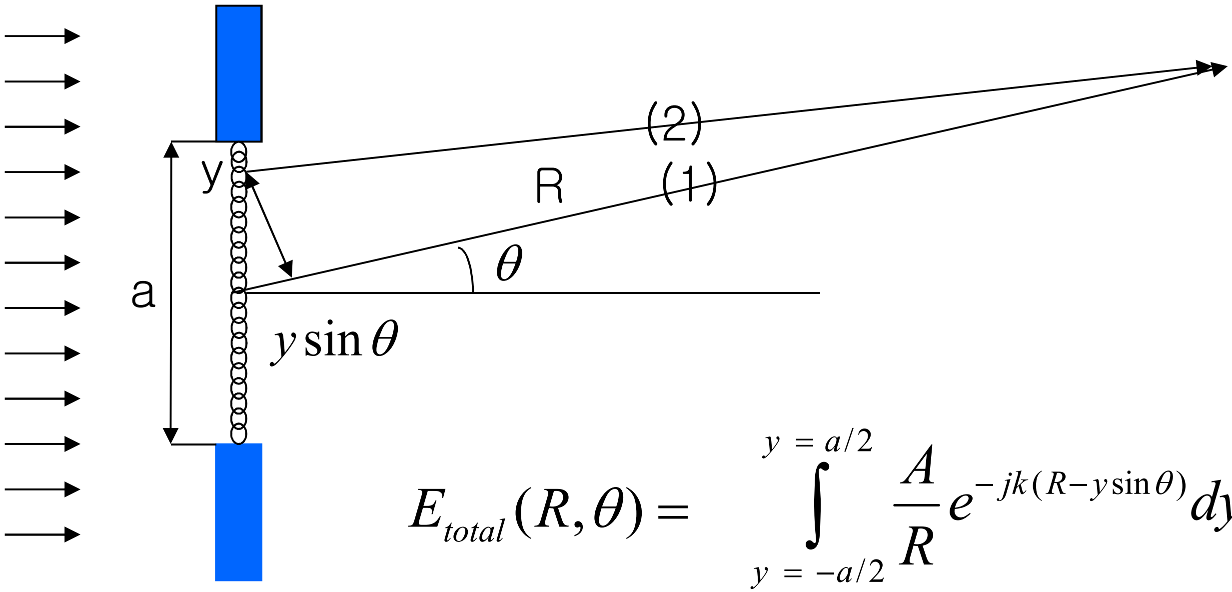
Si Photonics

Lecture 6: Diffraction and Diffraction Gratings

Woo-Young Choi

Dept. of Electrical and Electronic Engineering
Yonsei University

Lecture 6: Diffraction and Diffraction Gratings



(1) $\dfrac{A}{R}\exp(-jkR)$

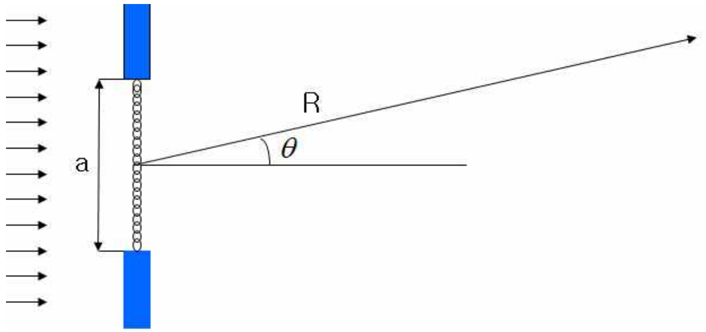
(2) $\dfrac{A}{R}\exp\left[-jk(R-y\sin\theta)\right]$

$$E_{total}(R,\theta)=\int_{y=-a/2}^{y=a/2}\frac{A}{R}e^{-jk(R-y\sin\theta)}dy=\frac{A}{R}e^{-jkR}\int_{y=-a/2}^{y=a/2}e^{jky\sin\theta}dy$$

Since interference is due to *phase difference* , ignore the constant phase term

$$E_{total}(R,\theta)=\frac{A}{R}\int_{y=-a/2}^{y=a/2}e^{jky\sin\theta}dy$$

Lecture 6: Diffraction and Diffraction Gratings



Evaluate $E_{total}(R,\theta) = \frac{A}{R}\int_{-a/2}^{a/2} e^{jky\sin\theta}dy$

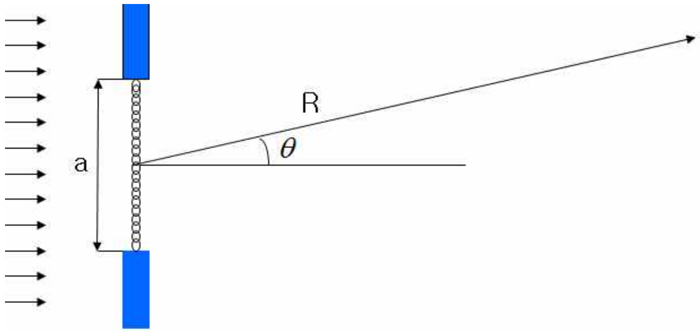
Let $y' = jky\sin\theta$ => $dy' = jk\sin\theta dy$

$$e^{jky\sin\theta}dy = e^{y'}\frac{dy'}{jk\sin\theta}$$

$$E_{total}(R,\theta) = \frac{A}{R}\int_{y'=-jk\frac{a}{2}\sin\theta}^{y'=jk\frac{a}{2}\sin\theta} e^{y'}\frac{dy'}{jk\sin\theta} = \frac{A}{R}\frac{1}{jk\sin\theta}\left(e^{jk\frac{a}{2}\sin\theta} - e^{-jk\frac{a}{2}\sin\theta}\right)$$

$$= \frac{A}{R}\frac{2j}{jk\sin\theta}\sin(k\frac{a}{2}\sin\theta) = \frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}$$

Lecture 6: Diffraction and Diffraction Gratings



$$E_{total}(R,\theta) = \frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}$$

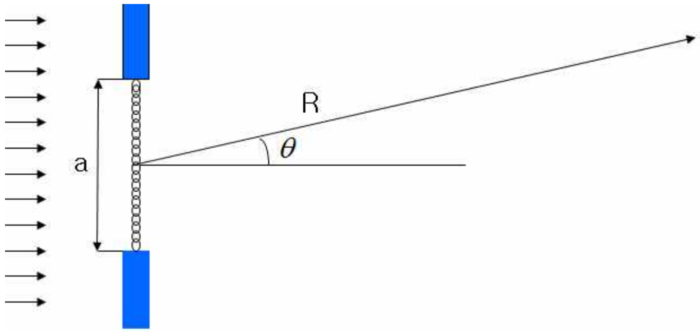
$$\frac{E_{total}(R,\theta)}{E_{total}(R,0)} = ?$$

$$E_{total}(R,0) = \frac{2A}{R}\frac{\cos(k\frac{a}{2}\sin\theta)k\frac{a}{2}\cos\theta\Big|_{\theta=0}}{k\cos\theta\Big|_{\theta=0}} = \frac{2A}{R}\frac{a}{2}$$

$$\therefore \frac{E_{total}(\theta)}{E_{total}(0)} = \frac{\frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}}{\frac{2A}{R}\frac{a}{2}} = \frac{\sin(k\frac{a}{2}\sin\theta)}{k\frac{a}{2}\sin\theta} = \frac{\sin(k_y\frac{a}{2})}{k_y\frac{a}{2}} \qquad k_y = k\sin\theta$$

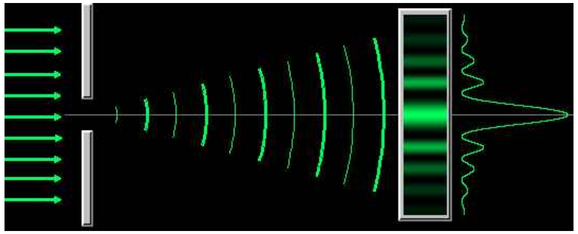
➔ sinc function

Lecture 6: Diffraction and Diffraction Gratings



$$E_{total}(R,\theta) = \frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}$$

$$\frac{E_{total}(R,\theta)}{E_{total}(R,0)} = ?$$

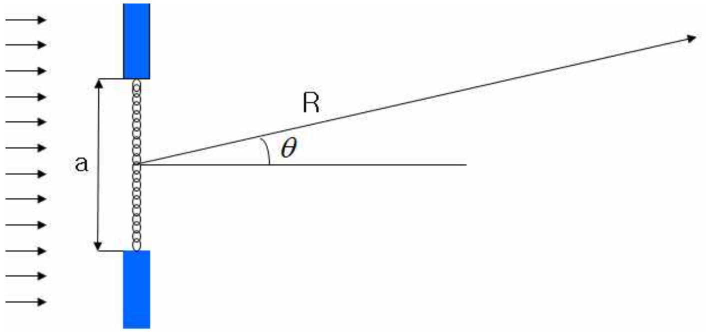


$$E_{total}(R,0) = \frac{2A}{R}\frac{\cos(k\frac{a}{2}\sin\theta)k\frac{a}{2}\cos\theta\Big|_{\theta=0}}{k\cos\theta\Big|_{\theta=0}} = \frac{2A}{R}\frac{a}{2}$$

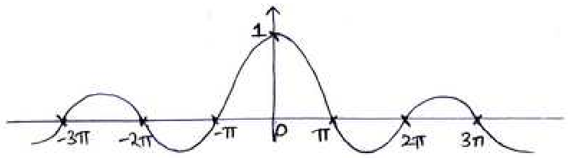
$$\therefore \frac{E_{total}(\theta)}{E_{total}(0)} = \frac{\frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}}{\frac{2A}{R}\frac{a}{2}} = \frac{\sin(k\frac{a}{2}\sin\theta)}{k\frac{a}{2}\sin\theta} = \frac{\sin(k_y\frac{a}{2})}{k_y\frac{a}{2}} \qquad k_y = k\sin\theta$$

➔ sinc function

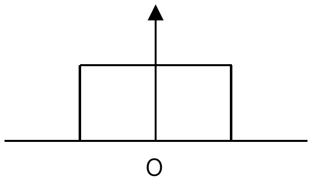
Lecture 6: Diffraction and Diffraction Gratings



$$\frac{E_{total}(\theta)}{E_{total}(0)} = \frac{\sin(k_y \frac{a}{2})}{k_y \frac{a}{2}}$$

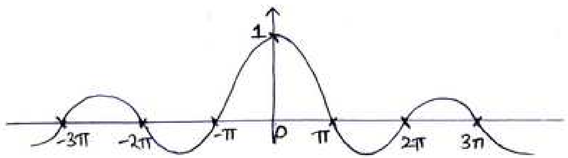


Source

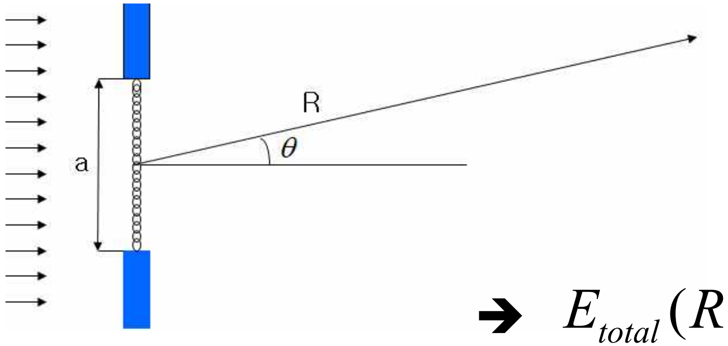


FT relationship

Diffraction



Lecture 6: Diffraction and Diffraction Gratings



$$E_{total}(R,\theta) = \frac{A}{R}\int_{y=-a/2}^{y=a/2} e^{jky\sin\theta}\,dy$$

$$\Rightarrow \quad E_{total}(R,k_y) = \frac{1}{R}\int_{y=-\infty}^{y=\infty} A(y)e^{jk_y y}\,dy \qquad \left(k_y = k\sin\theta\right)$$

From Signals and Systems

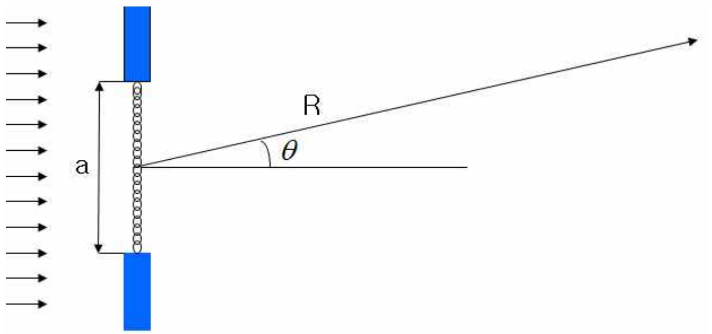
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega$$

$$\mathrm{f}(t) \quad <=> \quad F(\omega)$$

$$E_{total}(k_y) <=> A(y)$$

Diffraction of A(y) is F.T. of *A(y)*

Lecture 6: Diffraction and Diffraction Gratings



$$E_{total}(R,k_y) = \frac{1}{R}\int_{y=-\infty}^{y=\infty} A(y)e^{jk_y y}dy$$

Far-field diffraction

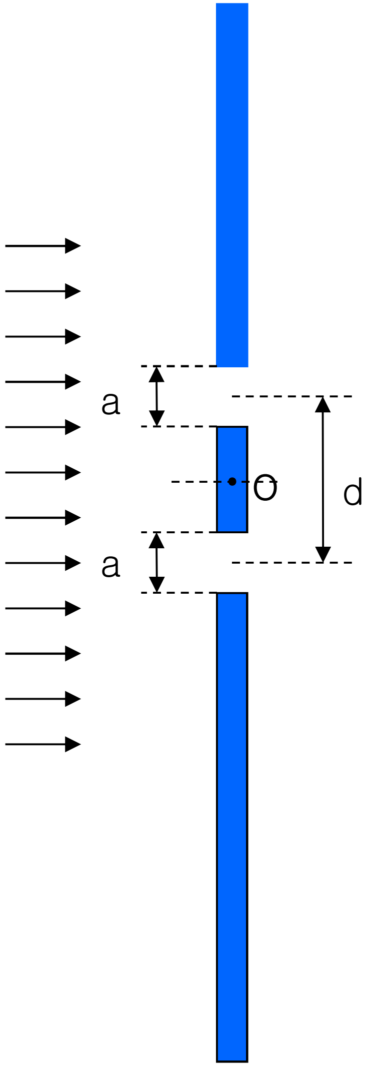
Fraunhoffer Diffraction

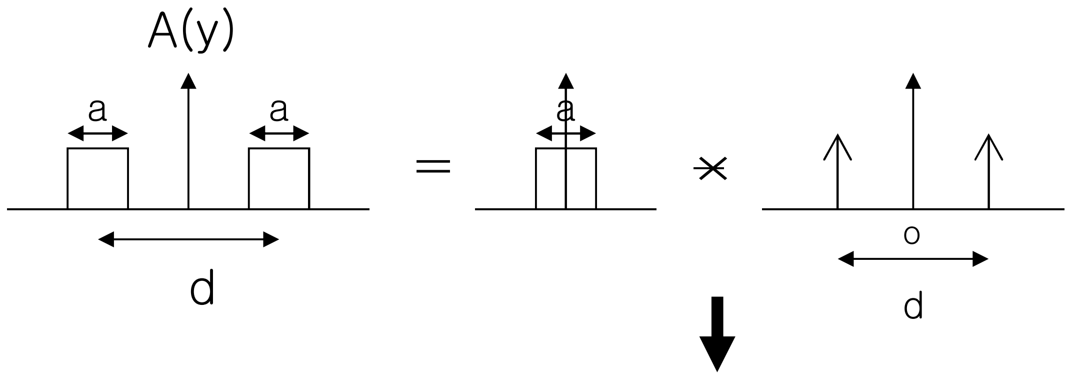
Jeseph Ritter von Fraunhoffer

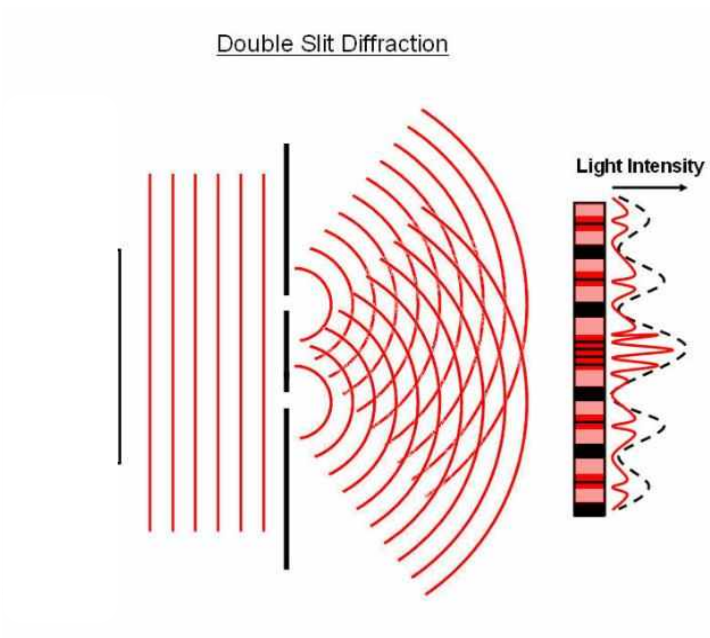
(1787-1826)

German physicist

Lecture 6: Diffraction and Diffraction Gratings



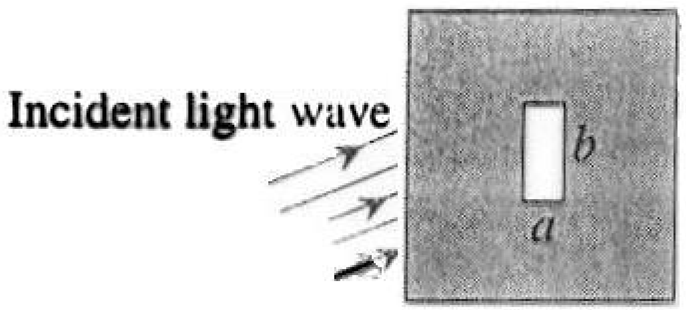


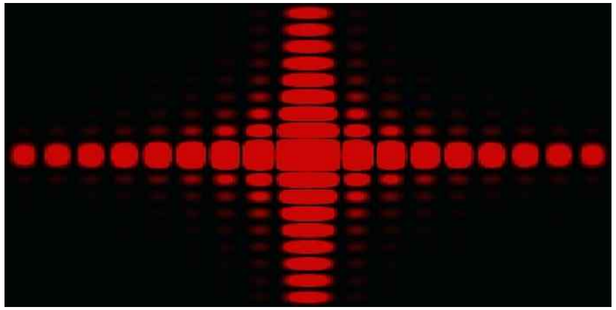


$$\frac{\sin(k_y \frac{a}{2})}{k_y \frac{a}{2}} \quad \text{X} \quad e^{jk_y \frac{d}{2}} + e^{-jk_y \frac{d}{2}} = 2\cos(k_y \frac{d}{2})$$

$$\frac{E(k_y)}{E(0)} = 2\cos(k_y \frac{d}{2}) \frac{\sin(\frac{k_y a}{2})}{\frac{k_y a}{2}}$$

Lecture 6: Diffraction and Diffraction Gratings



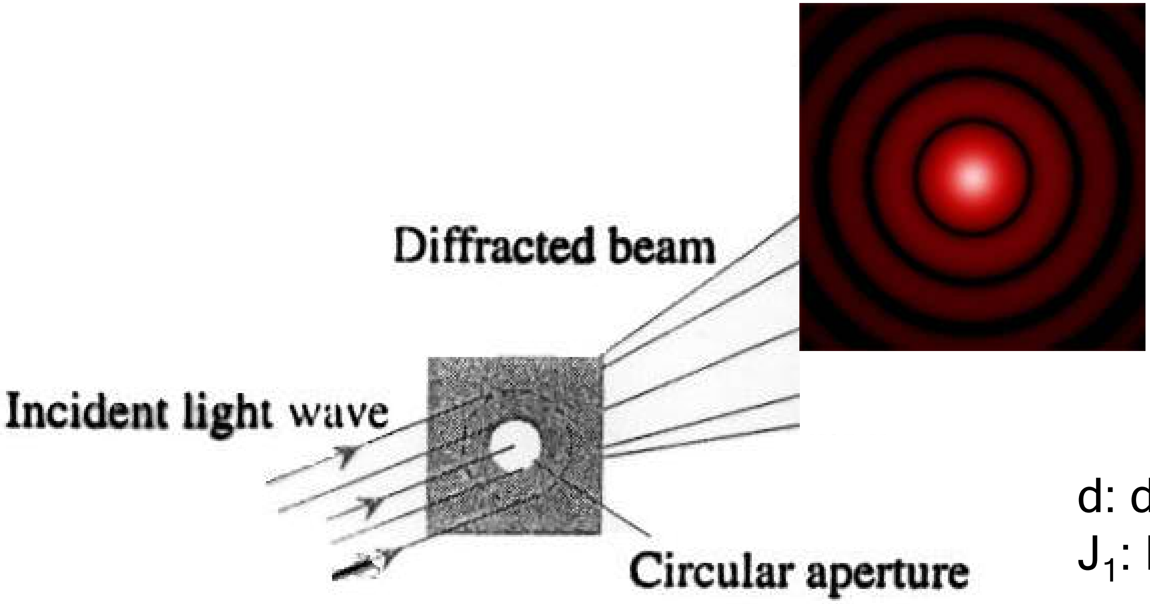


$$E_{total}(k_x, k_y) \sim \int_{y=-\infty}^{y=\infty} \int_{x=-\infty}^{x=\infty} A(x,y) e^{jk_x x} e^{jk_y y} dx dy = \int_{y=-\frac{b}{2}}^{y=\frac{b}{2}} \int_{x=-\frac{a}{2}}^{x=\frac{a}{2}} e^{jk_x x} e^{jk_y y} dx dy$$

$$= \int_{y=-\frac{b}{2}}^{y=\frac{b}{2}} e^{jk_x y} dy \int_{x=-\frac{a}{2}}^{x=\frac{a}{2}} e^{jk_x x} dx$$

$$\frac{E_{total}(k_x, k_y)}{E_{total}(0,0)} = \frac{\sin(k_y \frac{b}{2})}{k_y \frac{b}{2}} \frac{\sin(k_x \frac{a}{2})}{k_x \frac{a}{2}}$$

Lecture 6: Diffraction and Diffraction Gratings



Airy Disk

$$\frac{I(k_r)}{I_0} = \left(\frac{2J_1(k_r \, d/2)}{k_r \, d/2} \right)^2$$

d: diameter
J_1: Bessel function of first order

For the first dark ring,

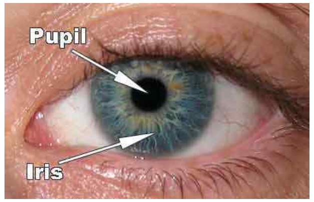
$$k_r \frac{d}{2} \sim 3.83 \quad \frac{2\pi}{\lambda}\sin(\theta)\frac{d}{2} \sim 3.83 \quad \sin\theta \sim \frac{3.83}{d}\frac{\lambda}{\pi} \sim 1.22\frac{\lambda}{d}$$

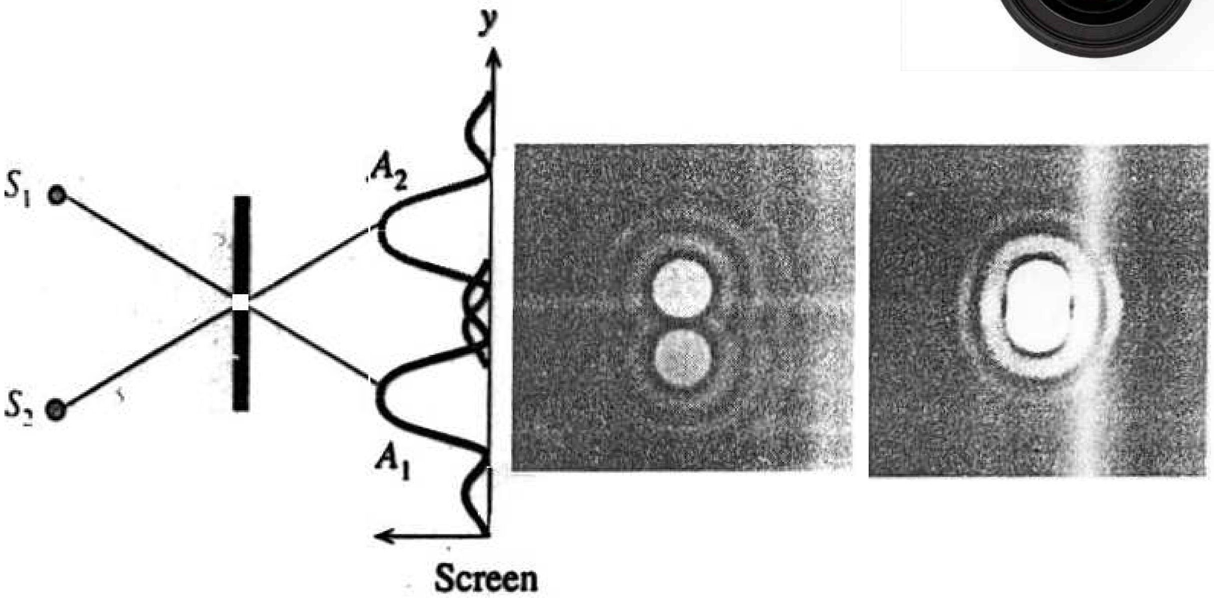
θ determines imaging resolution

Lecture 6: Diffraction and Diffraction Gratings

$$\sin\theta \sim 1.22\frac{\lambda}{d}$$

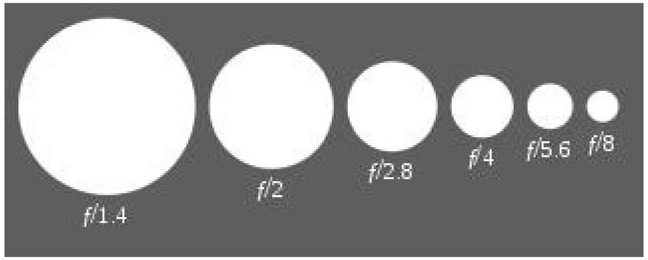
Many imaging systems have circular aperture





Larger d ➔ smaller θ ➔ Better resolution

(But smaller depth of field)



Lecture 6: Diffraction and Diffraction Gratings

Gran Telescopio Canarias (**GTC**)

Aperture diameter: 10.4 m
(World's largest optical telescope)

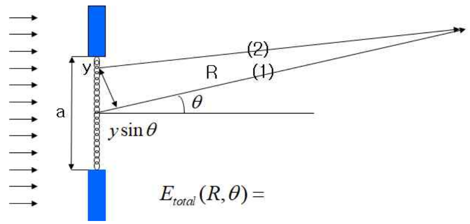
In Spain

Lecture 6: Diffraction and Diffraction Gratings

In our analysis,

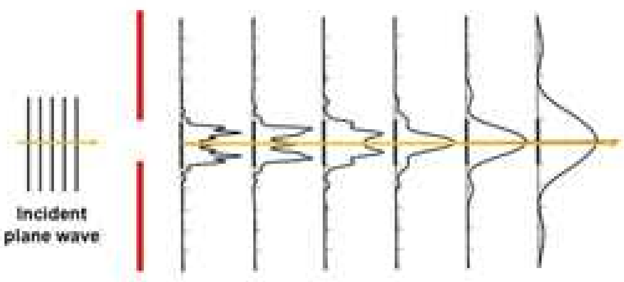
- Diffraction is observed at locations having same R

➔ In reality, observation is made at a flat surface. Consequently, there is additional phase shifts, requiring more complicated analysis



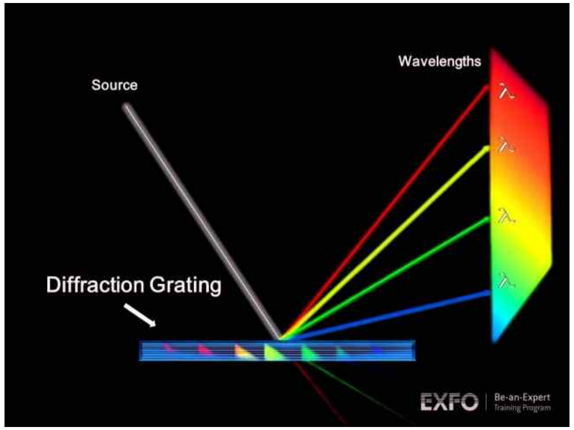
- It is assumed R>>λ (far field)

If not, FT relation cannot be used ➔ Near-field or Fresnel diffraction.

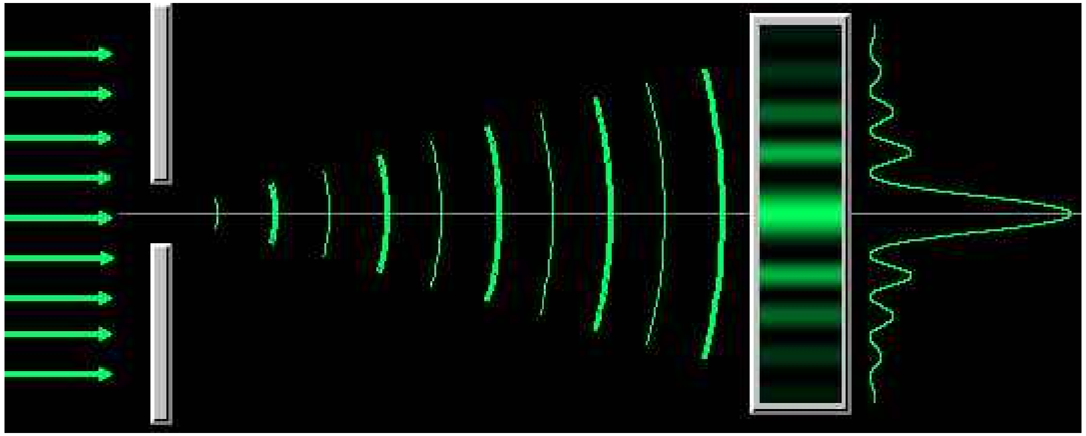


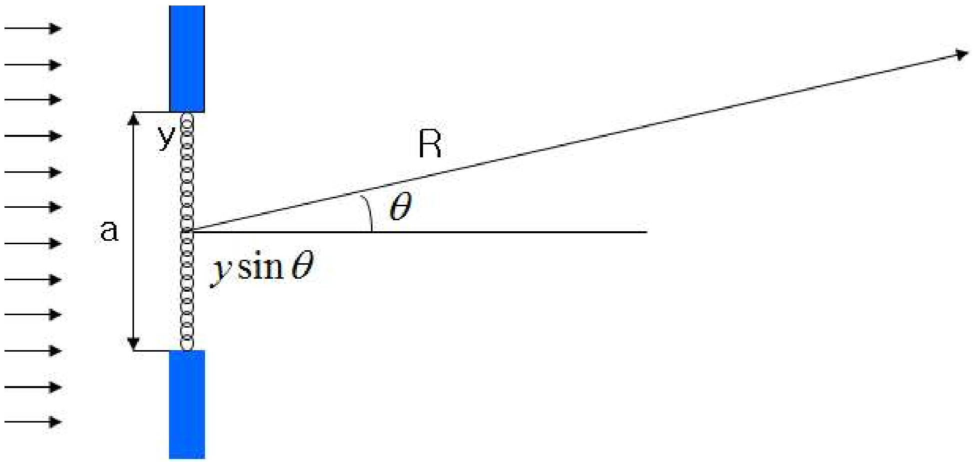
Augustin-Jean Fresnel
(1788~1827)
French Physicist

Lecture 6: Diffraction and Diffraction Gratings



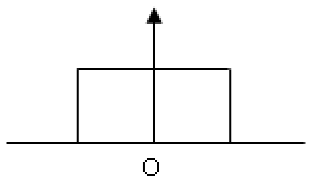
Lecture 6: Diffraction and Diffraction Gratings





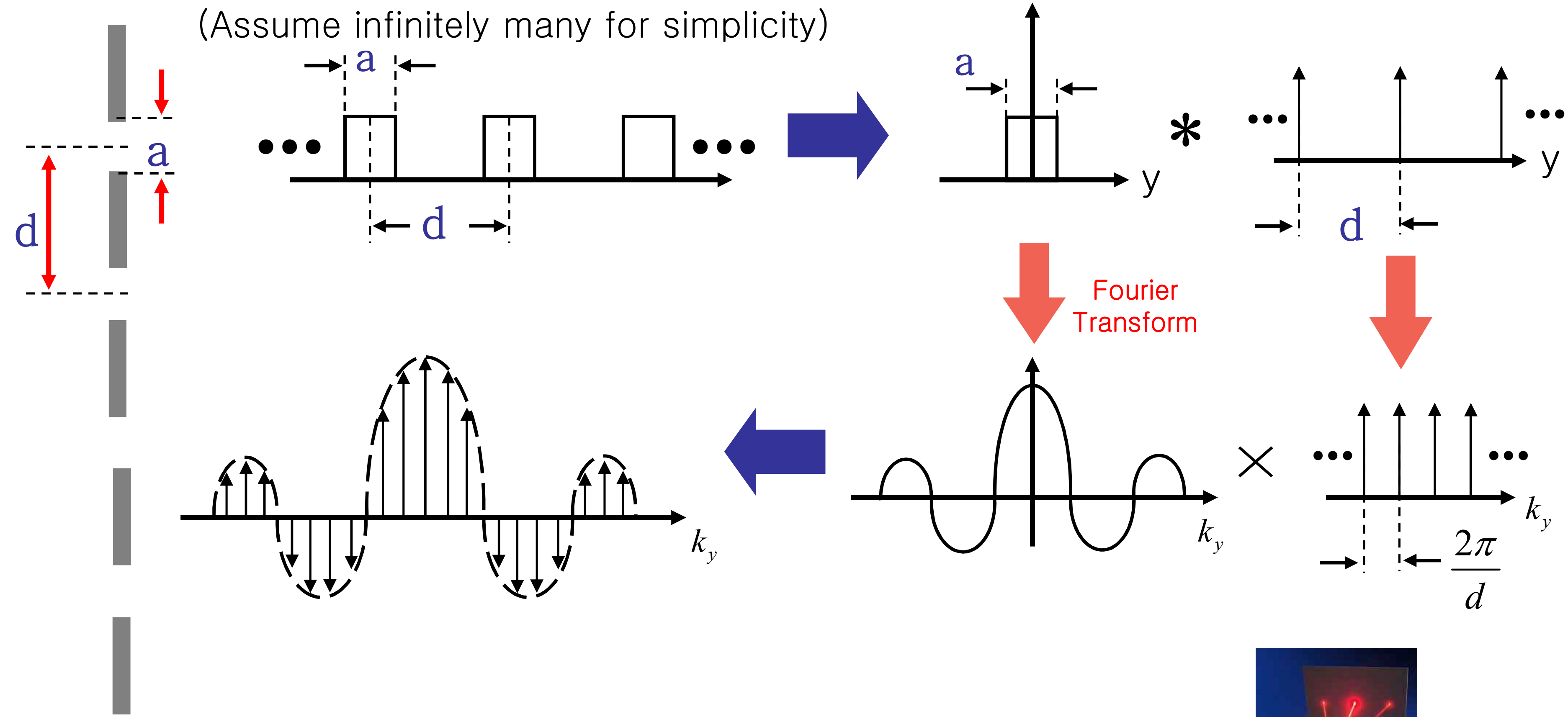
slit shape (y) ←→ far-field (k_y)

Fourier Transform



$$\frac{\sin\left(k_y \frac{a}{2}\right)}{k_y \frac{a}{2}}$$

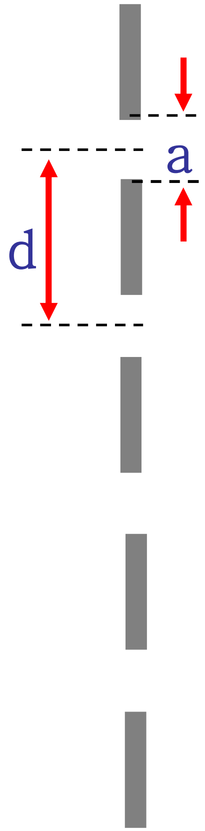
Lecture 6: Diffraction and Diffraction Gratings

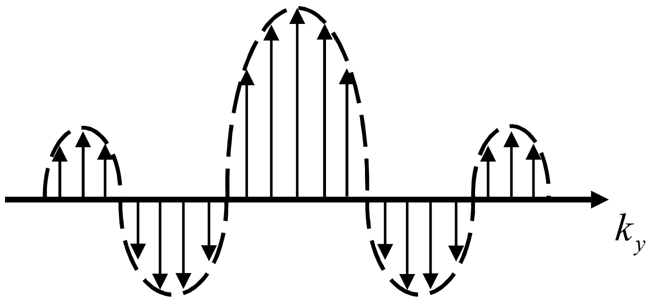


Diffracted light from periodic opening (Diffraction Grating)

==> Far-field only for discrete k_y's $\quad k_y = k\sin\theta = m\frac{2\pi}{d}$

Lecture 6: Diffraction and Diffraction Gratings



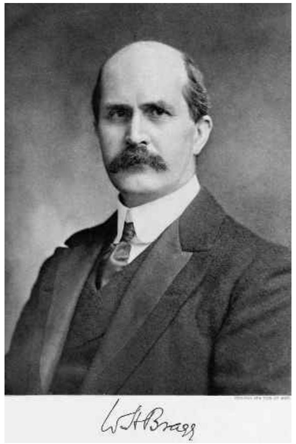


$$k_y = m\frac{2\pi}{d}$$

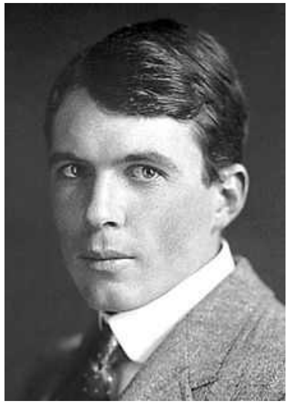
$$\frac{2\pi}{\lambda}\sin\theta = m\frac{2\pi}{d}$$

$$d\sin\theta = m\lambda$$

Grating equation (Bragg Condition)



William Henry Bragg
(1862-1942)



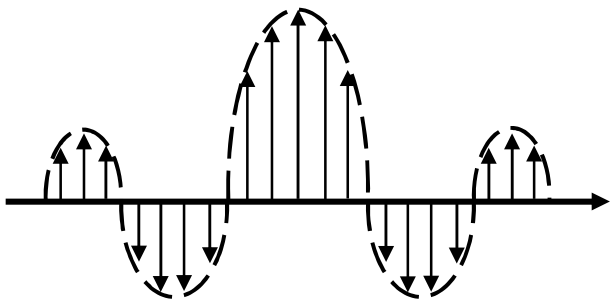
William Lawrence Bragg
(1890-1971)

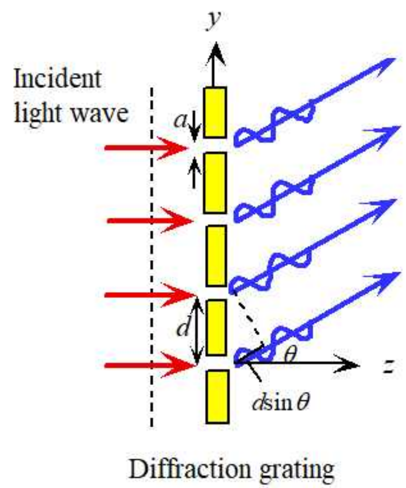
Nobel Prize in Physics (1915)

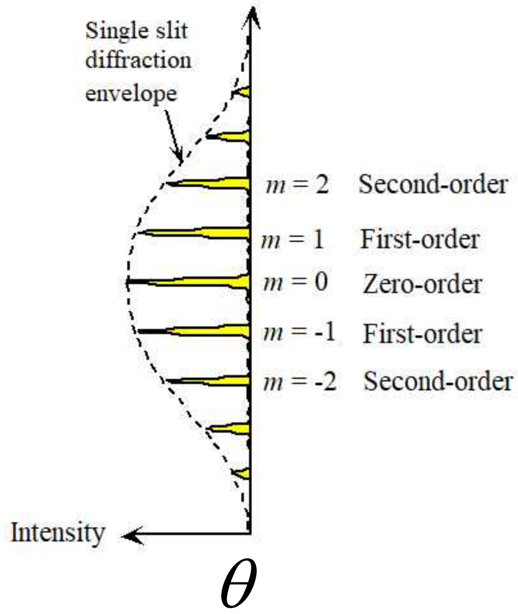
The only father-son joint Nobel winner

W. L. Bragg is the youngest Nobel Physics winner

Lecture 6: Diffraction and Diffraction Gratings





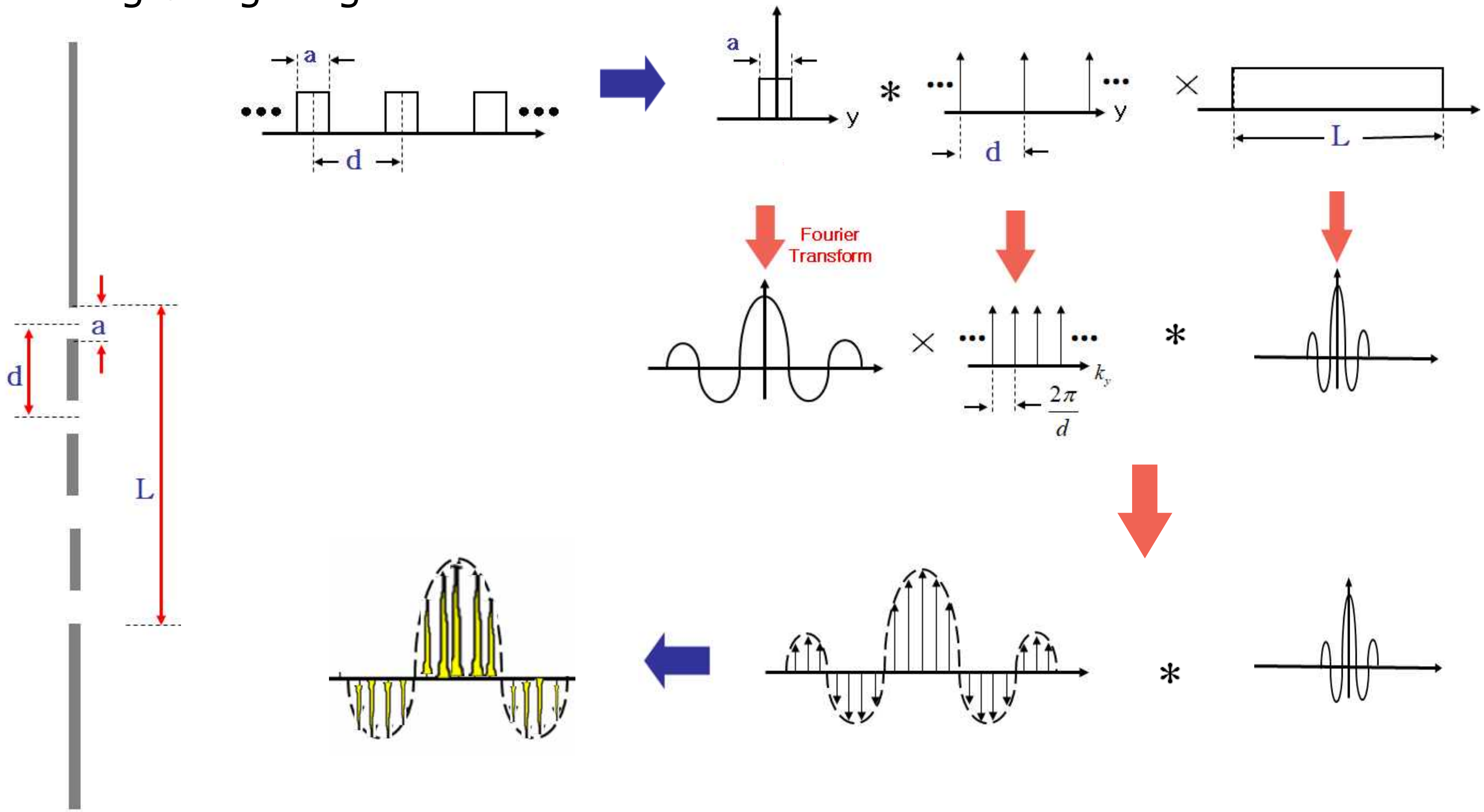


$$d \sin \theta = m\lambda$$

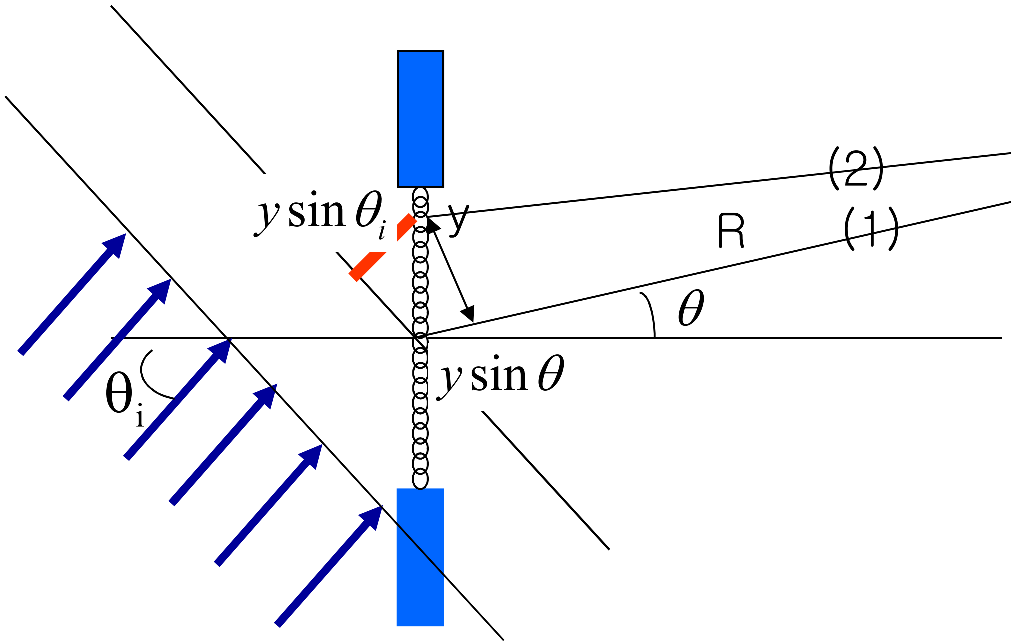
Width for each diffracted beam?

Lecture 6: Diffraction and Diffraction Gratings

Finite grating length



Input with tilted angle

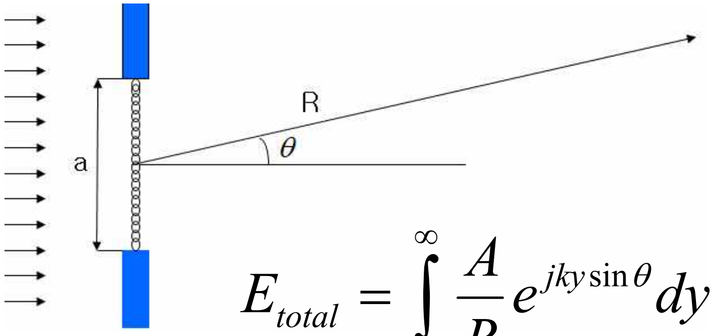


$$\frac{A}{R}e^{jky\sin\theta}$$

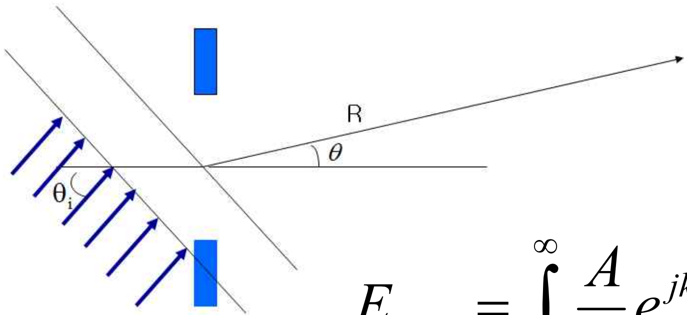
$$\Longrightarrow \frac{A}{R}e^{jky\sin\theta}e^{-jky\sin\theta_i}$$

$$E_{total}=\int_{-\infty}^{\infty}\frac{A}{R}e^{jky\sin\theta}dy \quad \Longrightarrow \quad E_{total}=\int_{-\infty}^{\infty}\frac{A}{R}e^{jky(\sin\theta-\sin\theta_i)}dy$$

Lecture 6: Diffraction and Diffraction Gratings



$$E_{total} = \int_{-\infty}^{\infty} \frac{A}{R} e^{jky\sin\theta} dy$$



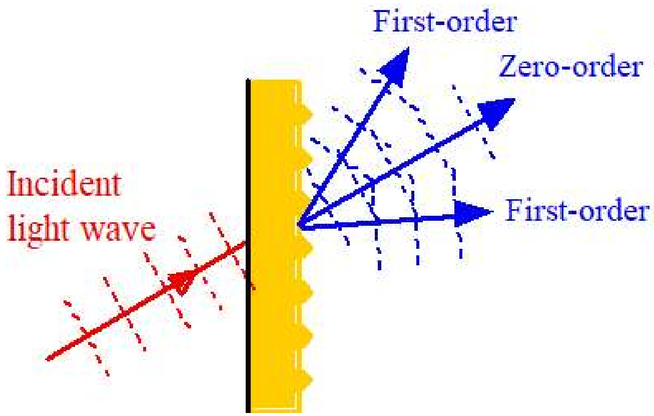
$$E_{total} = \int_{-\infty}^{\infty} \frac{A}{R} e^{jky(\sin\theta - \sin\theta_i)} dy$$

Normal Incidence on Grating

$$d\sin\theta = m\lambda$$

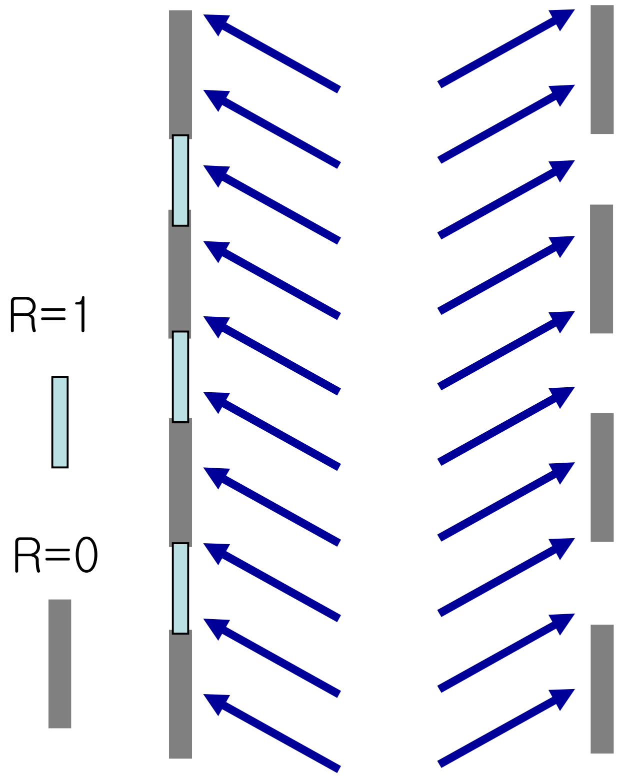
Tilted incidence on Grating

$$\mathrm{d}(\sin\theta - \sin\theta_i) = \mathrm{m}\cdot\lambda$$



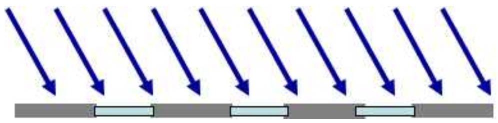
Lecture 6: Diffraction and Diffraction Gratings

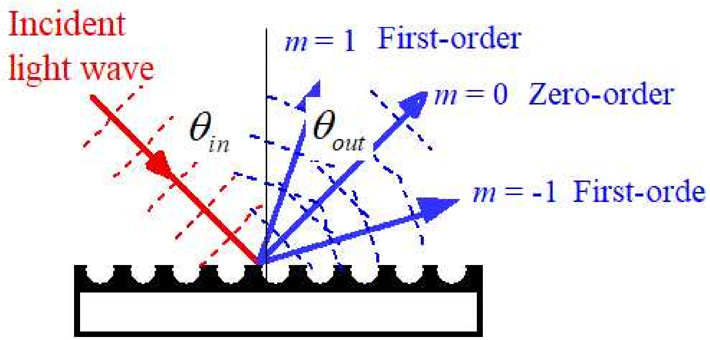
Reflection-type grating



Same diffraction equation

$$d\left(\sin\theta - \sin\theta_i\right) = m \cdot \lambda$$

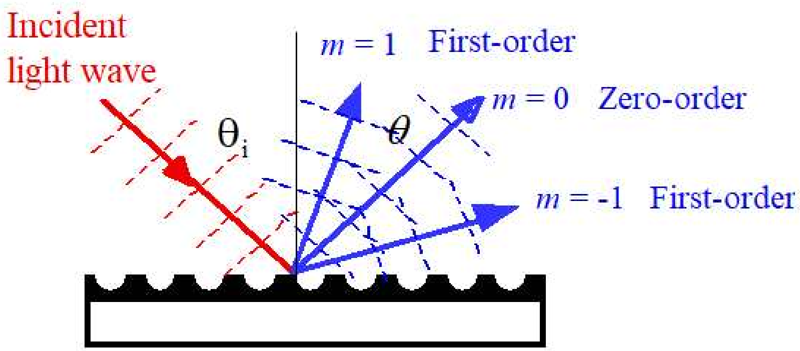




Reflection grating

Grating realized with periodic shaping of reflection surface

Lecture 6: Diffraction and Diffraction Gratings



Reflection grating

$$d\left(\sin\theta_{out} - \sin\theta_{in}\right) = m \cdot \lambda$$

$$\frac{2\pi}{\lambda}\left(\sin\theta_{out} - \sin\theta_{in}\right) = m \cdot \frac{2\pi}{d}$$

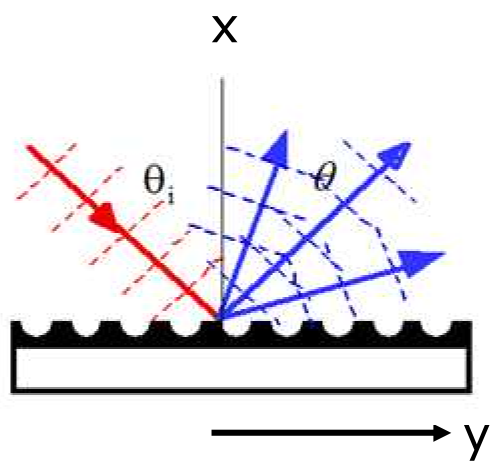
$$\frac{2\pi}{\lambda}\sin\theta_{out} = \frac{2\pi}{\lambda}\sin\theta_{in} + m \cdot \frac{2\pi}{d}$$

K-vector perspective?

$$k_{y,out} = k_{y,in} + m \cdot \frac{2\pi}{d}$$

Grating shifts $k_{y,in}$ by integer multiples of $2\pi/d$

Lecture 6: Diffraction and Diffraction Gratings



$$k_{y,out} = k_{y,in} + m \cdot \frac{2\pi}{d}$$

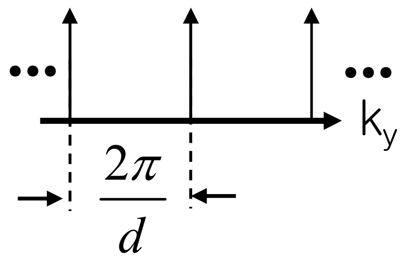
Grating imposes BC on the incident wave

$$E_r(x=0, y) = E_{in}(x=0, y) \text{x} f(y)$$

$$f(y) = \sum_m a_m \exp(jm\frac{2\pi}{d}y)$$

Far-Field Diffraction: F.T. of $E_{in}(x=0, y) \text{x} f(y)$

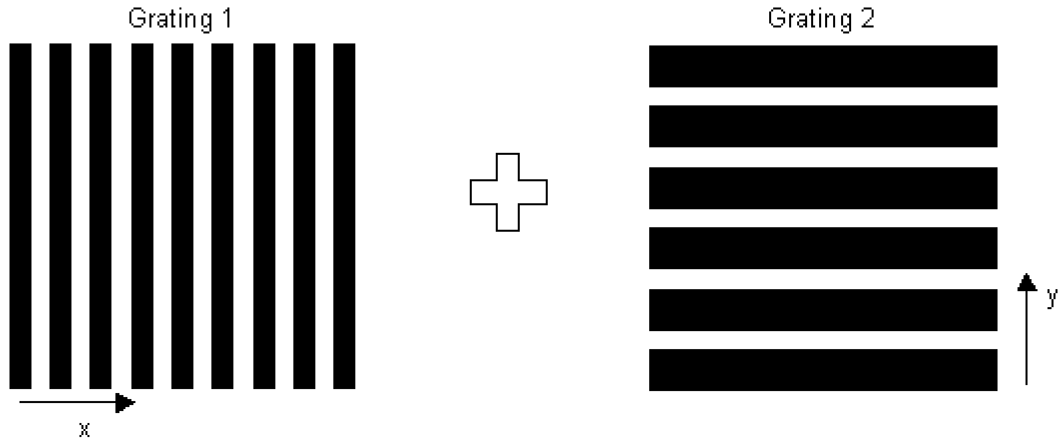
$$E(k_y)_{out} = E(k_y)_{in} * $$



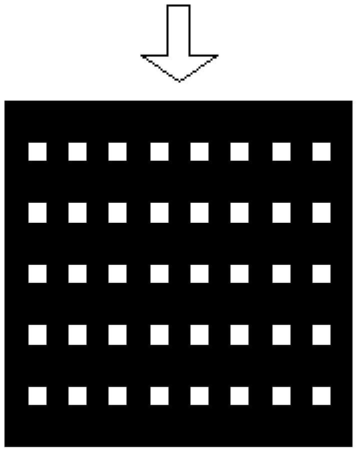
Spatial modulation ⬌ Sidebands formation

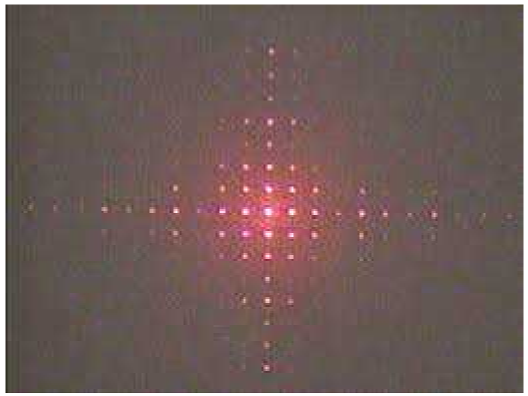
Lecture 6: Diffraction and Diffraction Gratings

2-D Diffraction Grating



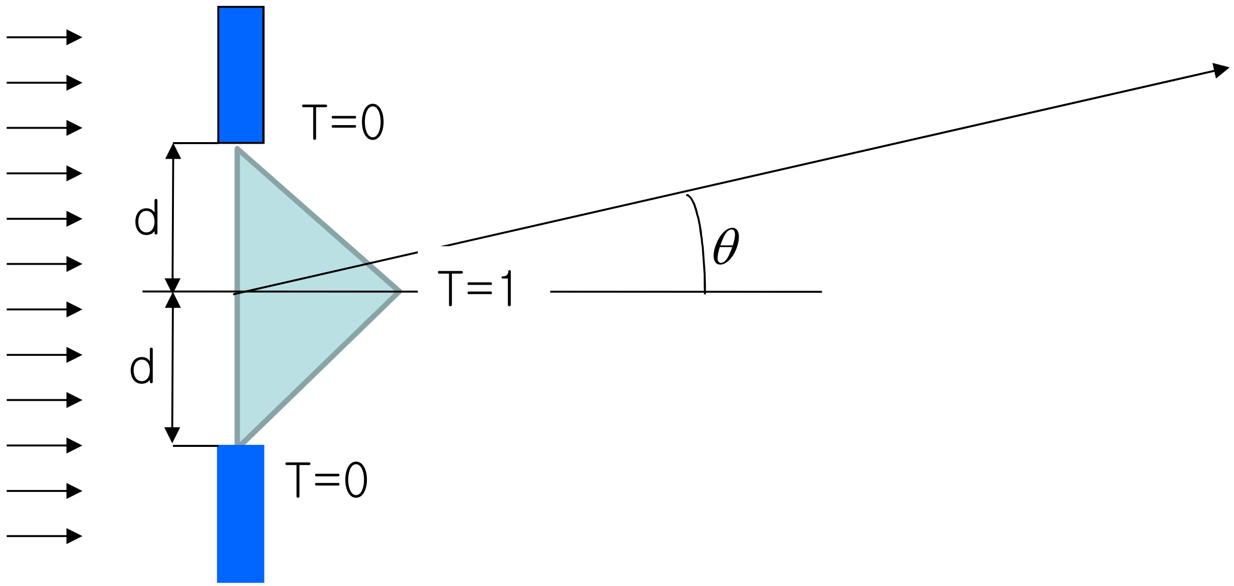
Take x and y separately





Lecture 6: Diffraction and Diffraction Gratings

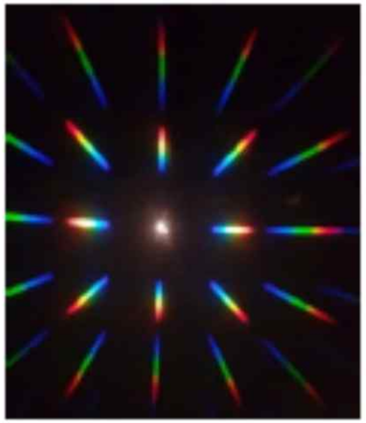
Homework(1): Determine the far-field diffraction intensity when light passes through an opening in which the amount of transmission linear varies from T=1 in the middle to 0 at top and bottom edges.



Lecture 6: Diffraction and Diffraction Gratings

Homework (2)

The following figure shows the moon looked through a two-dimensional diffraction grating. The grating period in both x-direction and y-direction is d.



(a) Why are there discrete bands of diffracted light?

(b) Why is the red further away from the image of the moon in the center than the blue within the same band?

(c) Explain how you can estimate the distance between the grating and the observation plane from above figure. Use d, the grating period, and x, the distance between the center and a point of a particular color whose wavelength is λ.