

Si Photonics

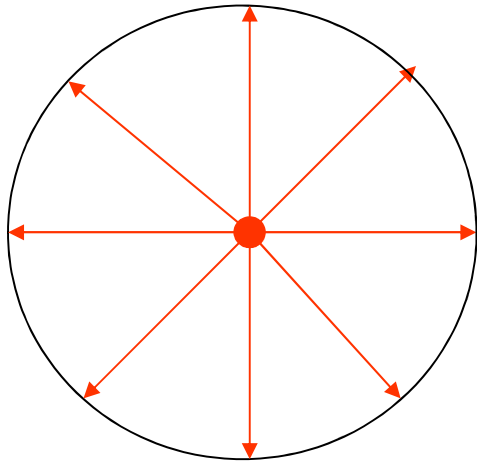
Lecture 5: Interference and Interferometers

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Lecture 5: Interference and Interferometers

Consider isotropic EM wave radiation by a point source.

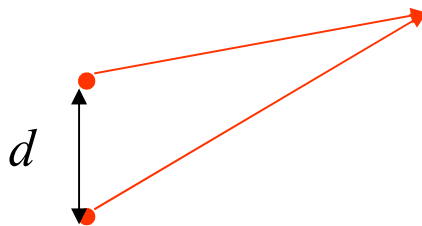


$$E \sim \frac{1}{R} e^{-jkR} \text{ (Spherical wave)}$$

Why $\frac{1}{R}$ dependence?

Because $\int |E|^2 R^2 \sin \theta d\theta d\phi$ should be constant.

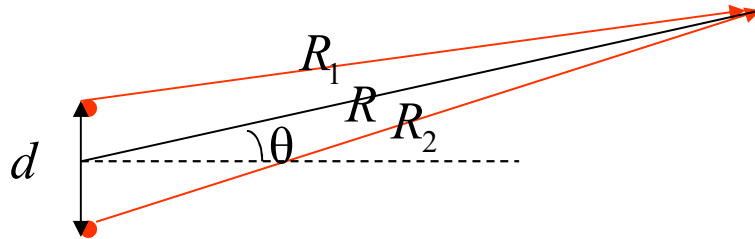
Two point sources separated by d



$$E_{\text{total}} = E_1 + E_2$$

(Assuming same E-field direction)

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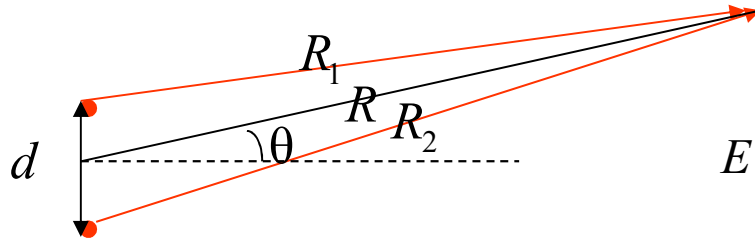


$$E(R, \theta) = E_1 + E_2 = \frac{A}{R_1} e^{-jkR_1} + \frac{A}{R_2} e^{-jkR_2}$$

$$\text{Assume } R \gg d \quad R_1 \approx R - \frac{d}{2} \sin \theta \quad R_2 \approx R + \frac{d}{2} \sin \theta$$

$$\begin{aligned} \therefore E(R, \theta) &\approx \frac{A}{R} e^{-jk(R - \frac{d}{2} \sin \theta)} + \frac{A}{R} e^{-jk(R + \frac{d}{2} \sin \theta)} \\ &= \frac{A}{R} e^{-jkR} \left(e^{jk \frac{d}{2} \sin \theta} + e^{-jk \frac{d}{2} \sin \theta} \right) = \frac{2A}{R} e^{-jkR} \cos\left(k \frac{d}{2} \sin \theta\right) \end{aligned}$$

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$$E(R, \theta) = \frac{2A}{R} e^{-jkR} \cos\left(k \frac{d}{2} \sin \theta\right)$$

$$|E|^2 = 4\left(\frac{A}{R}\right)^2 \cos^2\left(k \frac{d}{2} \sin \theta\right) \quad (\text{Intensity: } I) \Rightarrow \text{Max. and min. intensity conditions}$$

$$\text{For max., } k \frac{d}{2} \sin \theta = m\pi \quad kd \sin \theta = 2m\pi \quad \text{For min., } k \frac{d}{2} \sin \theta = \left(m + \frac{1}{2}\right)\pi \quad kd \sin \theta = (2m + 1)\pi$$

$kd \sin \theta$: phase difference

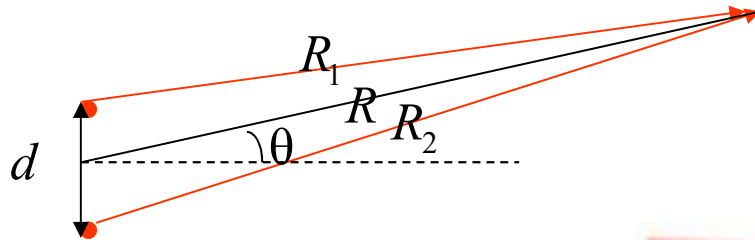
In phase

Out of phase

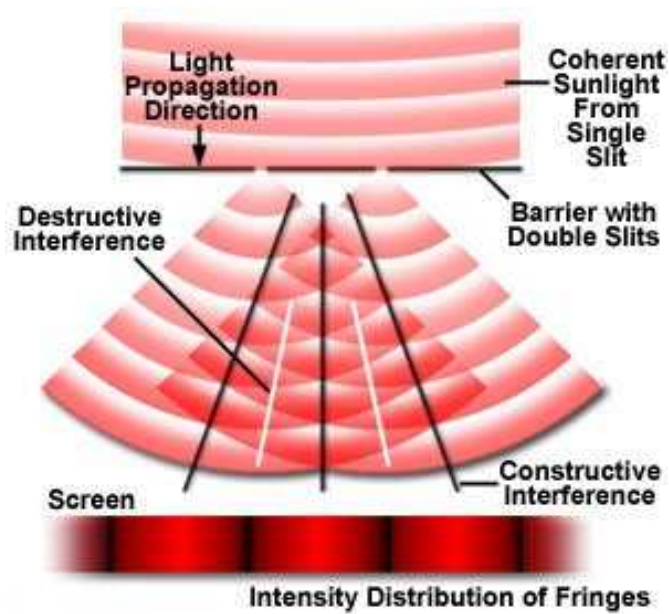
➔ Constructive interference

➔ Destructive interference

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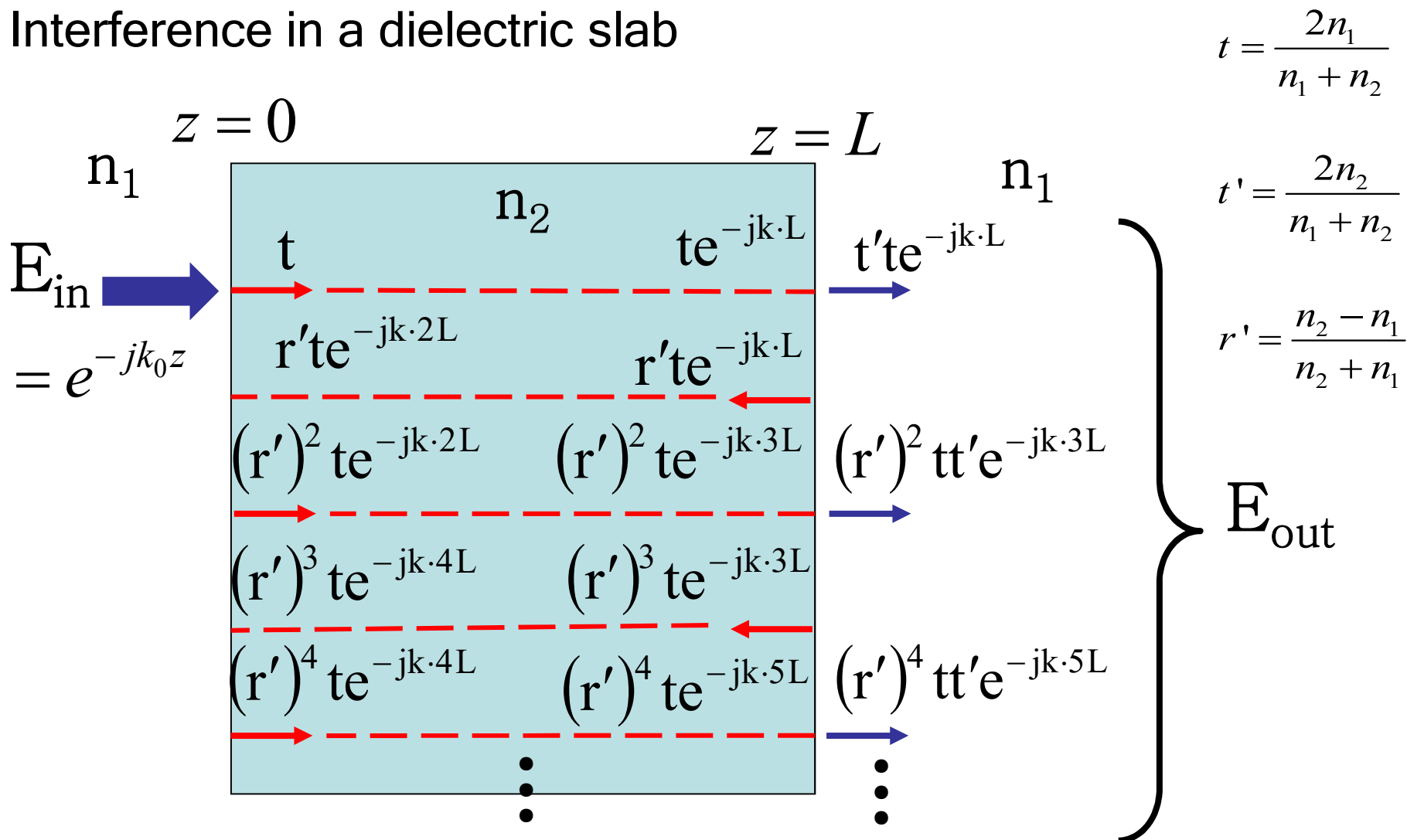
$$I(\text{Intensity}) = |E|^2 = 4\left(\frac{A}{R}\right)^2 \cos^2\left(k \frac{d}{2} \sin \theta\right)$$



Double Slit interference

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Interference in a dielectric slab



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$$E_{out} = tt'e^{-jk \cdot L} + (r')^2 tt'e^{-jk \cdot 3L} + (r')^4 tt'e^{-jk \cdot 5L} + \dots = \frac{tt'e^{-jk \cdot L}}{1 - (r')^2 e^{-j2kL}}$$

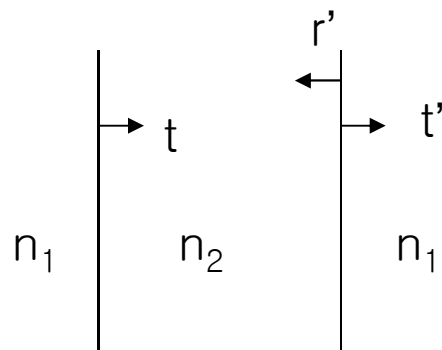
$$T = \frac{|E_t|^2}{|E_i|^2} = \frac{(tt')^2}{[1 - (r')^2 e^{-j2kL}][1 - (r')^2 e^{j2kL}]} = \frac{(tt')^2}{[1 - (r')^2]^2 + 4r'^2 \sin^2(kL)}$$

$$\begin{aligned} [1 - (r')^2 e^{-j2kL}][1 - (r')^2 e^{j2kL}] &= 1 - (r')^2 e^{j2kL} - (r')^2 e^{-j2kL} + (r')^4 \\ &= 1 - 2(r')^2 \cos(2kL) + (r')^4 \\ &= 1 - 2(r')^2(1 - 2\sin^2 kL) + (r')^4 \\ &= [1 - (r')^2]^2 + 4(r')^2 \sin^2(kL) \end{aligned}$$

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$$T = \frac{|E_t|^2}{|E_i|^2} = \frac{(tt')^2}{[1 - (r')^2]^2 + 4r'^2 \sin^2(kL)}$$

$$t = \frac{2n_1}{n_1 + n_2}, \quad r' = \frac{n_2 - n_1}{n_2 + n_1}, \quad t' = \frac{2n_2}{n_1 + n_2}$$



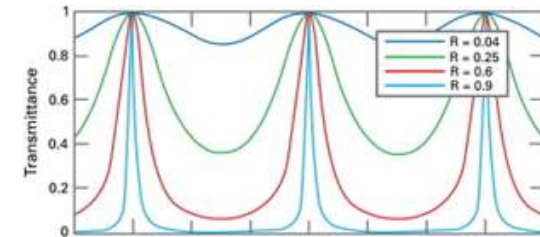
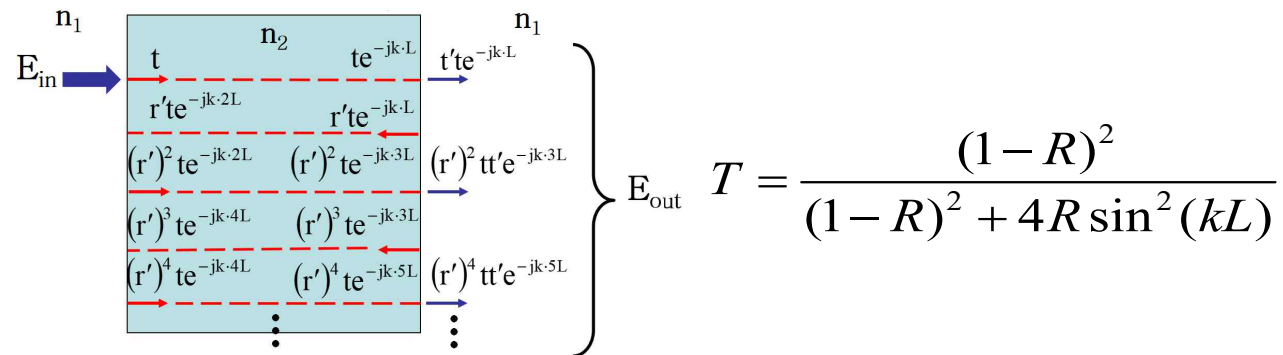
$$tt' = \frac{4n_1n_2}{(n_1 + n_2)^2}, \quad 1 - r'^2 = \frac{4n_1n_2}{(n_2 + n_1)^2}$$

Let $R = r'^2$

$$T = \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2(kL)}$$

Reflected Intensity Ratio: $\frac{4R \sin^2(kL)}{(1 - R)^2 + 4R \sin^2(kL)}$

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Filter

Max. Transmission: $\sin(kL) = 0 \Rightarrow T = 1$ (resonance)

$$kL = m\pi; \quad n_2 \frac{2\pi}{\lambda} L = m\pi \Rightarrow L = m \frac{\lambda}{2n_2} \text{ (half wavelength condition)}$$

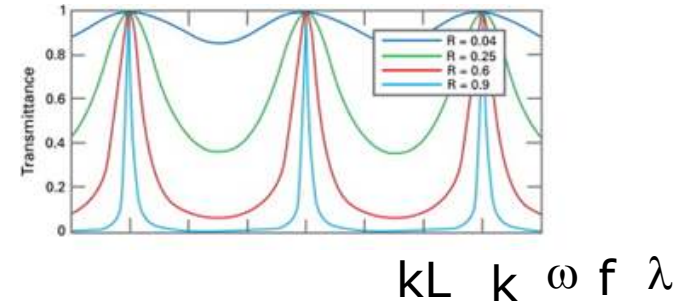
Min. Transmission: $\sin(kL) = 1$

$$kL = (m + \frac{1}{2})\pi; \quad n_2 \frac{2\pi}{\lambda} L = (m + \frac{1}{2})\pi \Rightarrow L = \frac{\lambda}{2n_2} (m + \frac{1}{2}) = \frac{\lambda}{n_2} (\frac{2m + 1}{4})$$

(quarter wavelength condition)

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$$T = \frac{(1-R)^2}{(1-R)^2 + 4R \sin^2(kL)}$$



Period? → Free Spectral Range

$$\Delta kL = \pi \quad 2\pi \text{ phase shift for round trip} \quad \Rightarrow \Delta k = \frac{\pi}{L}$$

$$\Delta \omega = ? \quad \text{Since } k = n_2 \frac{\omega}{c}, \Delta \omega = \frac{c}{n_2} \Delta k = \frac{c}{n_2} \frac{\pi}{L} \quad \Delta f = \frac{c}{2n_2 L} = \frac{1}{T};$$

$$T = \frac{2L}{c/n_2}; \text{ round-trip time}$$

$$\Delta \lambda = ? \quad \lambda = n_2 \frac{2\pi}{k} \quad \Delta \lambda \sim \frac{d\lambda}{dk} \Delta k = -n_2 \frac{2\pi}{k^2} \Delta k = -\frac{\lambda^2}{2n_2 L}$$

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$$T = \frac{(1-R)^2}{(1-R)^2 + 4R \sin^2(kL)}$$

Sharpness?

Determine peak width for $T=0.5$

FWHM (Full Width at Half Maximum):

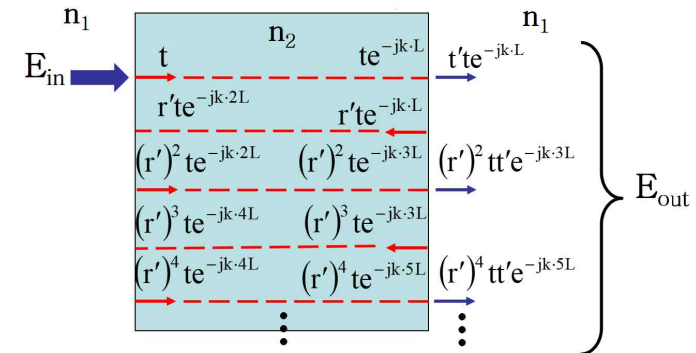
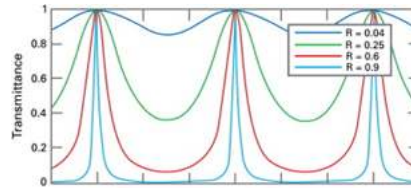
$$\frac{1}{2} = \frac{(1-R)^2}{(1-R)^2 + 4R \sin^2(\Delta kL)} = \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2(\Delta kL)} \quad \frac{4R}{(1-R)^2} \sin^2(\Delta kL) = 1$$

$$\Delta kL = \sin^{-1} \sqrt{\frac{(1-R)^2}{4R}} = \sin^{-1} \frac{(1-R)}{2\sqrt{R}}$$

$$\text{FWHM} \quad 2 \sin^{-1} \frac{(1-R)}{2\sqrt{R}} \sim \frac{(1-R)}{\sqrt{R}} \quad (\text{R close to 1 for many applications})$$

As R increases, FWHM decreases $\Rightarrow$ Larger Q-factor

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated per cycle}} \sim \frac{\omega_r}{\Delta\omega}$$



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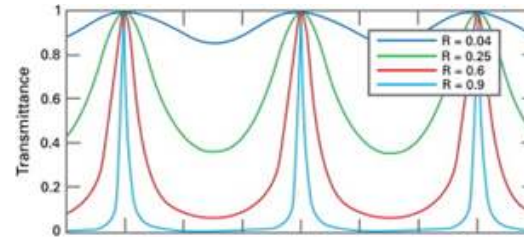
$$T = \frac{(1-R)^2}{(1-R)^2 + 4R \sin^2(kL)}$$

Finesse: FSR / FWHM

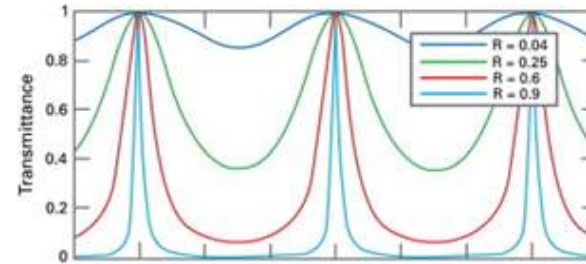
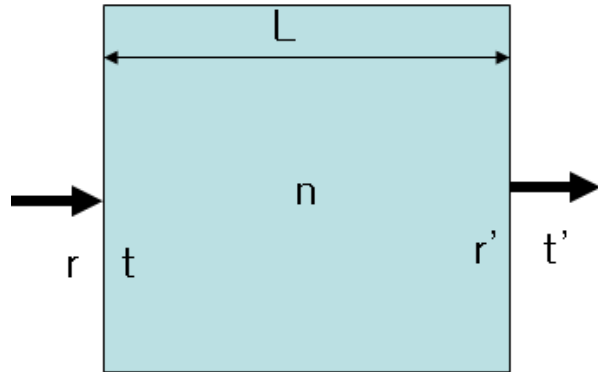
FSR: π

FWHM: $\frac{(1-R)}{\sqrt{R}}$

Finesse: $\frac{\pi\sqrt{R}}{(1-R)}$



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$$T = \frac{I_{out}}{I_{in}} = \frac{(1-R)^2}{(1-R)^2 + 4R \sin^2(kL)} \quad (R = r'^2)$$

Fabry-Perot Interferometer



Charles Fabry
(1867–1945)



Alfred Perot
(1863–1925)



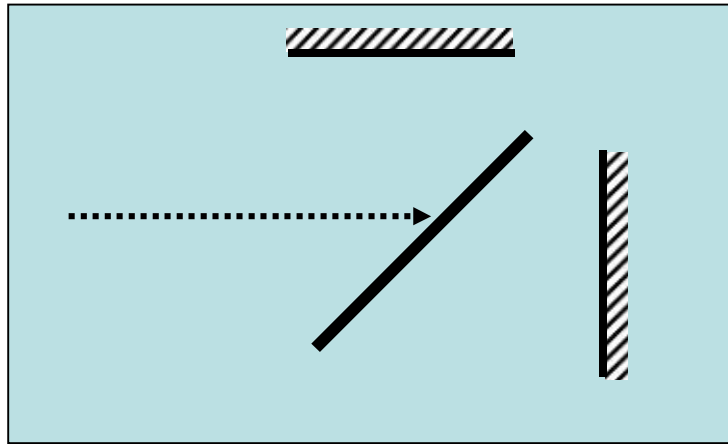
FP Interferometer: two parallel reflectors

Accurate determination of L or λ

➔ Precision instrumentation, Laser cavity

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Michelson Interferometer

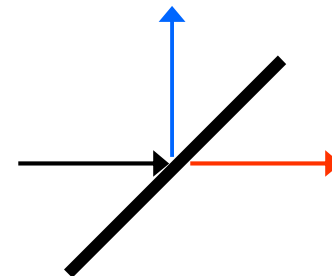


Albert Michelson
(1852~1931)
Nobel Prize in Physics in 1907

Beam Splitter:

Partially reflecting/transmitting mirror

Two mirrors and one beam splitter

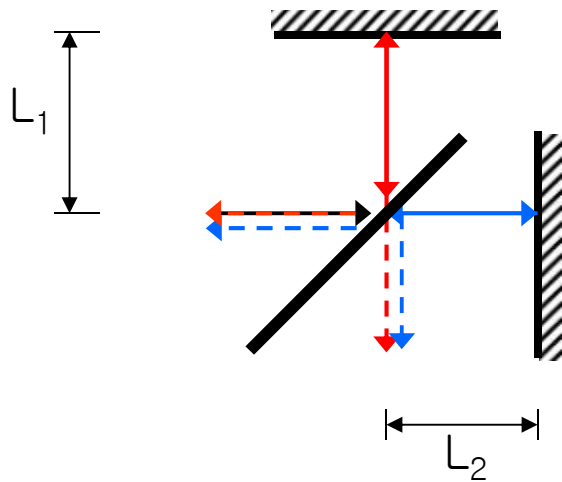


3-dB beam splitter

$$R = \frac{1}{2}, \quad T = \frac{1}{2}$$

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Michelson Interferometer:



Four outputs:

Side 1: r r
 2: t t

Bottom 1: r t
 2: t r

Assuming $E_{in} = 1$

$$E_{out, side} = re^{-jkl_1}(-1)e^{-jkl_1}r + te^{-jkl_2}(-1)e^{-jkl_2}t = -r^2e^{-j2kl_1} - t^2e^{-j2kl_2}$$

$$E_{out, bottom} = re^{-jkl_1}(-1)e^{-jkl_1}t + te^{-jkl_2}(-1)e^{-jkl_2}r = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

Lecture 5: Interference and Interferometers

$$E_{out, side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2} \quad E_{out, bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

$$\text{With } r = t = \frac{1}{\sqrt{2}},$$

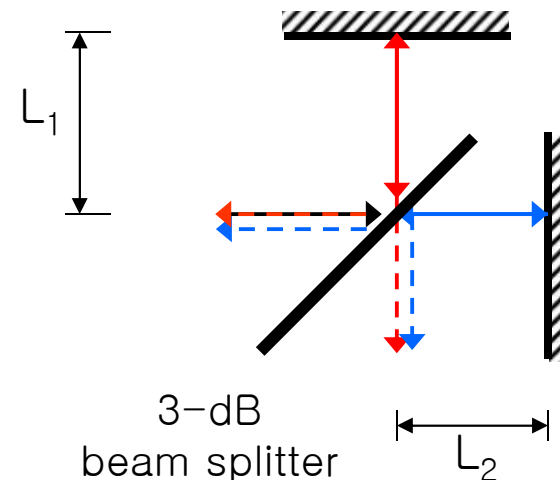
$$E_{out, side} = -\frac{1}{2} \left(e^{-j2kl_1} + e^{-j2kl_2} \right) = -\frac{1}{2} e^{-jk(l_1+l_2)} \left(e^{-jk(l_1-l_2)} + e^{jk(l_1-l_2)} \right)$$

$$I_{out, side} = |E_{out, side}|^2 = \cos^2[k(l_1 - l_2)]$$

$$\begin{aligned} E_{out, bottom} &= -\frac{1}{2} \left(e^{-j2kl_1} + e^{-j2kl_2} \right) \\ &= -\frac{1}{2} e^{-jk(l_1+l_2)} \left(e^{-jk(l_1-l_2)} + e^{jk(l_1-l_2)} \right) \end{aligned}$$

$$I_{out, bottom} = |E_{out, bottom}|^2 = \cos^2[k(l_1 - l_2)]$$

Against energy conservation!



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With $r = \frac{1}{\sqrt{2}}$, $t = j\frac{1}{\sqrt{2}}$

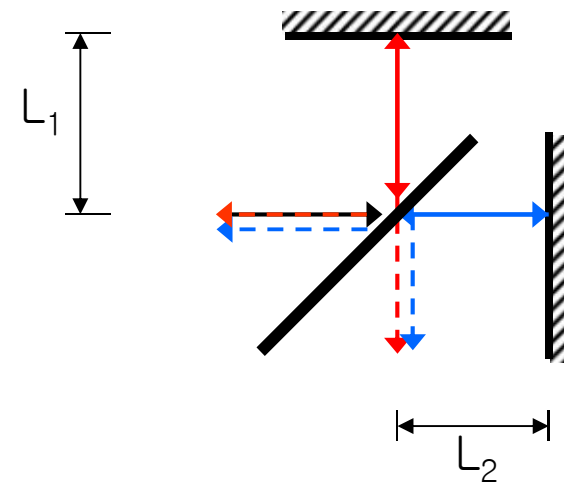
(r and t should have $\pi/2$ phase difference in order to satisfy energy conservation)

$$\begin{aligned} E_{out, side} &= -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2} \\ &= -\frac{1}{2} e^{-j2kl_1} + \frac{1}{2} e^{-j2kl_2} \end{aligned}$$

$$I_{out, side} = |E_{out, side}|^2 = \sin^2[k(l_1 - l_2)]$$

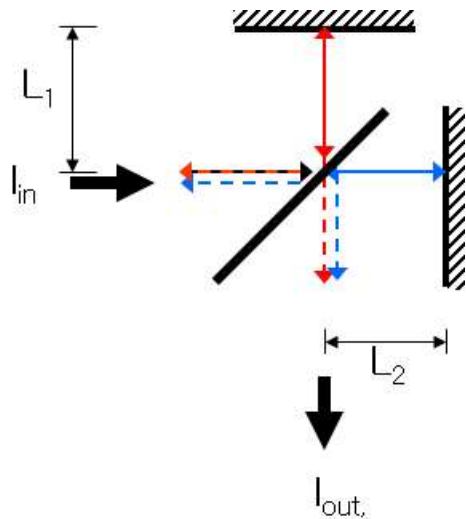
$$\begin{aligned} E_{out, bottom} &= -rte^{-j2kl_1} - rte^{-j2kl_2} \\ &= -\frac{j}{2} (e^{-j2kl_1} + e^{-j2kl_2}) \end{aligned}$$

$$I_{out, bottom} = |E_{out, bottom}|^2 = \cos^2[k(l_1 - l_2)]$$



Lecture 5: Interference and Interferometers

Michelson Interferometer

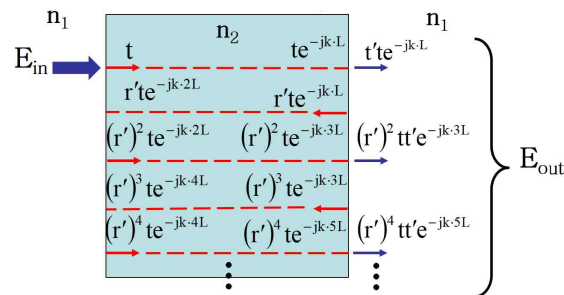


$$\frac{I_{out, bottom}}{I_{in}} = \cos^2[k(l_1 - l_2)]$$

$$\text{FSR: } \Delta k(l_1 - l_2) = \pi$$

$$\text{FWHM: } \Delta k(l_1 - l_2) = \frac{\pi}{2}$$

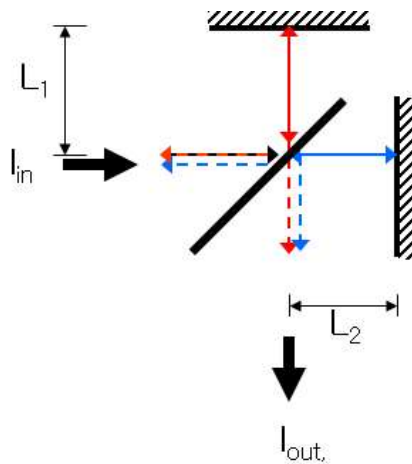
$$\text{Finesse: } 2$$



$$\text{Finesse: } \frac{\pi \sqrt{R}}{(1 - R)}$$

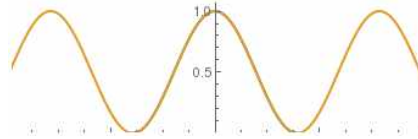
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Michelson Interferometer



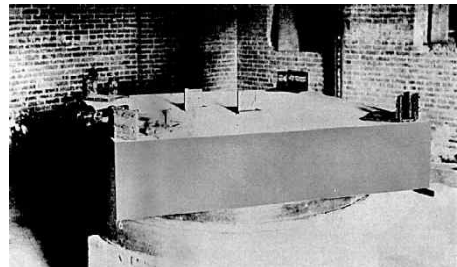
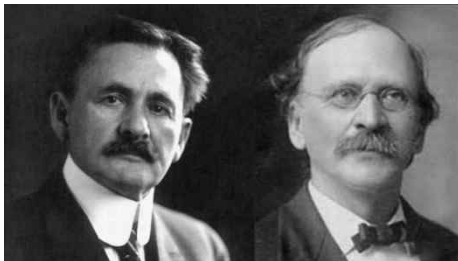
$$\frac{I_{out, bottom}}{I_{in}} = \cos^2[k(l_1 - l_2)]$$

$k(l_1 - l_2)$ can be measured very precisely



Resolution: $k\Delta l = \pi \quad \Delta l = \frac{\lambda}{2}$

Michelson–Morley experiment (1887)



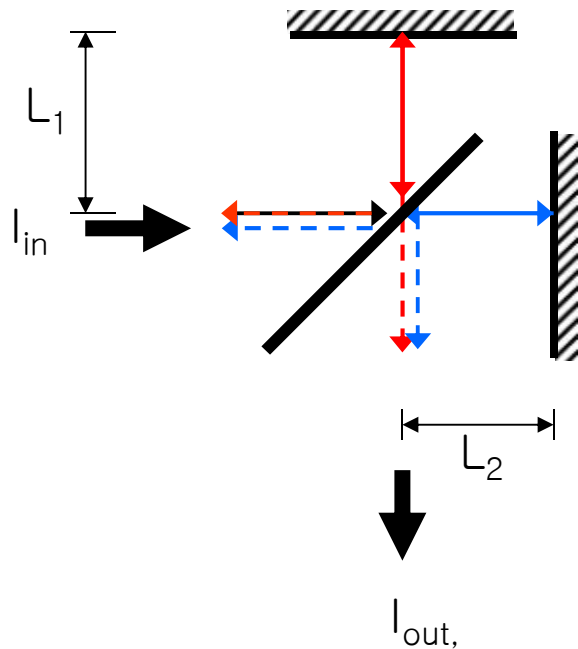
Looking for aether

Determined
speed of light is always the same

→ Einstein's special relativity

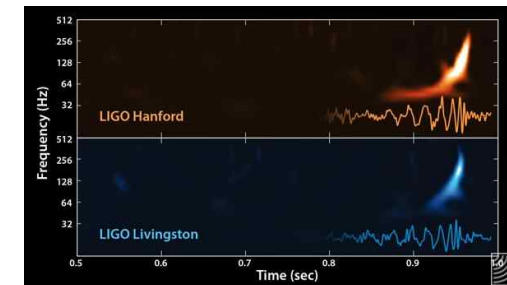
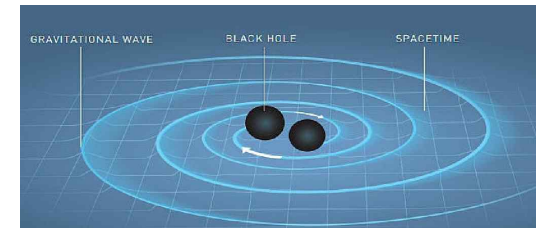
Lecture 5: Interference and Interferometers

Michelson Interferometer



$$\frac{I_{out, bottom}}{I_{in}} = \cos^2[k(l_1 - l_2)]$$

$k(l_1 - l_2)$ can be measured very precisely



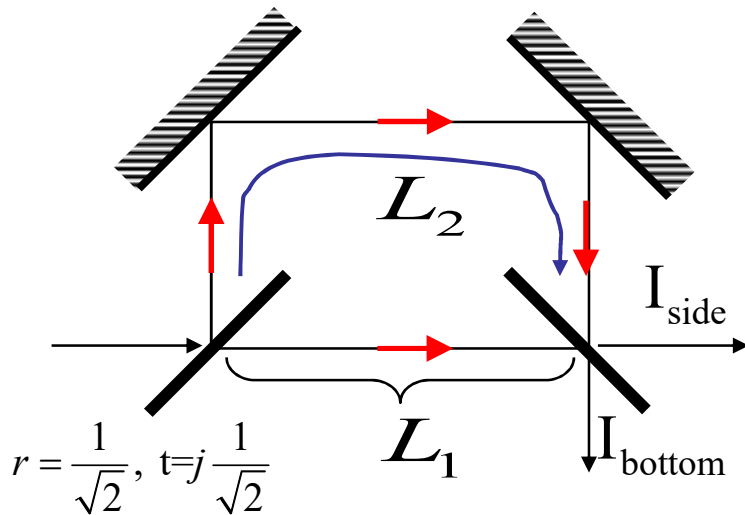
LIGO : Laser Interferometer Gravitational-wave Observatory

Einstein's General Relativity: Gravity changes space

2017 Nobel Prize in Physics

Lecture 5: Interference and Interferometers

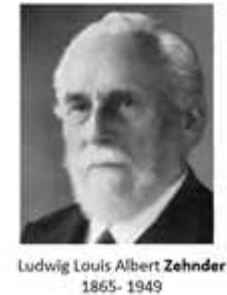
Mach-Zehnder Interferometer:



Ludwig Mach:
Austrian, Son of Ernst Mach



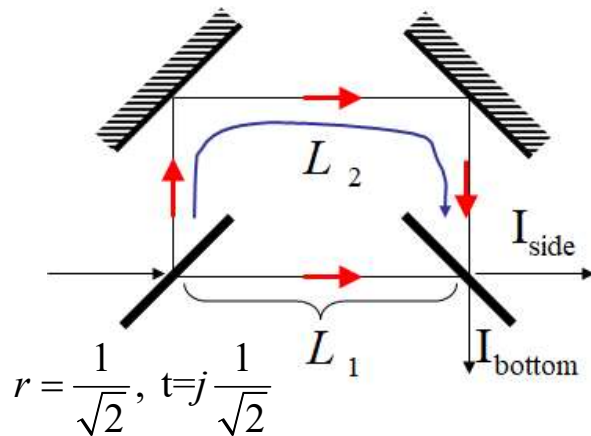
Ludwig Zehnder:
Swiss, Student of Röntgen



Lecture 5: Interference and Interferometers

Mach-Zehnder Interferometer

Assuming $E_{in} = 1$,

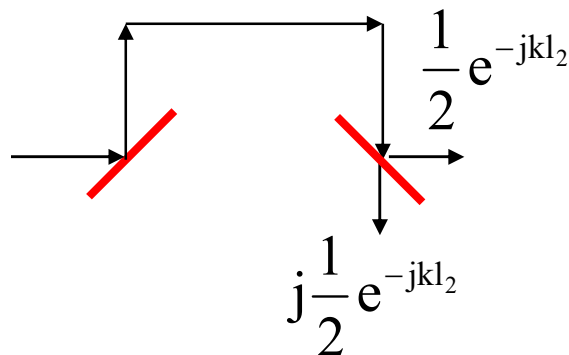


$$E_{out, side} = \frac{1}{2} \left(e^{-jkl_2} - e^{-jkl_1} \right) = \frac{1}{2} e^{-jk\frac{l_2+l_1}{2}} \left(e^{-jk\frac{l_2-l_1}{2}} - e^{jk\frac{l_2-l_1}{2}} \right)$$

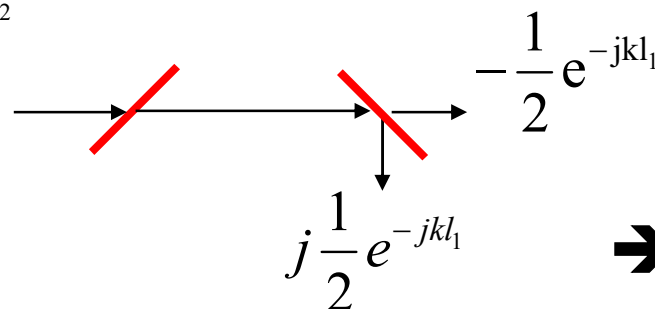
$$I_{out, side} = \sin^2 \left(k \frac{l_1 - l_2}{2} \right)$$

$$E_{out, bottom} = \frac{j}{2} \left(e^{-jkl_1} + e^{-jkl_2} \right) = \frac{j}{2} e^{-jk\frac{l_1+l_2}{2}} \left(e^{-jk\frac{l_1-l_2}{2}} + e^{jk\frac{l_1-l_2}{2}} \right)$$

Case#1



Case#2



➔ MZ Modulator

Lecture 5: Interference and Interferometers

Can any EM wave have interference?

- Only EM waves having clear phase relationship experience interference: coherent

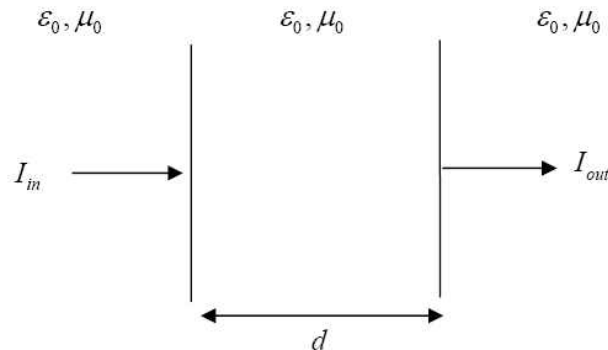
How large $(I_1 - I_2)$ can be?

- As long as two separated waves are coherent or within coherent length
Separated waves become incoherent due to intrinsic phase noises

Lecture 5: Interference and Interferometers

Homework (1)

Consider a Fabry–Perot interferometer made of two identical partially reflecting/transmitting mirrors as shown below. The mirrors transmit and reflect half of the incident power. (Or it has $r = 1/\sqrt{2}$ and $t = j/\sqrt{2}$).



- (a) What is I_{out}/I_{in} ? Give your answer as a function of $\sin(kd)$, where k is the wavenumber in the vacuum and d is the distance between two mirrors.
- (b) If the output power is plotted as a function of the frequency of the input light, what is the frequency separation between two adjacent peaks? Express your answer in terms of c , speed of light, d , and other fundamental parameters if required.
- (c) What is the finesse of this interferometer? Give a numerical answer.
[Finesse = (Free Spectral Range)/FWHM]

Lecture 5: Interference and Interferometers

Homework (2)

Determine I_{out}/I_{in} for an interferometer shown below. It is made of two beam splitters, two mirrors and a block of material having refractive index n and length l_1 placed in one arm. Assume there is no reflection from this block of material, both beam splitters have $r = 1/\sqrt{2}$, $t = j/\sqrt{2}$, and both mirrors have $r = -1$.

