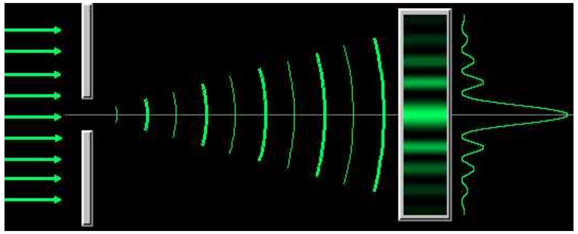
Si Photonics

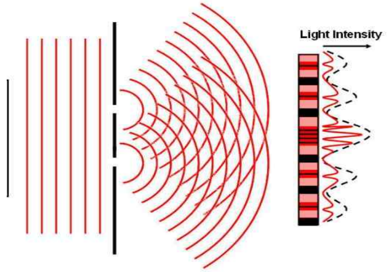
Lecture 7 : Dielectric Waveguides

Woo-Young Choi

Dept. of Electrical and Electronic Engineering
Yonsei University

Lecture 7: Dielectric Waveguides





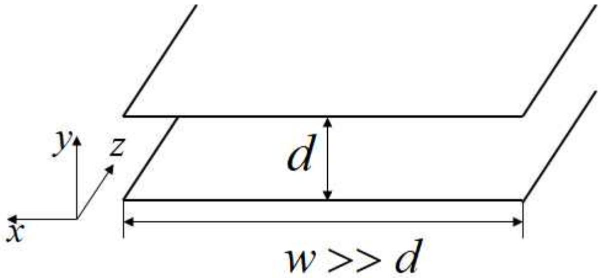
Diffraction changes beam pattern as it propagates

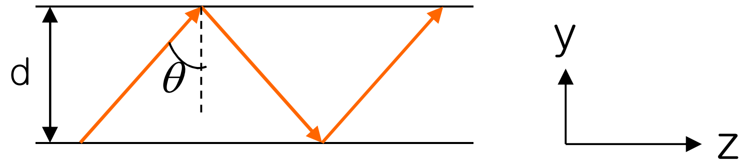
Ca EM waves propagate without diffraction?

➔ Waveguide

Lecture 7: Dielectric Waveguides

Metallic Waveguide





After one round trip in y-direction, the same magnitude and phase

$$e^{-jk_y d}(-1)e^{-jk_y d}(-1) = 1 \qquad (k_y = k\cos\theta)$$

$$\tan\theta = \frac{\beta}{k_y}$$

$$e^{-j2k_y d} = 1 \qquad 2k_y d = 2m\pi \text{ (m integer)}$$

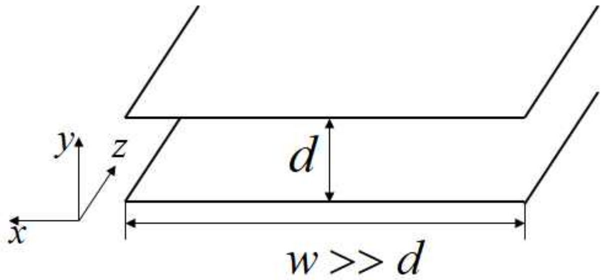
➔ Only certain θ's are allowed

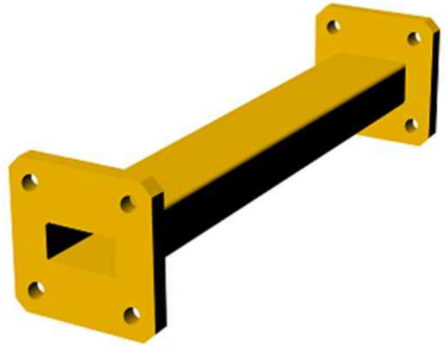
$$k_y = \frac{m\pi}{d} \qquad \beta(=k_z) = \sqrt{(nk_0)^2 - \left(\frac{m\pi}{d}\right)^2}$$

(m: mode index)

Each mode has its own θ

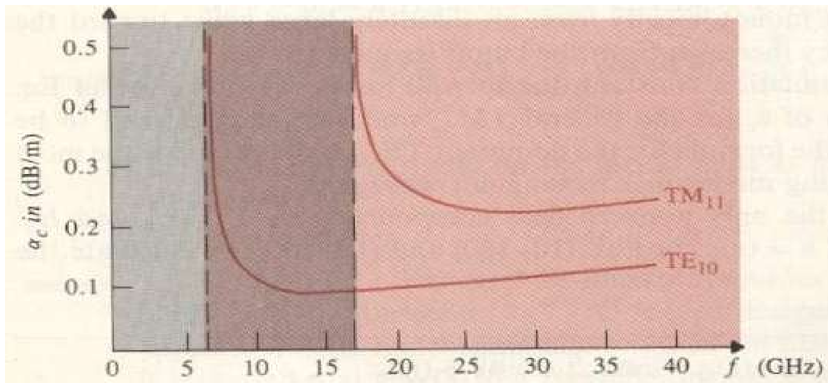
Lecture 7: Dielectric Waveguides





Metallic wavegudes are used for microwave signals

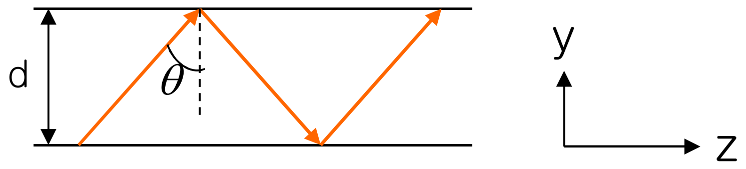
But too lossy for light propagation



➔ Dielectric waveguies with TIR (~ 0.2dB/km for optical fiber)

Lecture 7: Dielectric Waveguides

Metallic waveguide



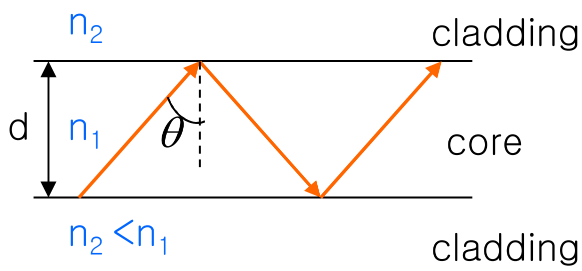
$$e^{-jk_yd}(-1)e^{-jk_yd}(-1) = e^{-j2k_yd} = 1$$

$$\therefore 2k_yd = 2m\pi \text{ and } k_y = \frac{m\pi}{d}$$

$$\beta(=k_z) = \sqrt{(nk_0)^2 - \left(\frac{m\pi}{d}\right)^2}$$

$$\tan\theta = \frac{\beta}{k_y}$$

Dielectric waveguide (TIR)



$$e^{-jk_yd}\ r_{\perp,//}\ e^{-jk_yd}\ r_{\perp,//} = 1$$

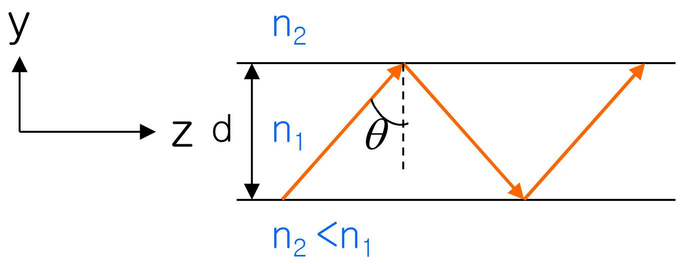
$$r_{\perp,//} = e^{j\phi_{\perp,//}}, \quad e^{-j2k_yd}e^{j2\phi_{\perp,//}} = 1$$

$$2k_yd - 2\phi_{\perp,//} = 2m\pi$$

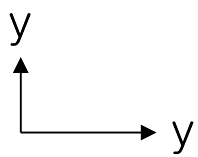
$$k_yd - \phi_{\perp,//} = m\pi$$

➔ Guidance condition for dielectric waveguide

Lecture 7: Dielectric Waveguides



$$k_y d - \phi_{\perp,//} = m\pi$$



$$(n = \frac{n_2}{n_1})$$

$$\phi_{\perp} : \tan(\frac{\phi_{\perp}}{2}) = \frac{(\sin^2\theta - n^2)^{1/2}}{\cos\theta}$$

➔Transverse Electric (TE)

E−field only in x−direction

$$\phi_{//} : \tan(\frac{\phi_{\parallel} + \pi}{2}) = \frac{(\sin^2\theta - n^2)^{1/2}}{n^2\cos\theta}$$

➔Transverse Magnetic (TM)

H−field only in x−direction

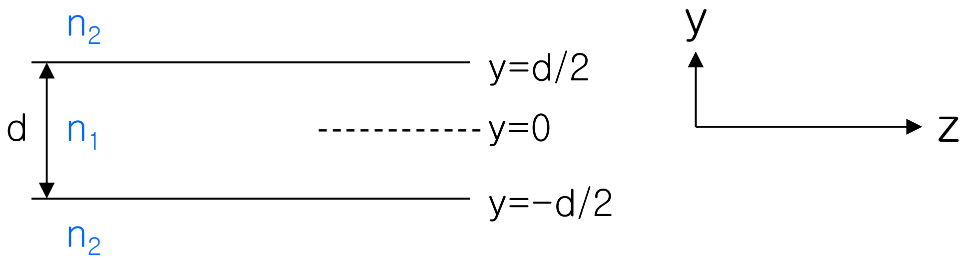
Numerically solve for θ for each mode ➔ k_y

$$\beta = \sqrt{(n_1 k_0)^2 - k_y{}^2}$$

E, H field profile for each guided mode?

Lecture 7: Dielectric Waveguides

(Planar Slab Waveguide)



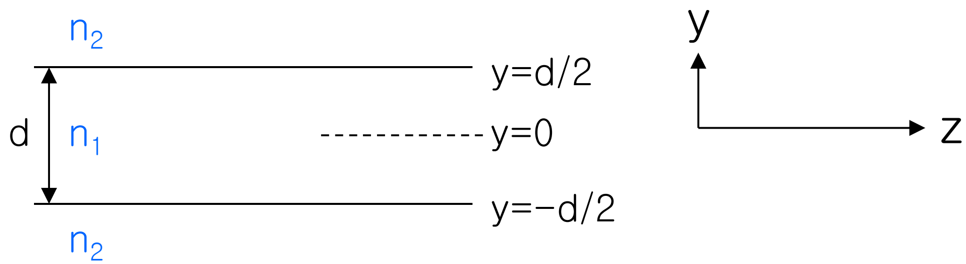
Full analysis starting from the wave equations $\nabla^2 \overline{E}(y,z,t) = \mu\varepsilon \dfrac{\partial^2 \overline{E}}{\partial t^2}(y,z,t)$

Assuming $\overline{E}\left(y,z,t\right) = \overline{E}(y)\cdot e^{-j\beta z}\cdot e^{j\omega t}$

$$\left(\frac{d^2}{dy^2}\overline{E}(y)\right)e^{-j\beta z}\cdot e^{j\omega t} - \beta^2\overline{E}(y)e^{-j\beta z}\cdot e^{j\omega t} = -\omega^2\mu\varepsilon(y)\overline{E}(y)e^{-j\beta z}\cdot e^{j\omega t}$$

$$\frac{d^2}{dy^2}\overline{E}(y) + \left[\omega^2\mu\varepsilon(y) - \beta^2\right]\overline{E}(y) = 0 \qquad \frac{d^2}{dy^2}\overline{E}(y) + \left[k^2(y) - \beta^2\right]\overline{E}(y) = 0$$

Lecture 7: Dielectric Waveguides



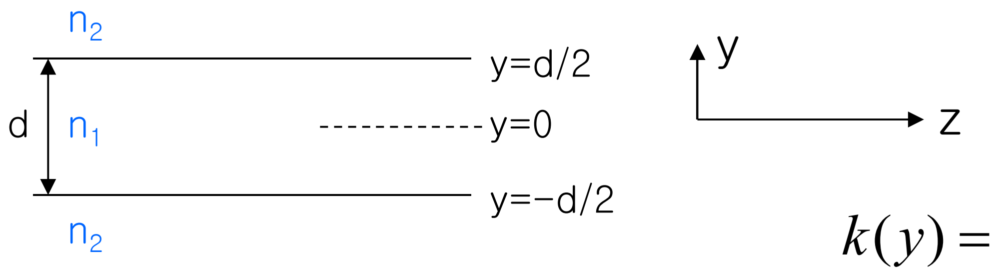
Assuming $\overline{E}\left(y,z,t\right)=\overline{E}(y)\cdot e^{-j\beta z}\cdot e^{j\omega t}$

$$\frac{d^2}{dy^2}\overline{E}(y)+\left[k^2(y)-\beta^2\right]\overline{E}(y)=0$$

For $\left|\mathrm{y}\right|>\frac{d}{2}$ (cladding) $\quad k(y)\ =n_2k_0$

For $\left|\mathrm{y}\right|<\frac{d}{2}$ (core) $\quad k(y)\ =n_1k_0$

Lecture 7: Dielectric Waveguides



$$k(y) = n_2 k_0 \ \text{for} \ |y| > \frac{d}{2}; \ \text{cladding}$$

$$k(y) = n_1 k_0 \ \text{for} \ |y| < \frac{d}{2}; \ \text{core}$$

$$\frac{d^2}{dy^2}\bar{E}(y) + \left[k^2(y) - \beta^2\right]\bar{E}(y) = 0$$

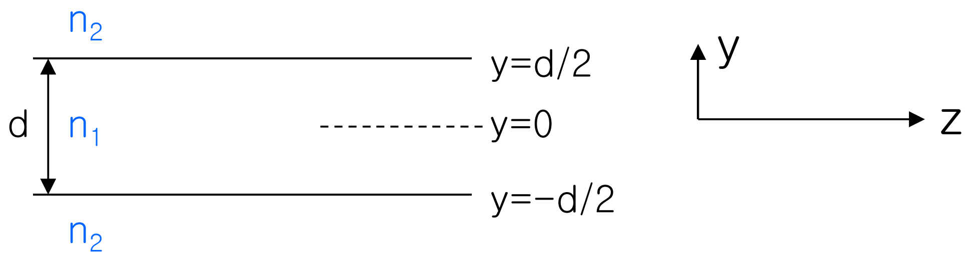
Consider TE Solution $\bar{E}(y) = \bar{x}\ E(y)$ (Perpendicular Polarization)

$$\frac{d^2}{dy^2}E(y) + \left[k^2(y) - \beta^2\right]E(y) = 0$$

Solve for β and $E(y)$ (Eigen value equation)

==> Discrete β's and correspodning $E(y)'s$

Lecture 7: Dielectric Waveguides



$$\frac{d^2E(y)}{dy^2} + (k^2(y) - \beta^2)E(y) = 0$$

$$k(y) = n_2 k_0 \text{ for } |\mathrm{y}| > \frac{d}{2};\ \text{cladding}$$

$$k(y) = n_1 k_0 \text{ for } |\mathrm{y}| < \frac{d}{2};\ \text{core}$$

Sign of $(k^2(y) - \beta^2)$ determines the solution type $(n_1 k_0 > \beta > n_2 k_0)$

In core, $k^2(y) - \beta^2 > 0$ => $E(y) \sim \sin(k_y y)$ or $\cos(k_y y)$

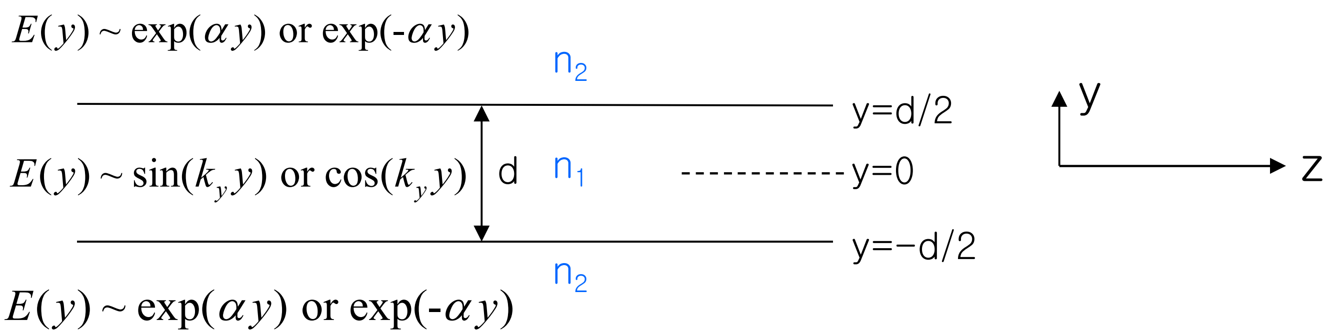
$$-k_y{}^2 E(y) + \left[(n_1 k_0)^2 - \beta^2\right]E(y) = 0 \qquad k_y = \sqrt{(n_1 k_0)^2 - \beta^2}$$

In cladding, $k^2(y) - \beta^2 < 0$ => $E(y) \sim \exp(\alpha y)$ or $\exp(-\alpha y)$

$$\alpha^2 E(y) + \left[(n_2 k_0)^2 - \beta^2\right]E(y) = 0 \qquad \alpha = \sqrt{\beta^2 - (n_2 k_0)^2}$$

Lecture 7: Dielectric Waveguides

$$\overline{E}(y,z,t) = \overline{x}E(y)\cdot e^{-j\beta z}\cdot e^{j\omega t}$$



$$y > \frac{d}{2} : E(y) = A\exp(\alpha y) + B\exp(-\alpha y)$$

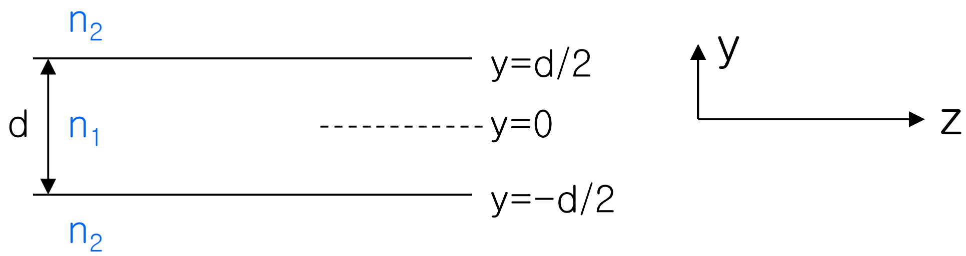
$$|y| < \frac{d}{2} : E(y) = C\sin(k_y y) + D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = E\exp(\alpha y) + F\exp(-\alpha y)$$

A=0 and F=0

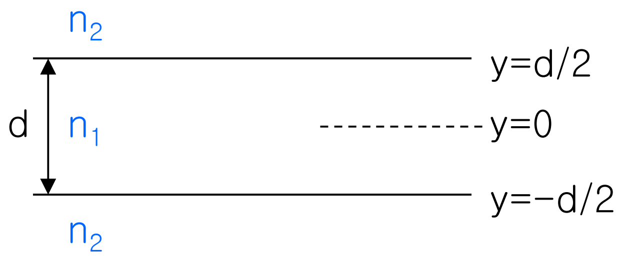
For easier analysis, divide the solutions into even and odd solutions

Lecture 7: Dielectric Waveguides



Even Solutions	Odd Solutions
$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$	$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$
$\lvert y \rvert < \frac{d}{2} : E(y) = D\cos(k_y y)$	$\lvert y \rvert < \frac{d}{2} : E(y) = D\sin(k_y y)$
$y < -\frac{d}{2} : E(y) = B\exp(\alpha y)$	$y < -\frac{d}{2} : E(y) = -B\exp(\alpha y)$
$(E = B)$	$(E = -B)$

Lecture 7: Dielectric Waveguides



Even Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B\exp(\alpha y)$$

$(E = B)$

Apply boundary conditions: $\overline{E}(y,z) = \overline{x}E(y)e^{-j\beta z}$

$E(y)$ is continuous at $y = \pm\frac{d}{2}$

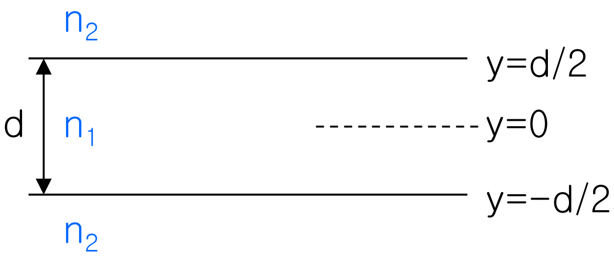
$H_{\tan}$ should be continuous at $y = \pm\frac{d}{2}$

$\overline{H}$ has $\overline{y}$ and $\overline{z}$ components, where $H_z \sim \frac{dE(y)}{dy}$

H_z should be continuous across the boundary

$\therefore E(y)$ and $\frac{dE(y)}{dy}$ are continuous at $y = \pm\frac{d}{2}$

Lecture 7: Dielectric Waveguides



Boundary conditions:

$$E(y) \text{ and } \frac{dE(y)}{dy} \text{ are continuous at } y = \pm\frac{d}{2}$$

Even Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B\exp(\alpha y)$$

$$(E = B)$$

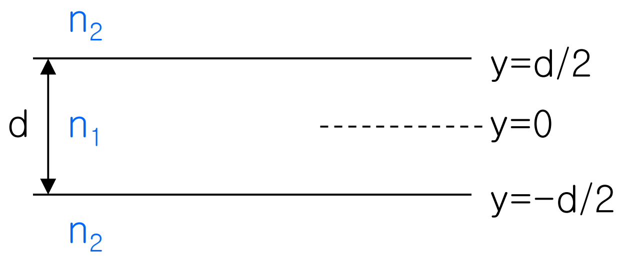
At $y = \dfrac{d}{2}$,

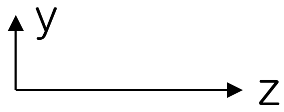
$$B\exp(-\alpha\frac{d}{2}) = D\cos(k_y\frac{d}{2}) \quad \text{-------} \ (1)$$

$$-\alpha B\exp(-\alpha\frac{d}{2}) = -k_y D\sin(k_y\frac{d}{2}) \quad \text{-----} \ (2)$$

$$\frac{(2)}{(1)} : \ \alpha = k_y\tan(k_y\frac{d}{2})$$

Lecture 7: Dielectric Waveguides





Apply boundary conditions.

$$E(y) \text{ and } \frac{dE(y)}{dy} \text{ are continuous at } y = \pm\frac{d}{2}$$

Odd Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\sin(k_y y)$$

$$y < -\frac{d}{2} : E(y) = -B\exp(\alpha y)$$

$$(E = -B)$$

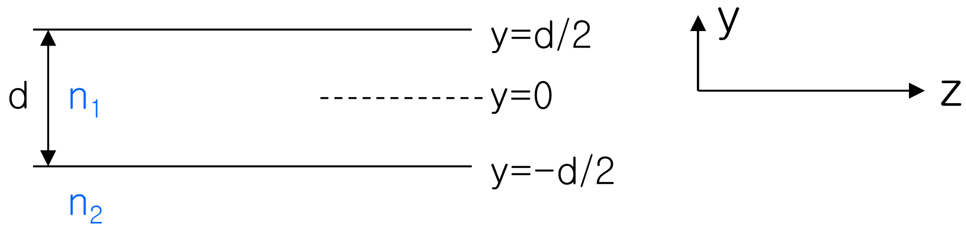
At $y = \frac{d}{2}$,

$$B\exp(-\alpha\frac{d}{2}) = D\sin(k_y\frac{d}{2}) \quad \text{-------- (1)}$$

$$-\alpha B\exp(-\alpha\frac{d}{2}) = k_y D\cos(k_y\frac{d}{2}) \quad \text{----- (2)}$$

$$\frac{(2)}{(1)}: \ \alpha = -k_y\cot(k_y\frac{d}{2}) = k_y\tan(k_y\frac{d}{2} - \frac{\pi}{2})$$

Lecture 7: Dielectric Waveguides



Even: $\alpha = k_y \tan(k_y \frac{d}{2})$

Odd: $\alpha = k_y \tan(k_y \frac{d}{2} - \frac{\pi}{2})$

(From p. 10)

Determine k_y and α that satisfy above conditions.

$$k_y = \sqrt{(n_1 k_0)^2 - \beta^2}$$

$$\alpha = \sqrt{\beta^2 - (n_2 k_0)^2}$$

For easier solutions, do following normalization:

Let $X = k_y \frac{d}{2}$, $Y = \alpha \frac{d}{2}$

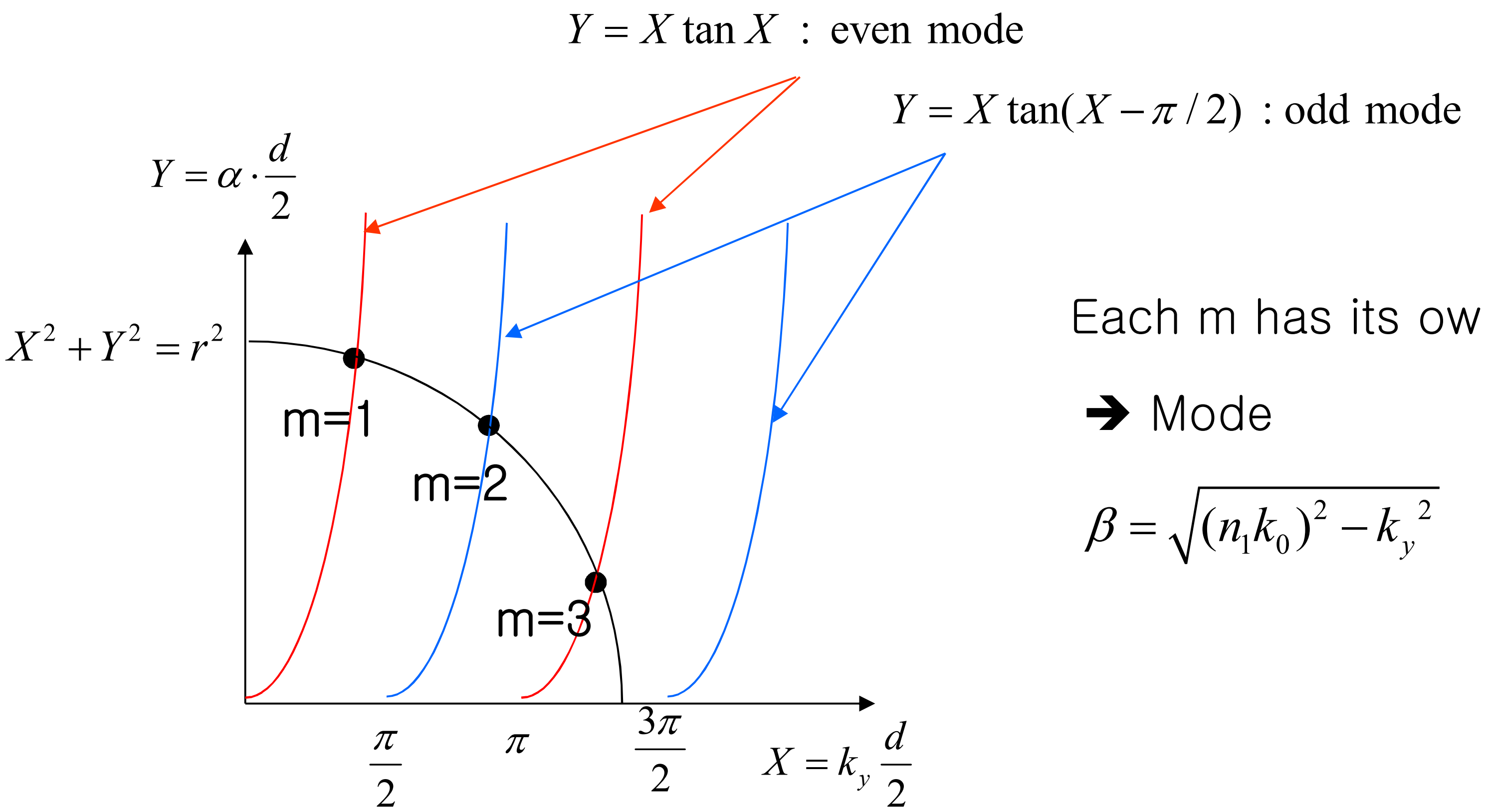
Then, $Y = X \tan X$ for even (1) $\quad Y = X \tan(X - \frac{\pi}{2})$ for odd (2)

$$X^2 + Y^2 = \left(\frac{d}{2}\right)^2 (k_y{}^2 + \alpha^2) = \left(\frac{d}{2}\right)^2 k_0{}^2 (n_1{}^2 - n_2{}^2) = r^2$$

$$X^2 + Y^2 = r^2 \, (3)$$

Lecture 7: Dielectric Waveguides

$Y = X \tan X$ for even $\quad Y = X \tan(X - \frac{\pi}{2})$ for odd $\quad X^2 + Y^2 = r^2$

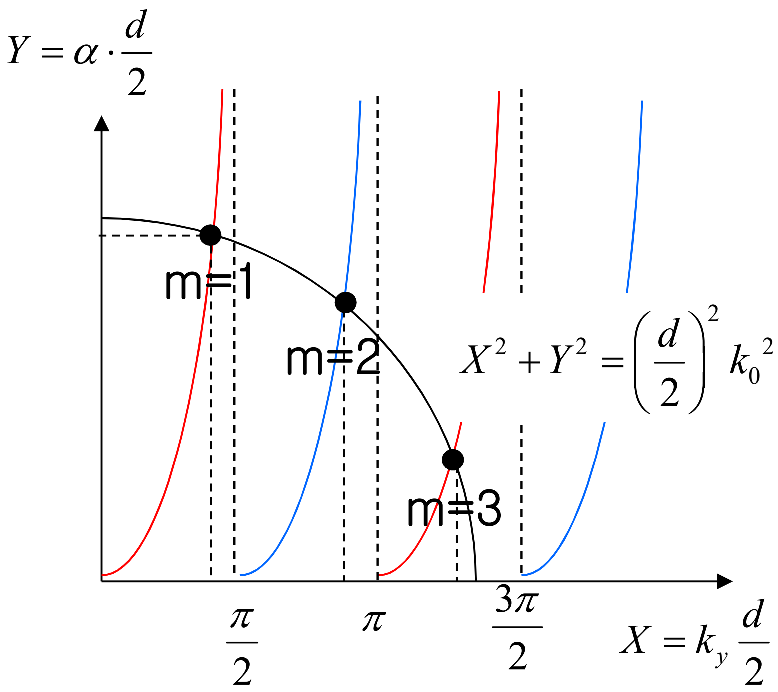


Each m has its own k_y, α

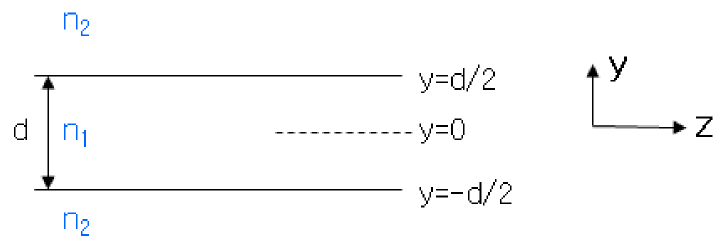
➔ Mode

$$\beta = \sqrt{(n_1 k_0)^2 - k_y^2}$$

Lecture 7: Dielectric Waveguides



$$X^2 + Y^2 = \left(\frac{d}{2}\right)^2 k_0^{\,2}(n_1^{\,2} - n_2^{\,2}) = r^2$$



Observations:

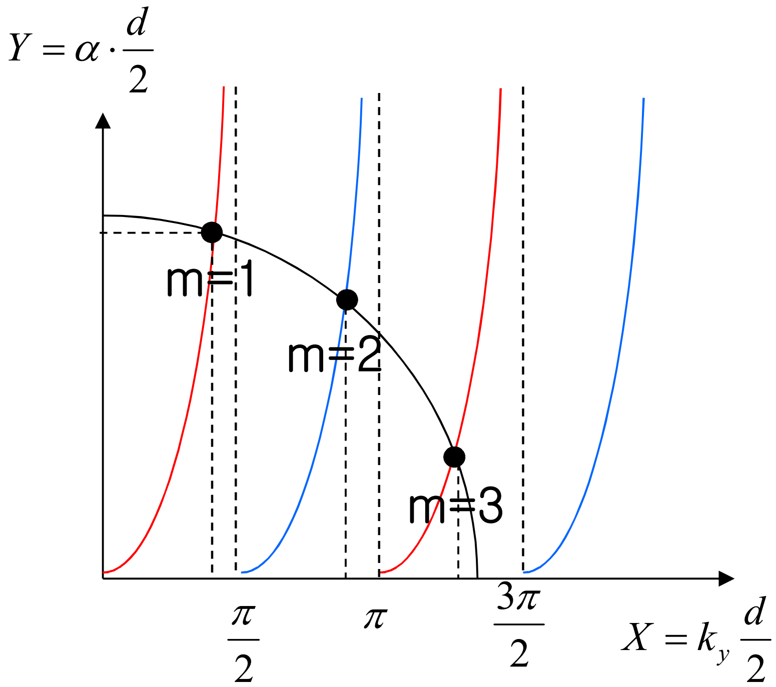
– Even, odd, even, odd ⋯ as r increases

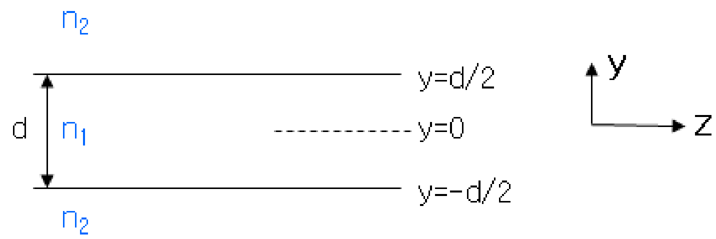
– With larger r, more modes

– There is at least one even TE mode

– Larger m, smaller α, β

Lecture 7: Dielectric Waveguides





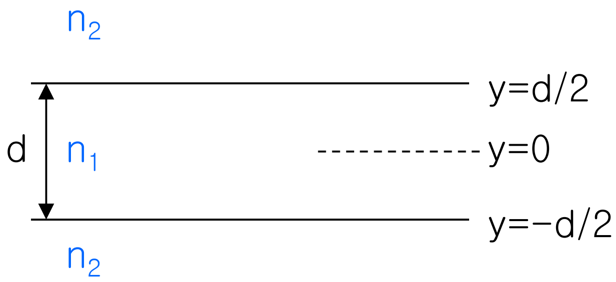
$$\overline{E}(y,z,t)=\overline{x}E(y)\cdot e^{-j\beta z}\cdot e^{j\omega t}$$

For $|y|<\frac{d}{2}$ $\quad E(y)=D\cos(k_y y) \qquad C\sin(k_y y)$

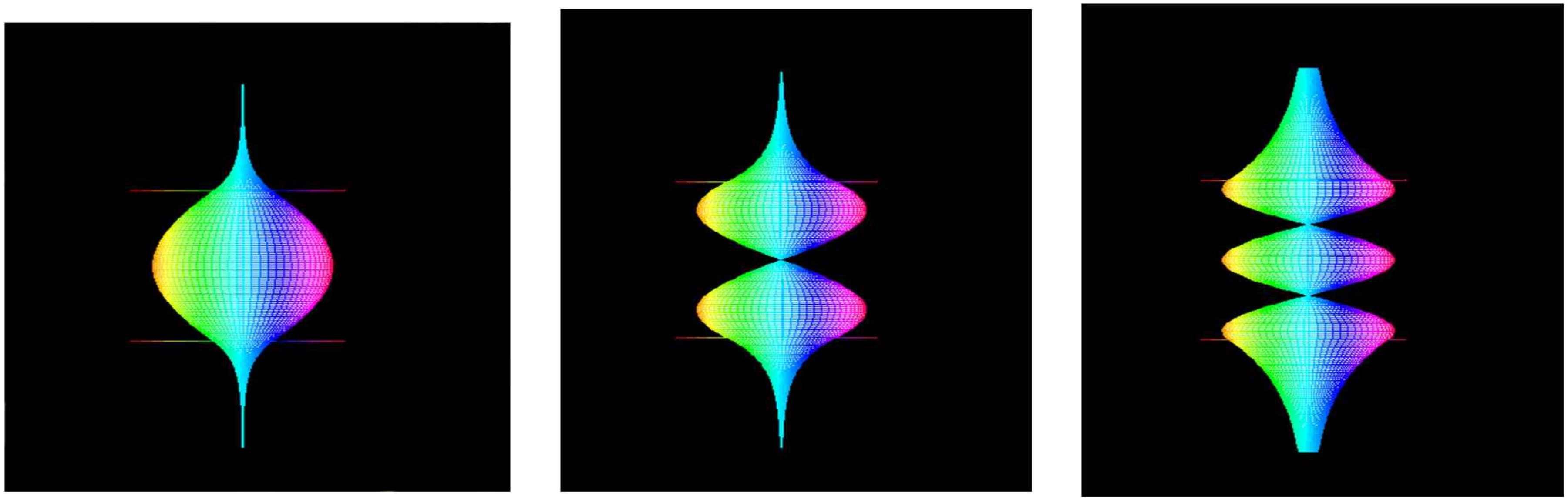
For $y>\frac{d}{2}$ $\quad E(y)=B\exp(-\alpha y) \qquad B\exp(-\alpha y)$

For $y<-\frac{d}{2}$ $\quad E(y)=B\exp(\alpha y) \qquad -B\exp(\alpha y)$

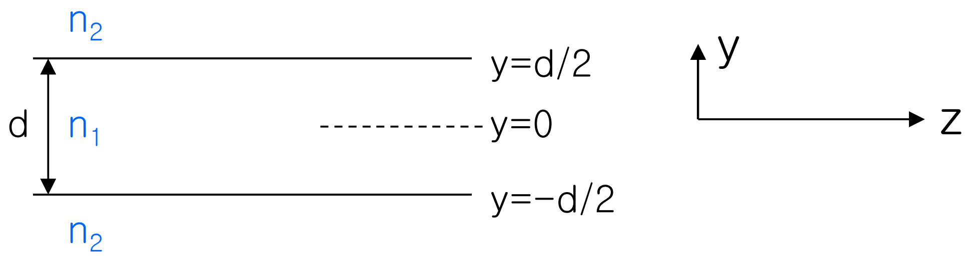
Lecture 7: Dielectric Waveguides

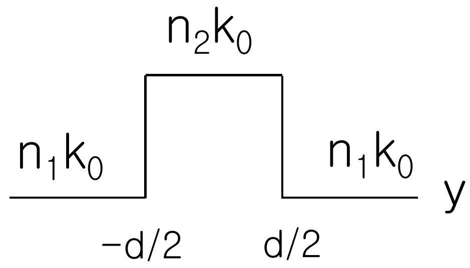


E(y) profile: n_1=1.5, n_2=1.495, d=10μm, λ=1μm



Lecture 7: Dielectric Waveguides



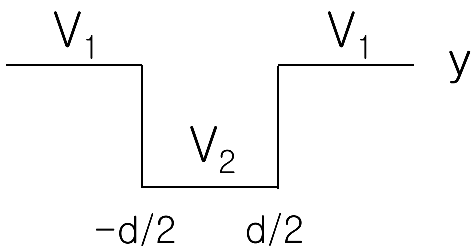


$$\frac{d^2E(y)}{dy^2}+(k^2(y)-\beta^2)E(y)=0$$

Schrödinger Equation in Quantum Mechanics

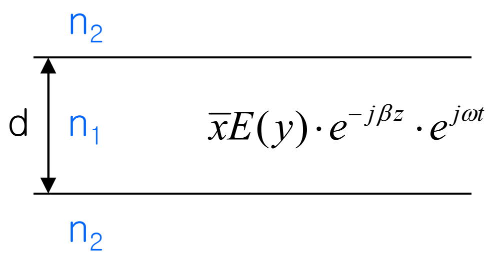
$$-\frac{\hbar^2}{2m}\nabla^2\psi(x,y,z)+V(x,y,z)\psi(x,y,z)=E\psi$$

$$\frac{d^2\psi(y)}{dy^2}+\frac{2m}{\hbar^2}\left[E-V(y)\right]\psi(y)=0$$



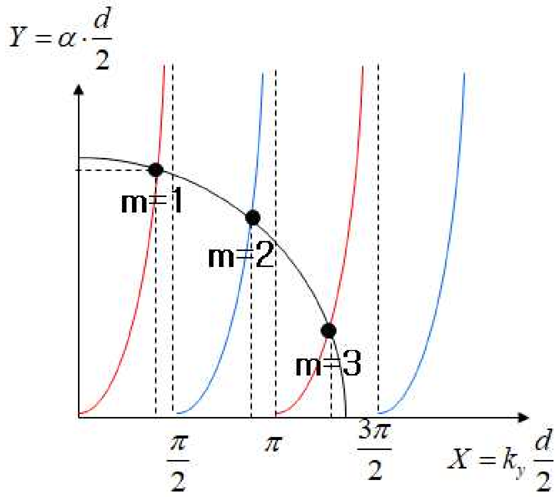
$$E(y) <==> \psi(y) \quad k(y) <==> V(y) \quad \beta <==> E$$

Lecture 7: Dielectric Waveguides



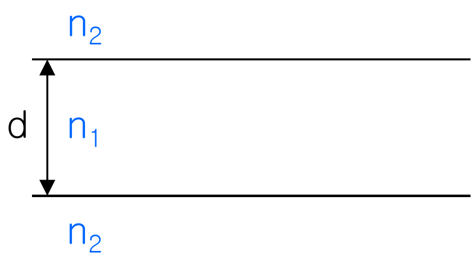
- Effective index: $n_{eff} = \beta/k_0$

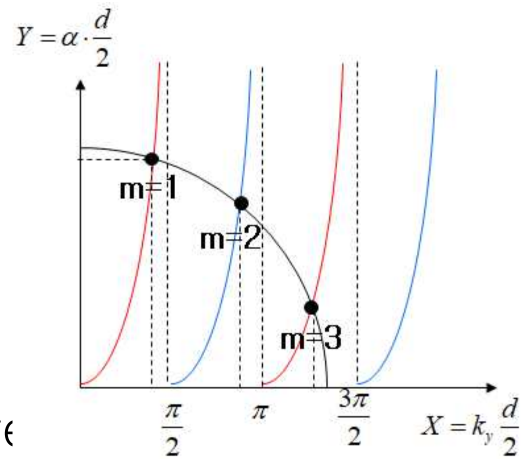
How does n_{eff} change for different modes?



Higher m ➔ Larger k_y ➔ Smaller β ➔ Smaller n_{eff}

Lecture 7: Dielectric Waveguides



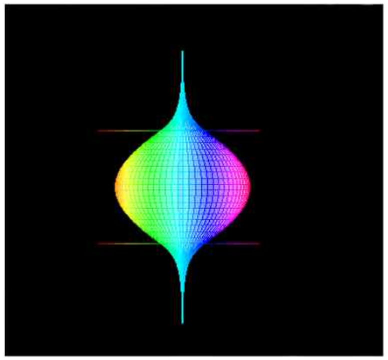


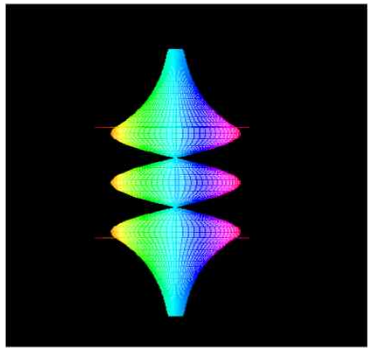
– Confinement factor: Γ

How much power is confined within the core

$$\Gamma = \frac{\text{Power inside core}}{\text{Total Power}} = \frac{\int_{y=-\frac{d}{2}}^{y=\frac{d}{2}} |E(y)|^2 \, dy}{\int_{y=-\infty}^{y=\infty} |E(y)|^2 \, dy}$$

How does Γ change for different modes?

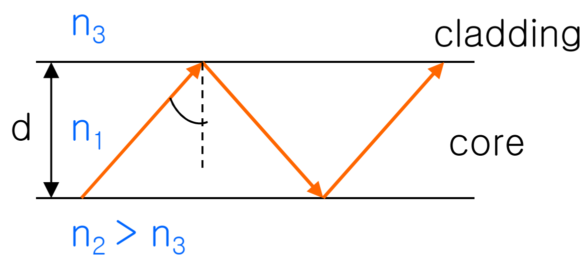




Higher m ➔ Smaller α

➔ Smaller Γ

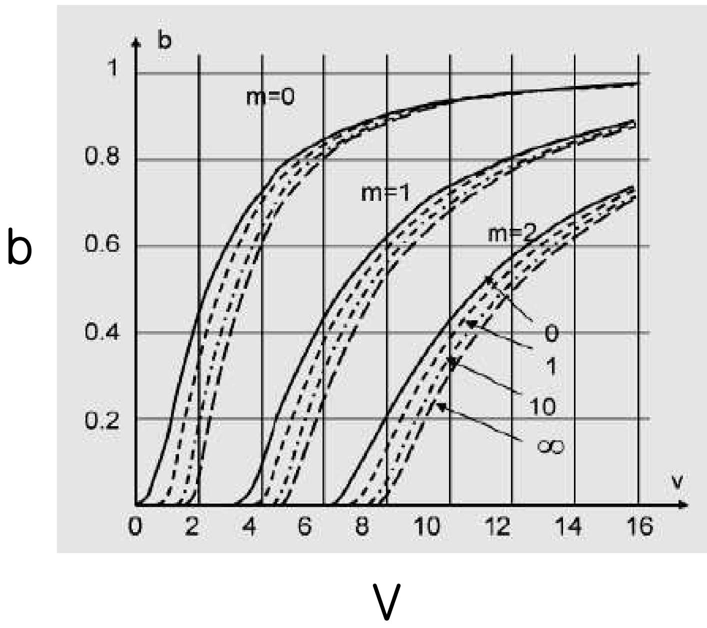
Lecture 7: Dielectric Waveguides



Asymmetric waveguide?

- Numerical solution
- Graphical solution

b-V diagram



$$V = k_0 d (n_1^2 - n_2^2)^{1/2}$$

(Normalized k)

$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}$$

(Asymmetry factor)

$$b = \frac{\left(\beta/k_0\right)^2 - n_2^2}{n_1^2 - n_2^2}$$

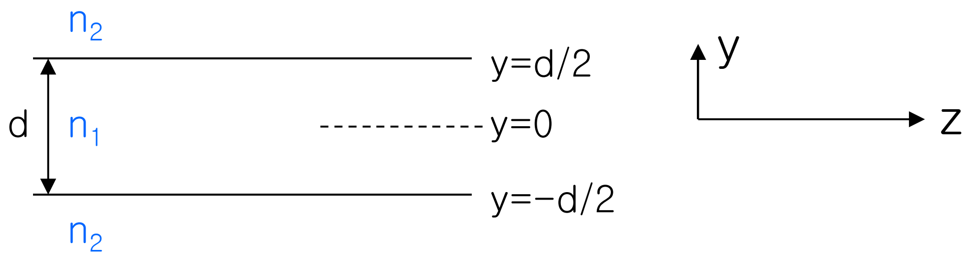
(Normalized β)

For a given waveguide: V,a ➔ b from the diagram

Then, determine β

Larger V, smaller a ➔ larger β

Lecture 7: Dielectric Waveguides



TE Solution (Perpendicular Polarization) $\overline{E}(y) = \overline{x}\ E(y)$

$$\frac{d^2}{dy^2}E(y) + \left[k^2(y) - \beta^2\right]E(y) = 0$$

How about parallel polarization? TM solution

$$\overline{H}(y) = \overline{x}\ H(y) \qquad \frac{d^2}{dy^2}H(y) + \left[k^2(y) - \beta^2\right]H(y) = 0$$

Slightly different from TE solution due to different boundary conditions

$TE : E(y)$ and $\dfrac{dE(y)}{dy}$ continuous at boundaries $\qquad TM : H(y)$ and $\dfrac{1}{\varepsilon}\dfrac{dH(y)}{dy}$ continuous at boundaries

TM solutions have very similar behavior as TE solutions

Typically, TM mode has lower effective index than TE mode

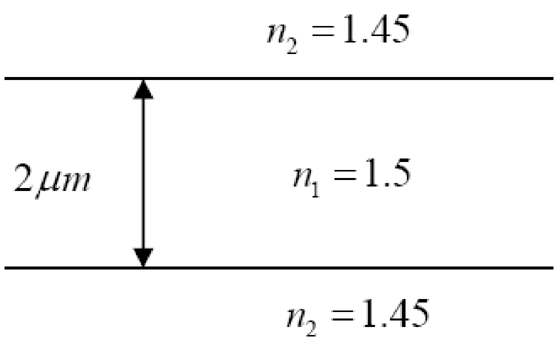
Homework 1

A symmetric three-layer waveguide is shown below. Consider only TE mode for this problem.

(a) Determine how many modes the waveguide can support for λ=1.0μm.

(b) Sketch the electric field intensity in the waveguide for each mode.

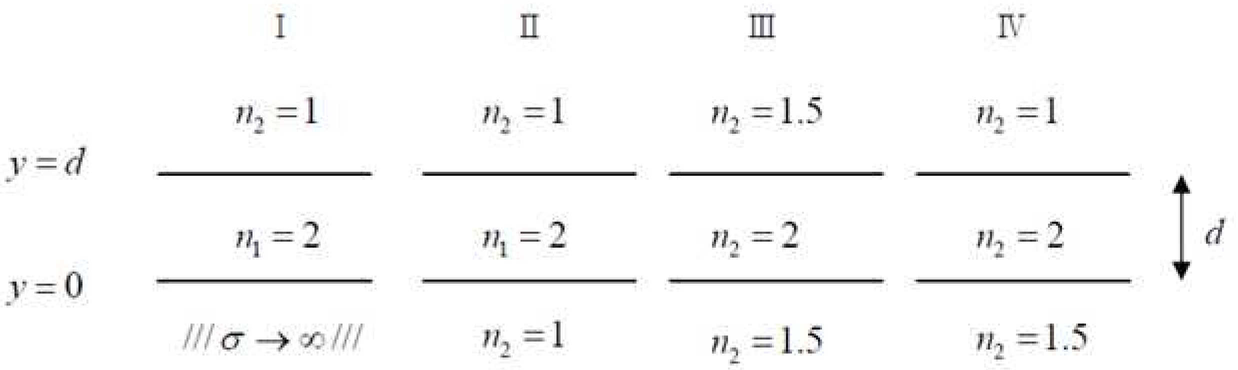
(c) When λ is increased, the number of modes the waveguide can support may change. What is the largest wavelength for which the mode number remains the same as what was obtained in (a)?



Lecture 7: Dielectric Waveguides

Homework 2

Several different types of waveguides having the same core material and thickness are shown below.



(a) If we sketch the fundamental mode power distribution for each waveguide, which waveguide has the largest y value for the peak power position? Explain.

(b) Which has higher confinement factor, Type II or Type III? Explain.