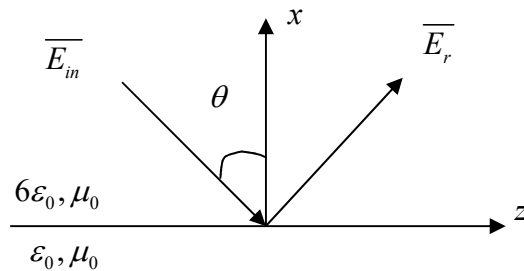
Lecture 4: Reflection and Transmission at Dielectric Interface

Homework (Due 9/20 Tuesday 11am)

An EM wave whose E-field given as

$$\underline{E} = (\underline{x} \sin\theta + \underline{y} + \underline{z} E_z) \exp(jk_x x) \exp(-jk_z z)$$

is obliquely incident on the dielectric interface as shown below.



- (a) Determine E_z .
- (b) If the reflected E-field has only y component, what is the incident angle?
- (c) Determine the expression for the transmitted E-field for the incident angle determined in (b).

Lecture 4: Reflection and Transmission at Dielectric Interface

- No class next week
- Make up classes during the self-study period (12/8 and 12/13)