

# Measurement of the carrier lifetime and linewidth enhancement factor of semiconductor optical amplifiers using their optical modulation responses

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## 1 Introduction

Semiconductor optical amplifiers (SOAs) are widely used in many optical signal-processing applications such as wavelength conversion and optical switching.<sup>1-3</sup> In order to model SOA characteristics accurately for these applications, it is needed to consider longitudinal variations in the carrier and photon densities using multisection rate equations.<sup>1</sup> For many applications, however, the spatially integrated SOA rate-equation model is sufficient and has been widely used.<sup>2,3</sup> In addition, numerical values for parameters in this SOA model have been determined by several methods.<sup>4-6</sup> The carrier lifetime was determined from the SOA frequency response to modulated bias currents, two optical input signals closely spaced in wavelength, or a cross-gain-modulated probe beam.<sup>3,4</sup> However, electrical parasitics are involved in the first method, and two optical sources are needed for the second and third methods. The SOA linewidth enhancement factor was determined from relative sizes of sidebands in optical spectra of AM and FM signals observed by the self-heterodyne technique<sup>3,4</sup> or with a high-resolution optical spectrum analyzer.<sup>5</sup>

In this paper, we demonstrate a simple method of determining SOA differential and effective carrier lifetimes and linewidth enhancement factors that are needed in the spatially integrated SOA rate-equation model. In our method, the frequency response of SOA self-gain-modulated signals and the fiber response of SOA output signals are measured first. Then, fitting the measurement results with the analytically derived modulation responses, numerical values are determined.

**Abstract.** We demonstrate a simple technique for determining numerical values of carrier lifetimes and linewidth enhancement factors in a semiconductor optical amplifier (SOA) for various SOA input optical powers. For this purpose we use SOA self-gain modulations and the conversion of frequency modulation to intensity modulation. © 2004 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.1795816]

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This paper is organized as follows. In Sec. 2, the theoretical analysis of the SOA frequency response to optically modulated input signals and the fiber response of SOA output signals are described. In Sec. 3, the measurement setup is described and experimental results are given. Finally, in Sec. 4, we summarize the results and conclude.

## 2 Analysis of SOA Frequency Response

Our analysis is based on the following spatially integrated rate-equation SOA model in which the optical signal intensity inside the SOA,  $P(t,z)$ , the integrated carrier density  $\sigma(t,z)$ , and the phase of optical field  $\phi(t,z)$ , are related in the following manner<sup>6</sup>:

$$P(t,z) = P(t,0) \exp\{\Gamma a[\sigma(t,z) - N_0 z]\}, \quad (1)$$

$$\frac{d\sigma(t,z)}{dt} = \frac{\sigma_0 - \sigma(t,z)}{\tau_s} - \frac{1}{\Gamma \hbar \omega_0 A_{\text{eff}}} \times (\exp\{\Gamma a[\sigma(t,z) - N_0 z]\} - 1) P(t,0), \quad (2)$$

$$\phi(t,z) = -\frac{\alpha}{2} \Gamma a[\sigma(t,z) - N_0 z], \quad (3)$$

where  $\sigma_0 = \tau_s I / (e A_{\text{eff}})$  and  $\sigma(t,z) = \int_0^z N(t,z') dz'$ . In these equations,  $N(t,z)$  is the SOA carrier density,  $P(t,0)$  is the input optical signal,  $a$  is the differential gain,  $N_0$  is the transparency carrier density,  $I$  is the injected current,  $e$  is the electron charge,  $A_{\text{eff}}$  is the effective area,  $\tau_s$  is the differential carrier lifetime,  $\hbar$  is the reduced Planck constant,

$\omega_0$  is the optical angular frequency,  $\Gamma$  is the confinement factor, and  $\alpha$  is the linewidth enhancement factor.

When harmonically modulated optical signals whose intensity is given as  $P(t,0) = P_0 + \Delta P_{in} \exp(-i\omega t) + \Delta P_{in}^* \exp(i\omega t)$  are injected into an SOA having length  $L$ , the SOA output optical power, the integrated carrier density, and the phase of the output optical field at  $z=L$  are also harmonically modulated and can be expressed as  $P(t,L) = P_{out} + \Delta P_{out} \exp(-i\omega t) + \Delta P_{out}^* \exp(i\omega t)$ ,  $\sigma(t,L) = \sigma_s + \Delta \sigma \exp(-i\omega t) + \Delta \sigma^* \exp(i\omega t)$ , and  $\phi(t,L) = \phi_{out} + \Delta \phi_{out} \exp(-i\omega t) + \Delta \phi_{out}^* \exp(i\omega t)$ . Inserting these expressions into Eqs. (1), (2), and (3), and performing the usual small-signal analysis, the following expressions can be obtained for  $\Delta P_{out}$ ,  $\Delta \sigma$ , and  $\Delta \phi_{out}$ :

$$\Delta P_{out} = G_{cw}(\Delta P_{in} + \Gamma a P_0 \Delta \sigma), \quad (4)$$

$$\Delta \sigma = \frac{-(G_{cw} - 1)/(\Gamma \hbar \omega_0 A_{eff})}{1/\tau_s + G_{cw} P_0 a / A_{eff} \hbar \omega_0 - i\omega} \Delta P_{in}, \quad (5)$$

$$\Delta \phi_{out} = -\frac{\alpha}{2} \Gamma a \Delta \sigma, \quad (6)$$

where  $G_{cw}$  represents the static gain and is defined as

$$G_{cw} = P_{out}/P_0 = \exp[\Gamma a(\sigma_s - N_0 L)], \quad (7)$$

Assuming  $G_{cw}$  is much larger than one, which is the usual case, the small-signal SOA gain  $T(\omega)$  can be expressed as

$$T(\omega) = \frac{\Delta P_{out}(\omega)}{\Delta P_{in}(\omega)} = G_{cw} \left( \frac{1/\tau_s - i\omega}{1/\tau_e - i\omega} \right), \quad (8)$$

and we have

$$|T(\omega)|^2 = G_{cw}^2 \frac{(1/\tau_s)^2 + \omega^2}{(1/\tau_e)^2 + \omega^2}, \quad (9)$$

where  $\tau_e$ , the effective carrier lifetime indicating how fast the carrier density can be modulated,<sup>3</sup> is given by

$$\frac{1}{\tau_e} = \frac{1}{\tau_s} + \frac{G_{cw} P_0 a}{\hbar \omega_0 A_{eff}}. \quad (10)$$

From the preceding results, it is possible to fit the measured response with Eq. (9) and determine the effective values of  $\tau_s$  and  $\tau_e$ . In addition, if we know the structure parameter  $A_{eff}$ , we can also estimate the differential gain  $a$  from Eq. (10).

When SOA output signals are injected into a fiber for propagation, they experience phase changes due to the fiber's chromatic dispersion. Assuming that the fiber loss and nonlinearities can be ignored, the small-signal fiber output intensity  $\Delta P_{fiber}$  is given as<sup>7</sup>

$$\Delta P_{fiber}(\omega) = \cos \theta \cdot \Delta P_{out}(\omega) - 2P_{out} \sin \theta \cdot \Delta \phi_{out}(\omega), \quad (11)$$

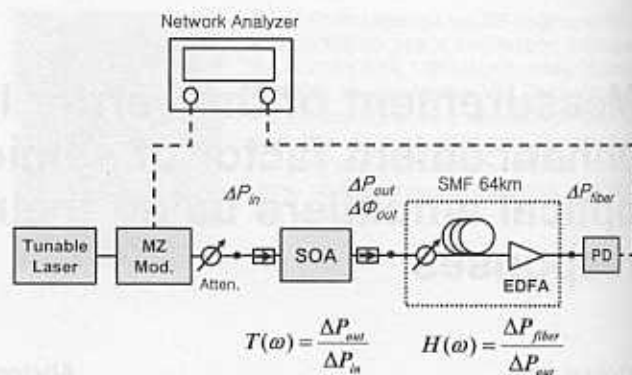


Fig. 1 Experimental setup.

where  $\theta = \omega^2 \pi \lambda^2 D L / 4 \pi c$ ;  $\Delta P_{out}$  and  $\Delta \phi_{out}$  are the small-signal SOA output intensity and the phase of the output optical field as given in Eqs. (4) and (6), respectively; and  $P_{out}$  is the static SOA optical output power injected into the fiber. In Eq. (11), the first term on the right side is responsible for IM-IM conversion, and the second term for FM-IM conversion. IM-IM conversion causes a dip at a frequency where  $\theta = \pi/2$  is satisfied, and FM-IM conversion compensates this dip by an amount depending on  $\Delta \phi_{out}$ .<sup>8</sup>

When SOA input signals are chirped, as in the case of Mach-Zehnder (MZ) modulator output, there are additional phase shifts due to this, and Eq. (6) should be modified as follows:<sup>9</sup>

$$\Delta \phi_{out} = \alpha_{mod} \frac{\Delta P_{in}}{2P_0} - \frac{\alpha}{2} \Gamma a \Delta \sigma, \quad (12)$$

where  $\alpha_{mod}$  is the chirp parameter for the MZ modulator.

Inserting Eqs. (4) and (12) into (11), we can obtain the following transfer function for SOA self-gain modulated signals transmitted through the fiber, assuming  $G_{cw}$  is much larger than one:

$$\begin{aligned} H(\omega) &= \frac{\Delta P_{fiber}(\omega)}{\Delta P_{out}(\omega)} \\ &= \cos \theta - \left( \frac{\alpha + \alpha_{mod}}{A_{eff} \hbar \omega_0 / a G_{cw} P_0 (1/\tau_s - i\omega)} + \alpha_{mod} \right) \\ &\quad \times \sin \theta. \end{aligned} \quad (13)$$

If the dispersion parameter  $D$  and  $\alpha_{mod}$  are known and the value of  $G_{cw} P_0 a / A_{eff} \hbar \omega_0$  is determined from Eq. (10) with the measured  $\tau_s$  and  $\tau_e$ , this transfer function can be used for the measurement of  $\alpha$ . Although similar techniques using FM-IM conversion have been used for measuring the linewidth enhancement factor in laser diodes,<sup>10,11</sup> no such treatment has been reported for the SOA.

### 3 Experiment and Results

Figure 1 shows the experimental setup used for our investigation. A commercial InP/InGaAsP bulk SOA is used, and the SOA bias current is set at 100 mA, which provides an unsaturated gain of about 25 dB. The SOA gain is depressed by 3 dB from the unsaturated gain when the input

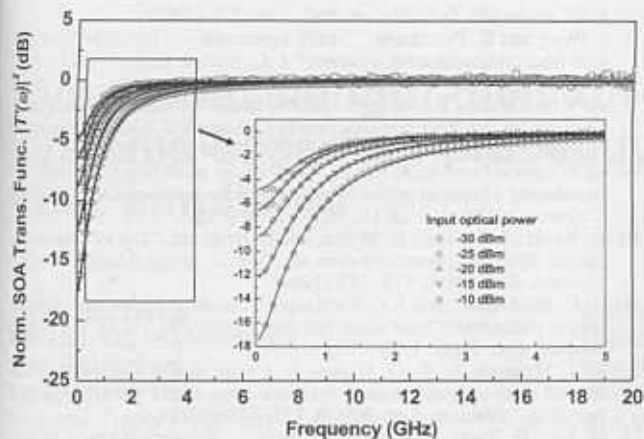


Fig. 2 Normalized optical modulation response  $|T'(\omega)|^2$  for various optical input powers to the SOA.

signal is  $-20$  dBm. Optical isolators are connected to input and output ports of SOA to suppress any optical feedback effects. The wavelength of the tunable laser is set to the SOA gain peak of  $1550$  nm. First, after injecting MZ-modulated signals into the SOA, we measure the optical modulation response of the SOA self-gain-modulated signals without fiber transmission in the frequency range of  $0.15$  to  $20$  GHz. By dividing this by the frequency response measured without the SOA, any parasitic effects can be eliminated and  $|T(\omega)|^2$  is obtained. The resulting frequency responses, normalized to the static gain  $G_{cw}$ , denoted by  $|T'(\omega)|^2$ , are shown in Fig. 2 for several input powers.

As expected from the analysis given in Sec. 2, the small-signal gain is reduced by the self-gain modulation at low modulation frequencies but reaches the static gain at high frequencies. When the input signal is modulated by the low frequency, the SOA carrier density changes in the opposite manner to the change in SOA input signal intensity. But if the input signal is modulated by high frequency, then the carrier density cannot follow the change of the input signal, and the input signal undergoes the average static gain. By fitting measured responses in Fig. 2 to Eq. (9) with the method of least squares, numerical values for  $\tau_s$  and  $\tau_e$  are determined. The results are shown in Table 1. The differential carrier lifetime increases as the input optical power increases, because it is inversely proportional to the differential carrier recombination rate and the carrier density is depleted with increasing input optical power. However, the

Table 1 Measured differential and effective carrier lifetime and linewidth enhancement factor.

Input power $P_0$ (dBm)	$G_{cw}$ (dB)	$\tau_s$ (ps)	$\tau_e$ (ps)	$\alpha$
-30	24.4	319	181	1.2
-25	23.1	374	175	2.4
-20	21.5	401	145	4.4
-15	18.5	479	116	5.9
-10	15.4	668	89	6.3

Table 2 Estimated differential gain and differential refractive index change.

Input power $P_0$ (dBm)	$a = dg/dN$ ( $\text{cm}^2$ )	$dn/dN$ ( $\text{cm}^2$ )
-30	$1.67 \times 10^{-15}$	$-2.50 \times 10^{-20}$
-25	$0.93 \times 10^{-16}$	$-2.77 \times 10^{-20}$
-20	$0.59 \times 10^{-15}$	$-3.27 \times 10^{-20}$
-15	$0.56 \times 10^{-15}$	$-4.12 \times 10^{-20}$
-10	$0.55 \times 10^{-15}$	$-4.21 \times 10^{-20}$

effective carrier lifetime decreases with increasing input optical power, as reported before.<sup>12</sup> Since  $A_{\text{eff}}$  is not known for our SOA, we assume its value to be  $1.5 \times 10^{-13} \text{ m}^2$ ,<sup>13</sup> and the resulting differential gain values are shown in Table 2. Because of the increased photon density, the differential gain is compressed with increasing SOA input power although the carrier density is decreased.

The frequency response  $H(\omega)$ , of dispersive fiber when intensity- and phase-modulated SOA output signals go through the dispersive fiber is given in Eq. (13) and is measured by dividing the optical modulation response after fiber transmission ( $64$  km) by that without the transmission, as shown in Fig. 1. The input optical power to the fiber is maintained at a level such that fiber nonlinearities are suppressed, and the fiber loss is compensated with an erbium-doped fiber amplifier. The residual chirp of the MZ modulator,  $\alpha_{\text{mod}}$ , is measured to be  $-0.15$  at the quadrature point, using the method given in Ref. 14. Figure 3 shows the measured results and the fit with Eq. (13). The previously determined values for  $G_{cw}P_0a/A_{\text{eff}}h\omega_0$  are used for fitting.

The resulting values of linewidth enhancement factors are listed in Table 1. The value of the linewidth enhancement factor is dependent on the level of a dip and increases as the input optical power to the SOA increases. This is due to the compression of the differential gain.

From the value of the linewidth enhancement factor and the differential gain  $a$ , we also calculate the differential refractive index change,  $dn/dN$ , using the relation<sup>15</sup>

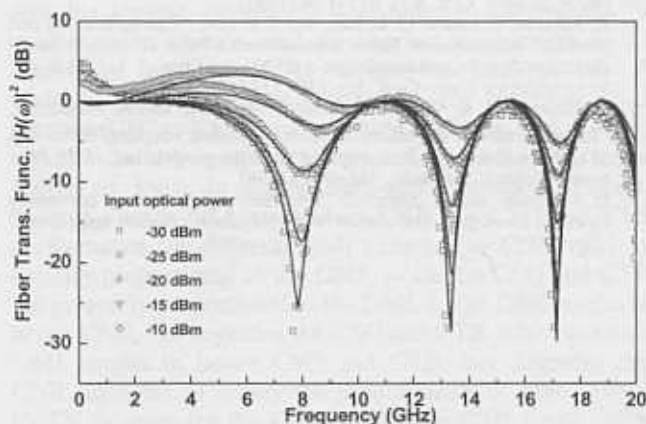


Fig. 3 Fiber response  $|H(\omega)|^2$  of SOA output signal for various optical input powers to SOA.

$$\alpha = -\frac{4\pi}{\lambda} \frac{dn/dN}{dg/dN} = -\frac{4\pi}{\lambda} \frac{dn/dN}{a}, \quad (14)$$

and list it in Table 2.

In the low frequency range, discrepancies between the measured data and the analytic results are observed. These are believed to be due to the limitation of the rate-equation model using the averaged carrier density and/or large non-linearity in the low frequency range, but further investigations are needed. However, these discrepancies do not affect the accuracy of estimated SOA linewidth enhancement factors, since the values of the linewidth enhancement factor are mostly dependent on the dip level in the frequency response. Although our present investigation is limited to estimating SOA parameters for different input power levels at a single input signal wavelength and bias current, it can be easily extended to determine influence of input signal wavelengths and bias currents, which are known to cause changes in linewidth enhancement factors.<sup>3,16</sup>

#### 4 Conclusion

We have demonstrated a simple measurement method for the SOA carrier lifetime and linewidth enhancement factor. We also estimated the effective values of the differential gain and the differential refractive index change as a function on the input optical power. Our measurement method uses a single optical source and simple fiber optic elements, and is expected to be useful for modeling SOAs in optical signal-processing applications.

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